Prince Edward Island
CANADA
Education and Early Years

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

MAT421A

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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for Grades 10-12 Mathematics* (2008) has been adopted as the basis for a revised high school mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential Graduation Learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

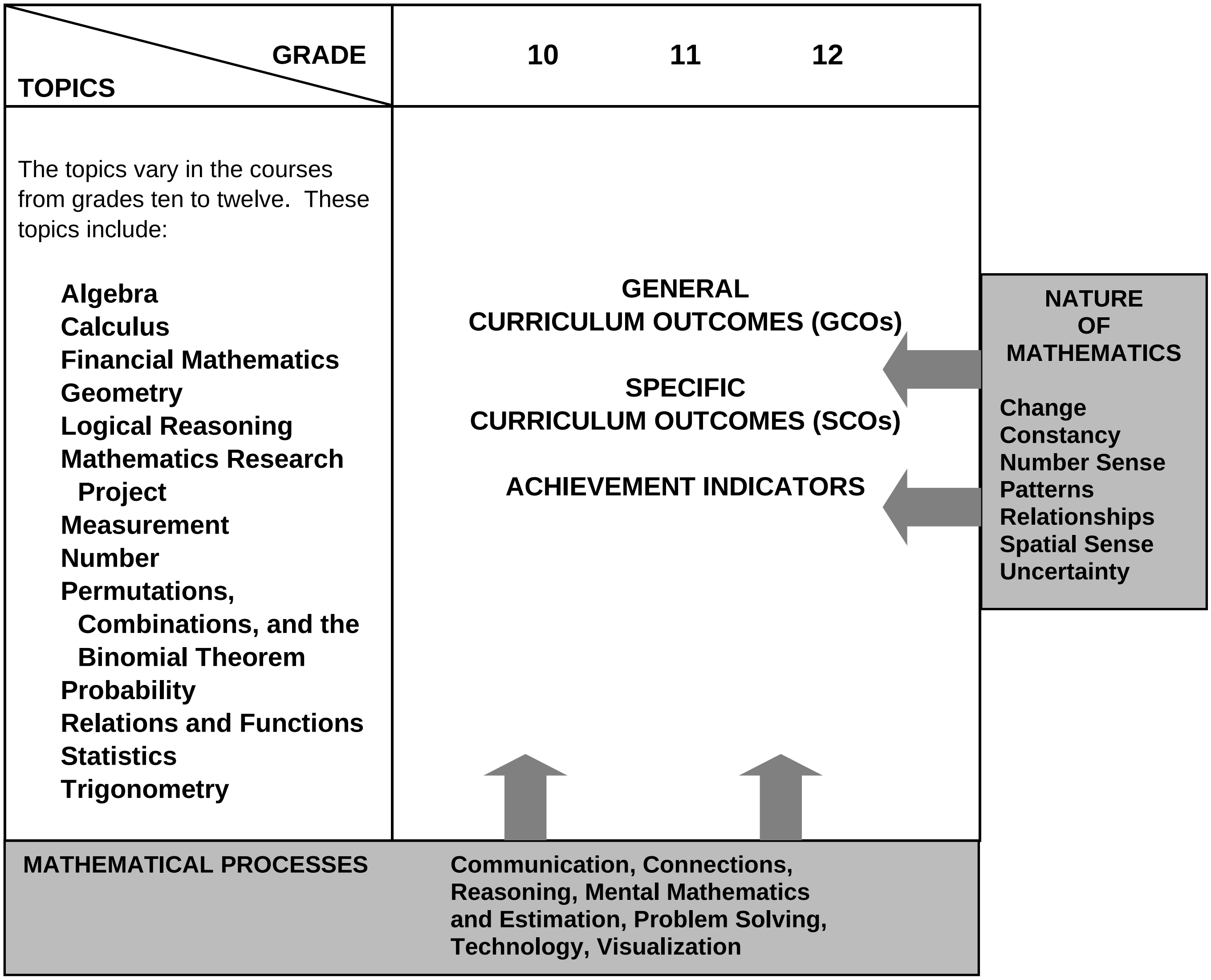
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for 10-12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



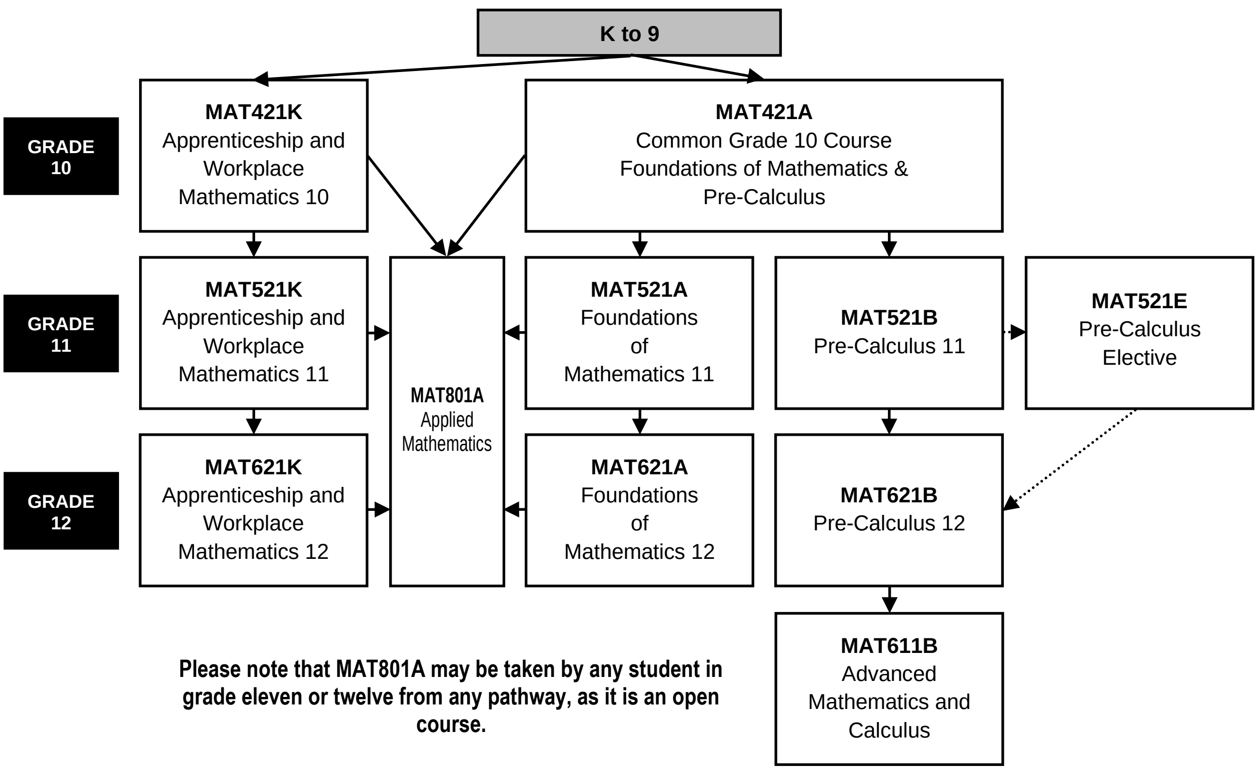
The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into a number of topics, as described above. These topics are not intended to be discrete units of instruction. The integration of outcomes across topics makes mathematical experiences meaningful. Students should make the connections among concepts both within and across topics. Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.

- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Pathways and Topics

The Prince Edward Island 10-12 mathematics curriculum includes pathways with corresponding topics rather than strands, which are found in the Prince Edward Island K-9 mathematics curriculum. Three pathways are available: **Apprenticeship and Workplace Mathematics**, **Foundations of Mathematics**, and **Pre-Calculus**. A common grade ten course (Foundations of Mathematics and Pre-Calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-Calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings. These pathways are illustrated in the diagram below:



Please note that MAT801A may be taken by any student in grade eleven or twelve from any pathway, as it is an open course.

The goals of all three pathways are to provide the prerequisite knowledge, skills, understandings, and attitudes for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study or for direct entry into the work force. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Apprenticeship and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics, and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, algebra and number, logical reasoning, relations and functions, statistics, probability, and a mathematics research project.

Pre-Calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, combinatorics, and introductory calculus.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

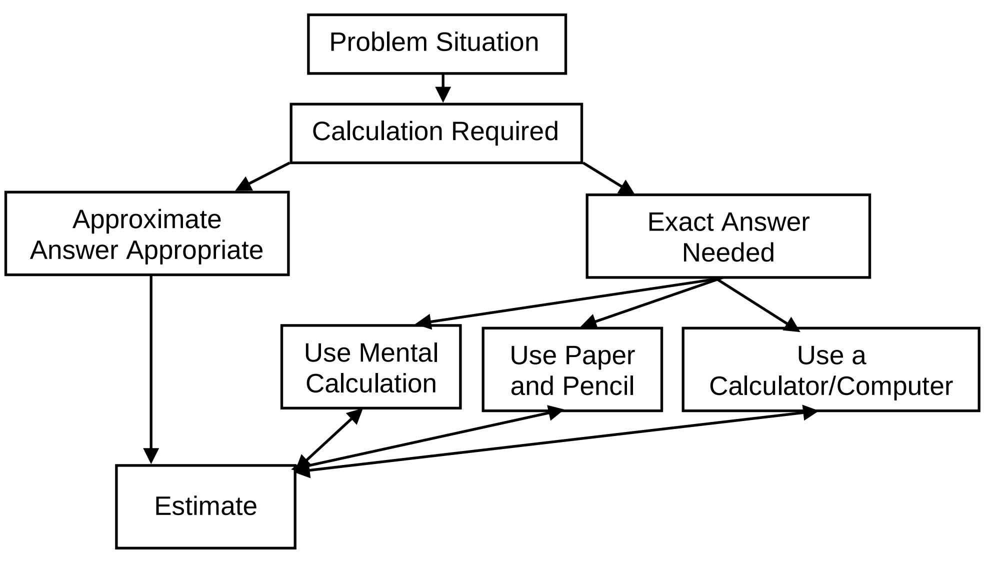
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you. . . ?" or "How could you. . . ?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.

- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all topics in mathematics and it is important that connections are made among topics. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical topics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate "communicating to learn mathematics and learning to communicate mathematically" (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

➢ Inquiry-Based Learning and Project Based Learning

Inquiry-based learning (IBL) allows students to explore, investigate, and construct new meaning from prior knowledge and from new information that is retrieved from other sources. It is not linear in nature, but promotes a continual looping back and forth throughout the process as students gather and process new information, redirect their inquiries, and continue through the process. Mathematical inquiry will require students to practise and refine their critical and creative-thinking skills. The terms *inquiry* and *research* are often used interchangeably within an educational context. While research often becomes the end-result of an inquiry process, it is the process itself that should be emphasized within an educational context. More information regarding the development of a mathematics research project is included in the appendix at the end of this document.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, teaching has been effective, or how best to address student learning needs. The quality of assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

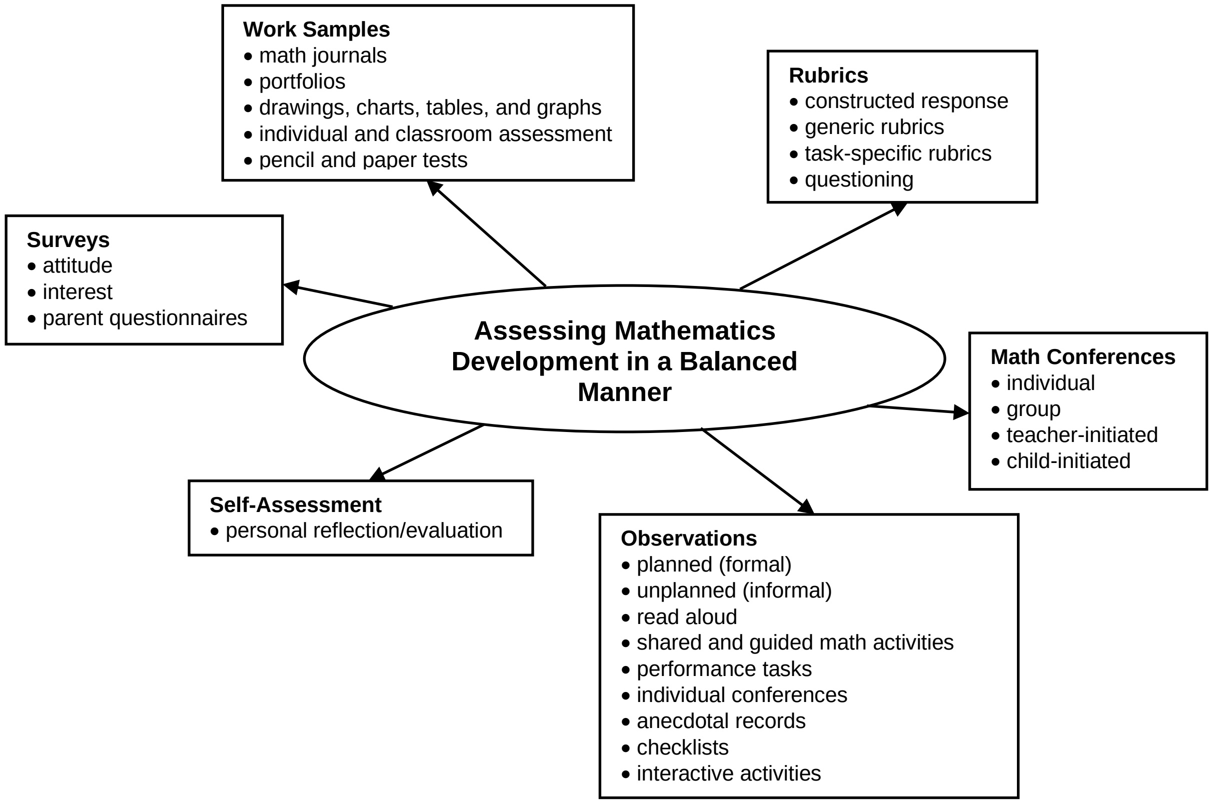
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children’s learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;

- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, phone calls, and electronic methods.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;

- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he or she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island high school mathematics curriculum are organized into a number of topics across the grades from ten to twelve. Each topic has associated with it a general curriculum outcome (GCO). They are overarching statements about what students are expected to learn in each topic from grades ten to twelve.

Topic	General Curriculum Outcome (GCO)
Algebra (A)	Develop algebraic reasoning.
Algebra and Number (AN)	Develop algebraic reasoning and number sense.
Calculus (C)	Develop introductory calculus reasoning.
Financial Mathematics (FM)	Develop number sense in financial applications.
Geometry (G)	Develop spatial sense.
Logical Reasoning (LR)	Develop logical reasoning.
Mathematics Research Project (MRP)	Develop an appreciation of the role of mathematics in society.
Measurement (M)	Develop spatial sense and proportional reasoning. **(*Foundations of Mathematics and Pre-Calculus*)**
	Develop spatial sense through direct and indirect measurement. **(*Apprenticeship and Workplace Mathematics*)**
Number (N)	Develop number sense and critical thinking skills.
Permutations, Combinations and Binomial Theorem (PC)	Develop algebraic and numeric reasoning that involves combinatorics.
Probability (P)	Develop critical thinking skills related to uncertainty.
Relations and Functions (RF)	Develop algebraic and graphical reasoning through the study of relations.
Statistics (S)	Develop statistical reasoning.
Trigonometry (T)	Develop trigonometric reasoning.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific curriculum outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding topic and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades nine to eleven which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;

- a list of the sections in *Foundations and Pre-Calculus Mathematics 10* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

In the second half of this document, a curriculum guide supplement is presented which follows the primary resource, *Foundations and Pre-Calculus Mathematics 10*. As well, an appendix is included which outlines the steps to follow in the development of an effective mathematics research project.

MEASUREMENT

SPECIFIC CURRICULUM OUTCOMES

M1 – Compare quantities within SI and Imperial units, using:
- **estimation strategies;**
- **measurement strategies.**

M2 – Apply proportional reasoning to solve problems involving conversions between and within SI and imperial units of measure.

M3 – Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:
- **right cones;**
- **right cylinders;**
- **right prisms;**
- **right pyramids;**
- **spheres.**

M4 – Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

MAT421A – Topic: Measurement (M)

GCO: Develop spatial sense and proportional reasoning.

<table>
<tr><th>GRADE 9</th><th>GRADE 10 – MAT421A</th><th>GRADE 11</th></tr>
<tr><td rowspan="2"></td><td rowspan="2">M1 Compare quantities within SI and Imperial units, using:
• estimation strategies;
• measurement strategies.</td><td>MAT521A</td></tr>
<tr><td>MAT521B</td></tr>
</table>

SCO: **M1 – Compare quantities within SI and Imperial units, using:**

- **estimation strategies;**
- **measurement strategies.**

[ME, PS, V]

Students who have achieved this outcome should be able to:

A. Provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard and mile, and explain the choices.

B. Compare SI and imperial units, using referents.

C. Estimate a linear measure, using a referent, and explain the process used.

D. Identify the most appropriate units used to determine a measurement in a problem-solving context.

E. Perform a linear measure using instruments such as rulers, calipers, or tape measures.

F. Describe and explain a personal strategy used to determine a linear measurement; *e.g.*, circumference of a bottle, length of a curve, perimeter of the base of an irregular 3-D object.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A C D)

1.2 (A C D E F)

1.3 (B)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: M1 – Compare quantities within SI and Imperial units, using:
- **estimation strategies;**
- **measurement strategies.**

[ME, PS, V]

Elaboration

Canada's official measurement system is SI (Système International d'Unités). Some SI units for linear measurement are listed in the following table.

UNIT	ABBREVIATION	MULTIPLYING FACTOR
kilometre	km	1000
hectometre	hm	100
decametre	dam	10
metre	m	1
decimetre	dm	0.1
centimetre	cm	0.01
millimetre	mm	0.001

Note that each unit in the SI measuring system is based on a power of 10. All linear measurements are derived from the metre. The most common units are the kilometre (km), metre (m), centimetre (cm), and the millimetre (mm). The kilometre is a large unit $(1\text{ km} = 1000\text{ m})$ and is suitable for measuring large distances. The millimetre is a small unit $(1\text{ mm} = 0.001\text{ m})$ and is suitable for measuring small distances.

The imperial system of measurement is widely used in the United States for measuring distances. Even though SI is Canada's official measurement system, some Canadian industries still use imperial units. The following units are the basic imperial units used for measuring distances.

inch (in)	
foot (ft)	$1\text{ ft} = 12\text{ in}$
yard (yd)	$1\text{ yd} = 3\text{ ft} = 36\text{ in}$
mile (mi)	$1\text{ mi} = 1760\text{ yd} = 5280\text{ ft}$

Various measuring instruments allow accurate measurement of distances in standard units. Personal referents can also be developed when estimating measurements. SI rulers, metre sticks, and measuring tapes give measurements to the nearest millimetre, or 0.1 cm. An SI caliper can accurately measure to the nearest tenth of a millimetre, or 0.01 cm, depending on its scales. An imperial ruler or measuring tape can measure distances to the nearest $\frac{1}{16}$ in. An imperial caliper can measure to the nearest $\frac{1}{1000}$ in.

A non-standard measuring unit can be used as a personal referent. Referents can help individuals estimate in standard units, such as SI or imperial units. For example, suppose that the width of a fingernail is used to approximate 1 cm. Then, something appears to be as wide as four fingernails can be estimated as being 4 cm wide.

MAT421A – Topic: Measurement (M)

GCO: Develop spatial sense and proportional reasoning.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
	M2 – Apply proportional reasoning to solve problems involving conversions between and within SI and imperial units of measure. .	**MAT521A** **M1** Solve problems that involve the application of rates. **M2** Solve problems that involve scale diagrams, using proportional reasoning. **MAT521B**

SCO: M2 – Apply proportional reasoning to solve problems involving conversions between and within SI and imperial units of measure.

[C, ME, PS]

Students who have achieved this outcome should be able to:

A. Explain how proportional reasoning can be used to convert a measurement within, or between, SI and imperial systems.

B. Solve a problem that involves the conversion of units within SI and imperial systems.

C. Solve a problem that involves the conversion of units between SI and imperial systems.

D. Verify, using unit analysis, a conversion within, or between, SI and imperial systems, and explain the conversion.

E. Explain, using mental mathematics, the reasonableness of a solution to a conversion problem.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A B C)

1.3 (A B C D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: M2 – Apply proportional reasoning to solve problems involving conversions between and within SI and imperial units of measure.
[C, ME, PS]

Elaboration

When solving problems involving measurement, it is crucial to work with the same units. It may be necessary to convert units within one measurement system (for example, inches to feet), or between imperial and SI units (for example, inches to centimetres). Students have not seen the SI scale in M1 prior to this course so it will need to be provided to students. Also, time needs to be spent converting between units in the SI measuring systems as conversions of this type have not been covered as a specific curriculum outcome elsewhere.

To convert from one measurement system to another, it is necessary to understand the relationships among the units of length in each system. All conversions involve proportional reasoning and unit analysis. Conversions between measurement systems may be approximate or exact. For example, the inch in the imperial system has been defined as exactly 2.54 cm.

The following are some common conversions:

Exact Conversions

$1 \text{ in} = 2.54 \text{ cm}$ $1 \text{ ft} = 30.48 \text{ cm}$ $1 \text{ yd} = 0.9144 \text{ m}$

Approximate Conversions

Rounded to four significant digits

$1 \text{ mm} \doteq 0.03937 \text{ in}$ $1 \text{ cm} \doteq 0.3937 \text{ in}$ $1 \text{ m} \doteq 1.094 \text{ yd}$

$1 \text{ m} \doteq 3.281 \text{ ft}$ $1 \text{ km} \doteq 0.6214 \text{ mi}$ $1 \text{ mi} \doteq 1.609 \text{ km}$

Please note that these conversions should be used when solving problems, as the answers in the textbook do not reflect the approximate conversions that are presented on page 18 in the textbook.

MAT421A – Topic: Measurement (M)

GCO: Develop spatial sense and proportional reasoning.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
SS1 Develop and apply the Pythagorean Theorem to solve problems. **SS2** Determine the surface area of composite 3-D objects to solve problems.	**M3** Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including: • right cones; • right cylinders; • right prisms; • right pyramids; • spheres.	**MAT521A** **M3** Demonstrate an understanding of the relationships between scale factors and areas of similar 2-D shapes, and the application of scale factor on 3-D objects. **MAT521B**

SCO: M3 – Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:
- **right cones;**
- **right cylinders;**
- **right prisms;**
- **right pyramids;**
- **spheres.**

[CN, PS, R, V]

Students who have achieved this outcome should be able to:

A. Sketch a diagram to represent a problem that involves surface area or volume.

B. Determine the surface area of a right cone, right cylinder, right prism, right pyramid or sphere, using an object or its labelled diagram.

C. Determine the volume of a right cone, right cylinder, right prism, right pyramid or sphere, using an object or its labelled diagram.

D. Determine an unknown dimension of a right cone, right cylinder, right prism, right pyramid or sphere, given the object's surface area or volume and the remaining dimensions.

E. Solve a problem that involves surface area or volume, given a diagram of a composite 3-D object.

F. Describe the relationship between the volumes of:
- right cones and right cylinders with the same base and height;
- right pyramids and right prisms with the same base and height.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.4 (A B D)

1.5 (A C D F)

1.6 (B C D)

1.7 (A D E)

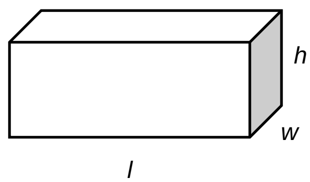
[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: **M3 – Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:**
- **right cones;**
- **right cylinders;**
- **right prisms;**
- **right pyramids;**
- **spheres.**

[CN, PS, R, V]

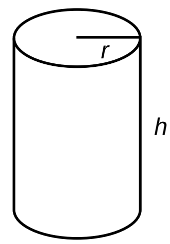
Elaboration

The surface area of a right rectangular prism and of a right cylinder can be calculated using the area of the bases (top and bottom) plus the lateral area. The volume of a right rectangular prism and a right cylinder can be calculated by multiplying the area of the base by the height.



$$SA = 2lw + 2lh + 2wh$$

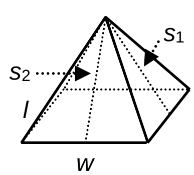
$$V = lwh$$



$$SA = 2\pi r^2 + 2\pi rh$$

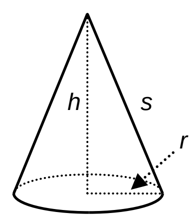
$$V = \pi r^2 h$$

The surface area of a right rectangular pyramid and of a right cone can be calculated using the area of the base plus the lateral area. The volume of a right rectangular pyramid is found by calculating one-third of the volume of its related right prism. The volume of a right cone is found by calculating one-third of the volume of its related right cylinder.



$$SA = lw + ls_1 + ws_2$$

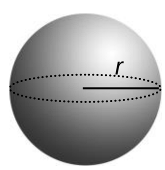
$$V = \frac{1}{3}lwh$$



$$SA = \pi r^2 + \pi rs$$

$$V = \frac{1}{3}\pi r^2 h$$

The surface area and the volume of a sphere both depend on the only the radius.



$$SA = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

MAT421A – Topic: Measurement (M)

GCO: Develop spatial sense and proportional reasoning.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
SS1 Develop and apply the Pythagorean Theorem to solve problems. **SS3** Demonstrate an understanding of similarity of triangles.	**M4** Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.	**MAT521A** **G1** Derive proofs that involve the properties of angles and triangles. **G2** Solve problems that involve the properties of angles and triangles. **G3** Solve problems that involve the cosine law and the sine law. **MAT521B** **T1** Demonstrate an understanding of angles in standard position (0^0 to 360^0). **T2** Solve problems, using the three primary trigonometric ratios for angles from 0^0 to 360^0 in standard position. **T3** Solve problems, using the cosine law and sine law, including the ambiguous case.

SCO: **M4 – Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.** **[C, CN, PS, R, T, V]**

Students who have achieved this outcome should be able to:

A. Explain the relationships between similar right triangles and the definitions of the primary trigonometric ratios.

B. Identify the hypotenuse of a right triangle and the opposite and adjacent sides for a given acute triangle in the triangle.

C. Solve right triangles, with or without technology.

D. Solve a problem that involves one or more right triangles by applying the primary trigonometric ratios or the Pythagorean theorem.

E. Solve a problem that involves indirect and direct measurement, using the trigonometric ratios, the Pythagorean theorem and measurement instruments such as a clinometer or a metre stick.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B D)

2.2 (B D)

2.3 (E)

2.4 (A B D)

2.5 (B D)

2.6 (B C D)

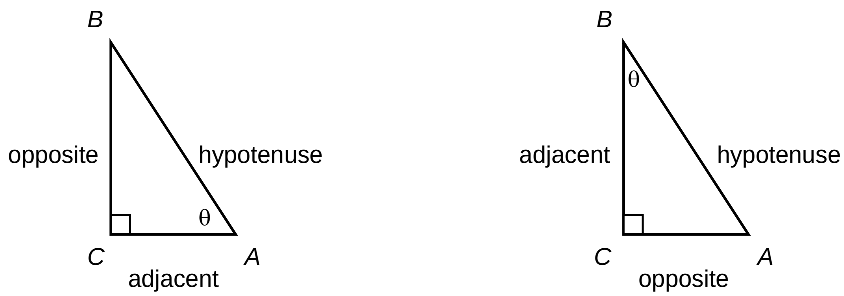
2.7 (B D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: M4 – Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]

Elaboration

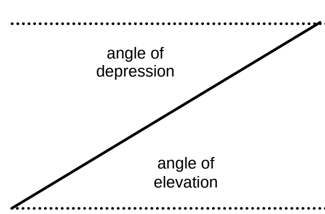
In similar triangles, corresponding angles are equal, and corresponding sides are in proportion. Therefore, the ratios of the lengths of corresponding sides are equal. The sides of a right triangle are labelled according to a reference angle, θ.



A trigonometric ratio is a ratio of the measures of two sides of a right triangle. The three primary trigonometric ratios are tangent, sine, and cosine. The short form for the tangent ratio of angle *A* is tan *A*. It is defined as $\tan A = \dfrac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$. The short form for the sine ratio of angle *A* is sin *A*. It is defined as $\sin A = \dfrac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$. The short form for the cosine ratio of angle *A* is cos *A*. It is defined as $\cos A = \dfrac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$. A mnemonic device such as **SOH – CAH – TOA** can be used to remember the three primary trigonometric ratios.

Students will also use the Pythagorean Theorem extensively when solving trigonometric problems. The Pythagorean Theorem may need to be reviewed, as it was originally taught in grade nine. Also, the concept of similar triangles, which was taught in grade nine, will be used as well.

A line of sight is an invisible line from one person or object to another person or object. Some applications of trigonometry involve an angle of elevation or an angle of depression. An *angle of elevation* is the angle formed by the horizontal and a line of sight above the horizontal. An *angle of depression* is the angle formed by the horizontal and a line of sight below the horizontal. As the diagram below shows, the angle of elevation and the angle of depression along the same line of sight are equal.



ALGEBRA AND NUMBER

SPECIFIC CURRICULUM OUTCOMES

AN1 – Demonstrate an understanding of factors of whole numbers by determining the:

- **prime factors;**
- **greatest common factor;**
- **least common multiple;**
- **square root;**
- **cube root.**

AN2 – Demonstrate an understanding of irrational numbers by:

- **representing, identifying and simplifying irrational numbers;**
- **ordering irrational numbers.**

AN3 – Demonstrate an understanding of powers with integral and rational exponents.

AN4 – Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.

AN5 – Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

MAT421A – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
N5 Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers). **N6** Determine the square root of positive rational numbers that are perfect squares. **N7** Determine an approximate square root of positive rational numbers that are non-perfect squares.	**AN1** Demonstrate an understanding of factors of whole numbers by determining the: • prime factors; • greatest common factor; • least common multiple; • square root; • cube root.	**MAT521A** **MAT521B** **AN2** Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands.

SCO: **AN1 – Demonstrate an understanding of factors of whole numbers by determining the:**

- **prime factors;**
- **greatest common factor;**
- **least common multiple;**
- **square root;**
- **cube root.**

[CN, ME, R]

Students who have achieved this outcome should be able to:

A. Determine the prime factors of a whole number.

B. Explain why the numbers 0 and 1 have no prime factors.

C. Determine, using a variety of strategies, the greatest common factor or the least common multiple of a set of whole numbers, and explain the process.

D. Determine, concretely, whether a given whole number is a perfect square, a perfect cube or neither.

E. Determine, using a variety of strategies, the square root of a perfect square, and explain the process.

F. Determine, using a variety of strategies, the cube root of a perfect cube, and explain the process.

G. Solve problems that involve prime factors, greatest common factors, least common multiples, square roots or cube roots.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (A B C G)

3.2 (D E F G)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: AN1 – Demonstrate an understanding of factors of whole numbers by determining the:
- **prime factors;**
- **greatest common factor;**
- **least common multiple;**
- **square root;**
- **cube root.**

[CN, ME, R]

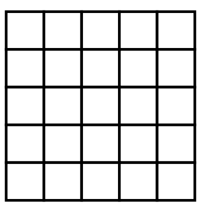
Elaboration

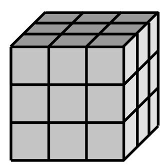
A number is prime is it has exactly two distinct factors. All natural numbers greater than one can be expressed as a unique product of prime factors, called the *prime factorization* of that number. The prime factorization can be used to find the greatest common factor and the least common multiple of a set of natural numbers. Please note that the concepts of greatest common factor and least common multiple are not fully developed previous to grade ten.

Perfect squares and square roots are linked to each other. The number 25 is a perfect square. It is formed by multiplying two factors of 5 together, that is, $(5)(5)=25$ or $5^2=25$. The square root of 25 is 5, or $\sqrt{25}=5$. Perfect cubes and cube roots are also linked to each other. The number 27 is a perfect cube. It is formed by multiplying three factors of 3 together, that is $(3)(3)(3)=27$ or $3^3=27$. The cube root of 27 is 3, or $\sqrt[3]{27}=3$.

To help students familiarize themselves with perfect squares and cubes, they should develop a table which includes the first twenty perfect squares and the first ten perfect cubes.

Perfect squares and perfect cubes can be modelled using manipulatives. For example, the perfect square 25 can be modelled as a square with side length 5, and the perfect cube 27 can be modelled as a cube with side length 3, as shown below.





MAT421A – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
N3 Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers; • solving problems that involve arithmetic operations on rational numbers.	**AN2** Demonstrate an understanding of irrational numbers by: • representing, identifying and simplifying irrational numbers; • ordering irrational numbers.	**MAT521A** **MAT521B**

SCO: **AN2 – Demonstrate an understanding of irrational numbers by:**

- **representing, identifying and simplifying irrational numbers;**
- **ordering irrational numbers.**

[CN, ME, R, V]

Students who have achieved this outcome should be able to:

A. Sort a set of numbers into rational and irrational numbers.

B. Determine an approximate value of a given irrational number.

C. Approximate the locations of irrational numbers on a number line, using a variety of strategies, and explain the reasoning.

D. Order a set of irrational numbers on a number line.

E. Express a radical as a mixed radical in simplest form (limited to numerical radicands).

F. Express a mixed radical as an entire radical (limited to numerical radicands).

G. Explain, using examples, the meaning of the index of a radical.

H. Represent, using a graphic organizer, the relationship among the subsets of the real numbers (natural, whole, integer, rational, irrational).

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (B G)

4.2 (A C D H)

4.3 (E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: AN2 – Demonstrate an understanding of irrational numbers by:
- **representing, identifying and simplifying irrational numbers;**
- **ordering irrational numbers.**

[CN, ME, R, V]

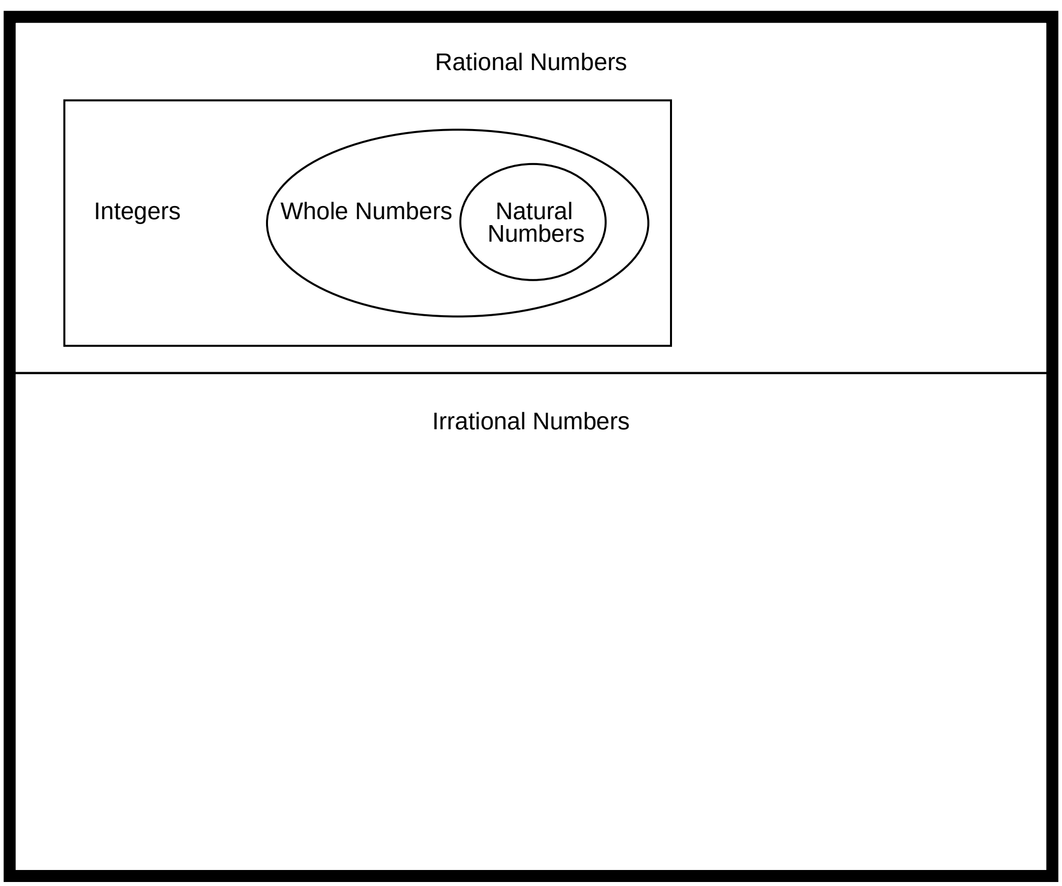
Elaboration

It is important that students understand each of the following sets of numbers:

- **Natural numbers** – the set of counting numbers {1, 2, 3, 4, 5, ...}
- **Whole numbers** – the set of counting numbers, together with zero {0, 1, 2, 3, 4, ...}
- **Integers** – the set of whole numbers and their opposites {..., –3, –2, –1, 0, 1, 2, 3, ...}
- **Rational numbers** – the set of all numbers that can be expressed as a fraction, *e.g.*, $\frac{3}{4}$, –5, 17.2
- **Irrational numbers** – the set of all numbers that cannot be expressed as a fraction, *e.g.*, $\sqrt{2}$, π, $\frac{\sqrt[3]{17}}{3}$
- **Real numbers –** the combined set of rational and irrational numbers

The following diagram shows the relationships among all of these sets of numbers:

Real Numbers



MAT421A – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
N1 Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by: • representing repeated multiplication using powers; • using patterns to show that a power with an exponent of zero is equal to one; • solving problems involving powers. **N2** Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.	**AN3** Demonstrate an understanding of powers with integral and rational exponents.	**MAT521A** **MAT521B**

SCO: AN3 – Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Explain, using patterns, why $a^{-n} = \frac{1}{a^n}$, $a \neq 0$.

B. Explain, using patterns, why $a^{1/n} = \sqrt[n]{a}$, $n > 0$.

C. Apply the exponent laws:

- $(a^m)(a^n) = a^{m+n}$;
- $a^m \div a^n = a^{m-n}$, $a \neq 0$;
- $(a^m)^n = a^{mn}$;
- $(ab)^m = a^m b^m$;
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$

to expressions with rational and variable bases, and integral and rational exponents, and explain the reasoning.

D. Express powers with rational exponents as radicals and vice versa.

E. Solve a problem that involves exponent laws or radicals.

F. Identify and correct errors in a simplification of an expression that involves powers.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.4 (B D E F)

4.5 (A E F)

4.6 (C E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: AN3 – Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]

Elaboration

This outcome continues that work that was done with whole number exponents in grade nine.

A power with a negative exponent can be written as a power with a positive exponent using the following principles:

$$a^{-n} = \frac{1}{a^n} \text{ and } \frac{1}{a^{-n}} = a^n, \text{ as long as } a \neq 0$$

A power with a fractional exponent can be written as a radical using the following principles:

$$a^{m/n} = \sqrt[n]{a^m} \text{ or } a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

These principles can be applied to the exponent laws previously learned in grade nine, as stated below.

EXPONENT NAME	EXPONENT LAW
Product of Powers	$(a^m)(a^n) = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = a^m b^m$
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
Zero Exponent	$a^0 = 1, a \neq 0$

MAT421A – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
PR6 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.	**AN4** Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.	**MAT521A** **MAT521B** **AN4** Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). **AN5** Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).

SCO: AN4 – Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. [CN, R, V]

Students who have achieved this outcome should be able to:

A. Model the multiplication of two given binomials, concretely and pictorially, and record the process symbolically.

B. Relate the multiplication of two binomial expressions to an area model.

C. Explain, using examples, the relationship between the multiplication of binomials and the multiplication of two-digit numbers.

D. Verify a polynomial product by substituting numbers for the variables.

E. Multiply two polynomials symbolically, and combine like terms in the product.

F. Generalize and explain a strategy for multiplication of polynomials.

G. Identify and explain errors in a solution for polynomial multiplication.

Note: It is intended that the emphasis of this outcome be on binomial by binomial multiplication, with extension to polynomial by polynomial to establish a general pattern for multiplication.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.5 (A B C)

3.6 (A B E)

3.7 (D E F G)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: AN4 – Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. [CN, R, V]

Elaboration

The distributive property is used to multiply polynomials together by multiplying each term in the first polynomial by each term in the second polynomial, and then collecting like terms. For example:

$$\begin{aligned}(3x-2)(4x+5) &= (3x)(4x+5)-2(4x+5)\\ &= 12x^2+15x-8x-10\\ &= 12x^2+7x-10\end{aligned}$$

$$\begin{aligned}(c-3)(4c^2-c+6) &= c(4c^2-c+6)-3(4c^2-c+6)\\ &= 4c^3-c^2+6c-12c^2+3c-18\\ &= 4c^3-13c^2+9c-18\end{aligned}$$

When both polynomials are binomials, the acronym **FOIL** can be used to remember the order in which the terms are multiplied. The acronym reminds us to multiply the **F**irst terms in each binomial, next the **O**utside terms, then the **I**nside terms, and then finally the **L**ast terms in each binomial.

Please note that it may be necessary to review the operations of addition and multiplication of polynomials, which were taught in grade nine, before teaching this outcome.

MAT421A – Topic: Algebra and Number (AN)

GCO: Develop algebraic reasoning and number sense.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
PR6 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.	**AN5** Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.	**MAT521A** **MAT521B** **RF1** Factor polynomials of the form: • $ax^2 + bx + c$, $a \neq 0$; • $a^2x^2 - b^2y^2$, $a \neq 0$, $b \neq 0$; where *a*, *b* and *c* are rational numbers.

SCO: AN5 – Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. [C, CN, R, V]

Students who have achieved this outcome should be able to:

A. Determine the common factors in the terms of a polynomial, and express the polynomial in factored form.

B. Model the factoring of a trinomial, concretely or pictorially, and record the process symbolically.

C. Factor a polynomial that is a difference of squares, and explain why it is a special case of trinomial factoring where $b = 0$.

D. Identify and explain errors in a polynomial factorization.

E. Factor a polynomial, and verify by multiplying the factors.

F. Explain, using examples, the relationship between multiplication and factoring of polynomials.

G. Generalize and explain strategies used to factor a trinomial.

H. Express a polynomial as a product of its factors.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.3 (A B D E)

3.4 (B F)

3.5 (B D E F H)

3.6 (B D E F G H)

3.8 (C E H)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: AN5 – Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. [C, CN, R, V]

Elaboration

Students should understand that factoring an algebraic expression is the reverse of multiplying polynomials, which is a process similar to division. This will be the first time that students will have factored polynomials.

To find the GCF of a polynomial, find the GCF of the coefficients and the variables separately. To find the GCF of the variables, select the lowest exponent that appears in the polynomial for each variable. Then, to factor the GCF from the polynomial, simply divide each term by the GCF. The factored polynomial will then be a product of the GCF and the sum or difference of the remaining factors. For example, $2m^3n^2 - 8m^2n + 12mn^4 = 2mn\left(m^2n - 4m + 6n^3\right)$. A common factor can be any polynomial. For example, $a(x+2) - b(x+2) = (x+2)(a-b)$.

To factor a trinomial of the form $x^2 + bx + c$, find two integers with a product of *c* and a sum of *b*. Use these two integers to write the factored form, which will be the product of *x* plus one integer, by *x* plus the other integer. For example, to factor $x^2 + 12x + 27$, find two integers with a product of 27 and a sum of 12. Since these integers are 3 and 9, the factored form of the trinomial is $(x+3)(x+9)$.

To factor a trinomial of the form $ax^2 + bx + c$, find two integers with a product of *ac* and a sum of *b*. Then, expand the middle term as a sum of these two integers. Finally, factor by grouping and removing common factors. For example, to factor $2x^2 + 5x - 3$, find two integers with a product of $(2)(-3)$, or –6, and a sum of 5. Since these integers are –1 and 6, we can expand the trinomial to $2x^2 - x + 6x - 3$. After grouping and removing common factors, we get $x(2x-1) + 3(2x-1)$, which factors to $(2x-1)(x+3)$.

Students should be aware that there are many trinomial expressions that do not factor. As well, algebra tiles can be particularly helpful for students who have difficulty factoring trinomial expressions.

Some polynomials are a result of special products. When factoring, use the patterns that formed these products.

- **Difference of Squares**
  The expression is a binomial where both terms are perfect squares and the operation between the terms is subtraction. In general, $a^2 - b^2 = (a+b)(a-b)$.

- **Perfect Square Trinomial**
  The expression is a trinomial where the first and last terms are perfect squares and the middle term is twice the product of the square root of the first and last terms. In general, $a^2 + 2ab + b^2 = (a+b)^2$.

RELATIONS AND FUNCTIONS

SPECIFIC CURRICULUM OUTCOMES

RF1 – Interpret and explain the relationships among data, graphs and situations.

RF2 – Demonstrate an understanding of relations and functions.

RF3 – Demonstrate an understanding of slope with respect to:
- **rise and run;**
- **line segments and lines;**
- **rate of change;**
- **parallel lines;**
- **perpendicular lines.**

RF4 – Describe and represent linear relations, using:
- **words;**
- **ordered pairs;**
- **tables of values;**
- **graphs;**
- **equations.**

RF5 – Determine the characteristics of the graphs of linear relations, including the:
- **intercepts;**
- **slope;**
- **domain;**
- **range.**

RF6 – Relate linear relations expressed in:
- **slope-intercept form** $\left[y = mx + b\right]$**;**
- **general form** $\left[Ax + By + C = 0\right]$**;**
- **slope-point form** $\left[y - y_1 = m\left(x - x_1\right)\right]$

to their graphs.

RF7 – Determine the equation of a linear relation, given:
- **a graph;**
- **a point and the slope;**
- **two points;**
- **a point and the equation of a parallel or perpendicular line**

to solve problems.

RF8 – Represent a linear function using function notation.

RF9 – Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

MAT421A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
	RF1 Interpret and explain the relationships among data, graphs and situations.	**MAT521A** **MAT521B**

SCO: **RF1 – Interpret and explain the relationships among data, graphs and situations. [C, CN, R, T, V]**

Students who have achieved this outcome should be able to:

A. Graph, with or without technology, a set of data, and determine the restrictions on the domain and range.

B. Explain why data points should or should not be connected on the graph for a situation.

C. Describe a possible situation for a given graph.

D. Sketch a possible graph for a given situation.

E. Determine, and express in a variety of ways, the domain and range of a graph, a set of ordered pairs, or a table of values.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.1 (E)

5.2 (E)

5.3 (C D)

5.4 (A B E)

5.5 (B E)

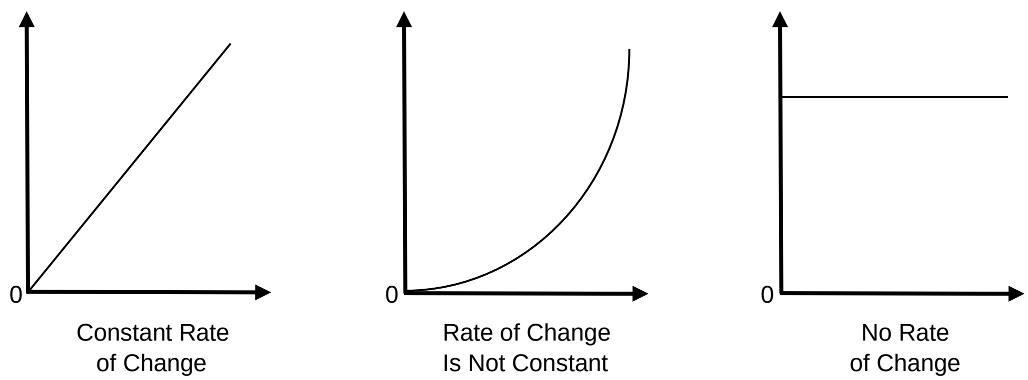
[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: RF1 – Interpret and explain the relationships among data, graphs and situations. [C, CN, R, T, V]

Elaboration

A graph is an effective way to show the relationship between two quantities. A constant rate of change is represented graphically by a straight line. The steepness of the line indicates the rate at which one quantity is changing in relation to the other.

Not all relationships are represented by a straight line. A curve shows that the rate of change is not constant. A horizontal line means that there is no rate of change, since every value on the horizontal axis is related to the same value on the vertical axis.



MAT421A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
	RF2 Demonstrate an understanding of relations and functions.	**MAT521A**
		MAT521B

SCO: **RF2 – Demonstrate an understanding of relations and functions. [C, R, V]**

Students who have achieved this outcome should be able to:

A. Explain, using examples, why some relations are not functions but all functions are relations.

B. Determine if a set of ordered pairs represents a function.

C. Sort a set of graphs as functions or non-functions.

D. Generalize and explain rules for determining whether graphs and sets of ordered pairs represent functions.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.2 (A B D)

5.4 (D)

5.5 (C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF2 – Demonstrate an understanding of relations and functions. [C, R, V]

Elaboration

A relation is any set of ordered pairs. A function is a relation where each value in the domain corresponds to exactly one value in the range. As a result, all functions are relations, but not all relations are functions.

For example, the relation which relates a person with his or her age would be a function, since each person has a unique age at any given time. However the relation which relates a person to one of his or her biological parents would not be a function, since each person has two biological parents.

MAT421A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
	RF3 Demonstrate an understanding of slope with respect to: • rise and run; • line segments and lines; • rate of change; • parallel lines; • perpendicular lines.	**MAT521A** **MAT521B**

SCO: **RF3 – Demonstrate an understanding of slope with respect to:**

- **rise and run;**
- **line segments and lines;**
- **rate of change;**
- **parallel lines;**
- **perpendicular lines.**

[PS, R, V]

Students who have achieved this outcome should be able to:

A. Determine the slope of a line segment by measuring or calculating the rise and run.

B. Classify lines in a given set as having positive or negative slopes.

C. Explain the meaning of the slope of a horizontal or vertical line.

D. Explain why the slope of a line can be determined by using any two points on that line.

E. Explain, using examples, slope as a rate of change.

F. Draw a line, given its slope and a point on the line.

G. Determine another point on a line, given the slope and a point on the line.

H. Generalize and apply a rule for determining whether two lines are parallel or perpendicular.

I. Solve a contextual problem involving slope.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A B C D E F G I)

6.2 (H I)

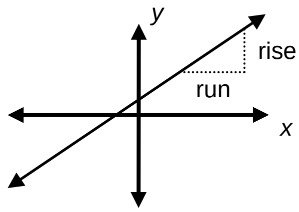
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF3 – Demonstrate an understanding of slope with respect to:
- **rise and run;**
- **line segments and lines;**
- **rate of change;**
- **parallel lines;**
- **perpendicular lines.**

[PS, R, V]

Elaboration

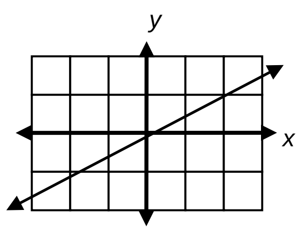
The slope of a line or line segment indicates how steep the line is. The slope of a line is the ratio of the rise to the run.



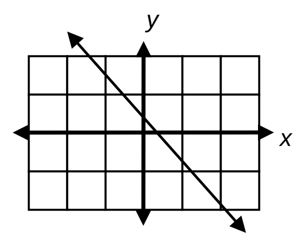
$$m = \frac{\text{rise}}{\text{run}}$$

The slope of a line can be determined using two points on the line, (x_1, y_1) and (x_2, y_2), by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_1 \neq x_2$. The slope indicates the rate of change of a linear relation.

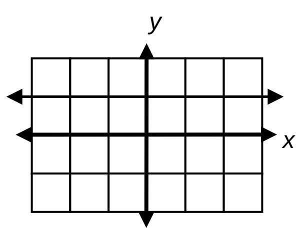
The sign of the slope indicates the direction of the line. A line or line segment that rises from the left to right has a positive slope. A line or line segment that falls from left to right has a negative slope. Also, horizontal lines have zero slope and vertical lines have undefined slopes.



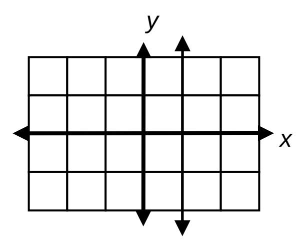
Positive Slope



Negative Slope



Zero Slope



Undefined Slope

Parallel lines have the same slopes but different intercepts. All horizontal lines, which have zero slope, are parallel to each other, and all vertical lines, which have undefined slope, are parallel to each other.

The slopes of oblique perpendicular lines are negative reciprocals of each other. The product of negative reciprocals is –1. A vertical line, which has an undefined slope, and a horizontal line, which has zero slope, are also perpendicular to each other.

MAT421A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
PR1 Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.	**RF4** Describe and represent linear relations, using: • words; • ordered pairs; • tables of values; • graphs; • equations.	**MAT521A** **MAT521B** **RF2** Graph and analyse absolute value functions (limited to linear and quadratic functions) to solve problems. **RF7** Solve problems that involve linear and quadratic inequalities in two variables. **RF11** Graph and analyse reciprocal functions (limited to the reciprocal of linear functions).

SCO: **RF4 – Describe and represent linear relations, using:**

- **words;**
- **ordered pairs;**
- **tables of values;**
- **graphs;**
- **equations.**

[C, CN, R, V]

Students who have achieved this outcome should be able to:

A. Identify independent and dependent variables in a given context.

B. Determine whether a situation represents a linear relation, and explain why or why not.

C. Determine whether a graph represents a linear relation, and explain why or why not.

D. Determine whether a table of values or a set of ordered pairs represents a linear relation, and explain why or why not.

E. Draw a graph from a set of ordered pairs within a given situation, and determine whether the relationship between the variables is linear.

F. Determine whether an equation represents a linear relation, and explain why or why not.

G. Match corresponding representations of linear relations.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.2 (A)

5.5 (A)

5.6 (A B C D E F G)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF4 – Describe and represent linear relations, using:

- **words;**
- **ordered pairs;**
- **tables of values;**
- **graphs;**
- **equations.**

[C, CN, R, V]

Elaboration

A relation can be presented in a variety of ways. For example:

Words: Three times the length of your ear, *e*, is equal to the length of your face, *f*, from chin to hairline.

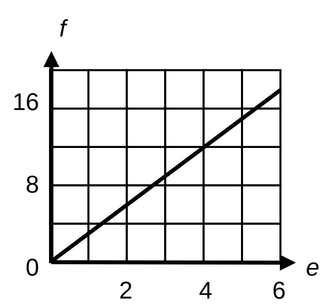
Equation: $f = 3e$

Ordered Pairs: $(4,12)$, $(4.5,13.5)$, $(5,15)$, $(5.5,16.5)$, $(6,18)$, $(6.5,19.5)$

Table of Values:

EAR LENGTH, *e* (cm)	FACE LENGTH, *f* (cm)
4	12
4.5	13.5
5	15
5.5	16.5
6	18
6.5	19.5

Graph:



There are a number of ways to determine whether a relation is a linear relation or a non-linear relation:

- Linear relations have graphs that are straight lines.
- In the table of values of a linear relation, values of *y* increase or decrease by a constant amount as values of *x* increase or decrease by a constant amount.
- When a linear relation is written as an equation, it will contain one or two variables and there will be no term whose degree is higher than one.

MAT421A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
PR3 Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.	**RF5** Determine the characteristics of the graphs of linear relations, including the: • intercepts; • slope; • domain; • range.	**MAT521A** **RF2** Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex; • intercepts; • domain and range; • axis of symmetry.
		MAT521B **RF3** Analyse quadratic functions of the form $y = a(x-p)^2 + q$ and determine the: • vertex; • domain and range; • direction of opening; • axis of symmetry; • *x*- and *y*-intercepts. **RF4** Analyse quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including: • vertex; • domain and range; • direction of opening; • axis of symmetry; • *x*- and *y*-intercepts and to solve problems.

SCO: **RF5 – Determine the characteristics of the graphs of linear relations, including the:**

- **intercepts;**
- **slope;**
- **domain;**
- **range.**

[CN, PS, R, V]

Students who have achieved this outcome should be able to:

A. Determine the intercepts of the graph of a linear relation, and state the intercepts as values or ordered pairs.
B. Determine the slope of the graph of a linear relation.
C. Determine the domain and range of the graph of a linear relation.
D. Sketch a linear relation that has one intercept, two intercepts or an infinite number of intercepts.
E. Identify the graph that corresponds to a given slope and *y*-intercept.
F. Identify the slope and *y*-intercept that correspond to a given graph.
G. Solve a contextual problem that involves intercepts, slope, domain or range of a linear relation.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.7 (A C E G) **6.1 (B D)** **6.4 (E F G)**

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: RF5 – Determine the characteristics of the graphs of linear relations, including the:

- **intercepts;**
- **slope;**
- **domain;**
- **range.**

[CN, PS, R, V]

Elaboration

The *x*-intercept of a line is the *x*-coordinate of the point where the line crosses the *x*-axis. It is found by substituting $y = 0$ into the equation of the line and solving for *x.* The *y*-intercept of a line is the *y*-coordinate of the point where the line crosses the *y*-axis. It is found by substituting $x = 0$ into the equation of the line and solving for *y.*

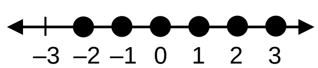
The slope of a line is found by writing the equation of the line in slope-intercept form, $y = mx + b$. The slope, *m*, will be the coefficient of the linear term, *mx*.

When comparing two quantities, the words domain and range are used to describe the values that are appropriate for the relation. In a set of ordered pairs, the domain is the set of the first elements of each pair, and the range is the set of the second elements. On a graph, values of the domain are plotted against the horizontal axis. Values of the range are plotted against the vertical axis.

There are a variety of ways to express the domain and the range of a relation:

Words: all integers greater than or equal to –2, and less than or equal to 3

Number Line:



Set Notation: $\{-2 \le n \le 3,\ n \in \text{integer}\}$

List: $\{-2,\ -1,\ 0,\ 1,\ 2,\ 3\}$

MAT421A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
	RF6 Relate linear relations expressed in: • slope-intercept form $[y = mx + b]$; • general form $[Ax + By + C = 0]$; • slope-point form $[y - y_1 = m(x - x_1)]$ to their graphs.	**MAT521A** **MAT521B**

SCO: **RF6 – Relate linear relations expressed in:**

- **slope-intercept form $[y = mx + b]$;**
- **general form $[Ax + By + C = 0]$;**
- **slope-point form $[y - y_1 = m(x - x_1)]$**

to their graphs. [CN, R, T, V]

Students who have achieved this outcome should be able to:

A. Express a linear relation in different forms, and compare the graphs.

B. Rewrite a linear relation in either slope-intercept or general form.

C. Generalize and explain strategies for graphing a linear relation in slope-intercept, general or slope-point form.

D. Graph, with and without technology, a linear relation given in slope-intercept, general or slope-point form, and explain the strategy used to create the graph.

E. Identify equivalent linear relations from a list of linear relations.

F. Match a set of linear relations to their graphs.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.3 (C D)

6.4 (C D F)

6.5 (B C D F)

6.6 (A B C D E F)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF6 – Relate linear relations expressed in:

- **slope-intercept form $[y = mx + b]$;**
- **general form $[Ax + By + C = 0]$;**
- **slope-point form $[y - y_1 = m(x - x_1)]$**

to their graphs. [CN, R, T, V]

Elaboration

The slope-intercept form of a linear equation is $y = mx + b$, where *m* represents the slope and *b* represents the *y*-intercept. This form is obtained by solving the given linear equation for *y*.

The standard form of a linear equation is $Ax + By = C$, where *A*, *B* and *C* are integers, and *A* and *B* are not both zero. By convention, *A* is a whole number.

The general form of a linear equation is $Ax + By + C = 0$, where *A*, *B* and *C* are integers, and *A* and *B* are not both zero. By convention, *A* is a whole number.

For a non-vertical line through the point (x_1, y_1) with slope *m*, the equation of the line can be written in slope-point form as $y - y_1 = m(x - x_1)$. Any point on the line can be used when determining the equation of the line in slope-point form.

Linear equations can be converted from one form to another by applying the rules of algebra.

MAT421A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
	RF7 Determine the equation of a linear relation, given: • a graph; • a point and the slope; • two points; • a point and the equation of a parallel or perpendicular line to solve problems.	**MAT521A** **MAT521B**

SCO: **RF7 – Determine the equation of a linear relation, given:**

- **a graph;**
- **a point and the slope;**
- **two points;**
- **a point and the equation of a parallel or perpendicular line**

to solve problems. **[CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. Determine the slope and *y*-intercept of a given linear relation from its graph, and write the equation in the form $y = mx + b$.

B. Write the equation of a linear relation, given its slope and the coordinates of a point on the line, and explain the reasoning.

C. Write the equation of a linear relation, given the coordinates of two points on the line, and explain the reasoning.

D. Write the equation of a linear relation, given the coordinates of a point on the line and the equation of a parallel or perpendicular line, and explain the reasoning.

E. Graph linear data generated from a context, and write the equation of the resulting line.

F. Solve a problem, using the equation of a linear relation.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.4 (A F)

6.5 (B C D F)

6.6 (E F)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF7 – Determine the equation of a linear relation, given:

- **a graph;**
- **a point and the slope;**
- **two points;**
- **a point and the equation of a parallel or perpendicular line**

to solve problems. [CN, PS, R, V]

Elaboration

To write the equation of a straight-line graph, use the following two constraints:

- the rate of change or slope, *m*;
- the *y*-intercept. If $(0, b)$ is the point where the line crosses the *y*-axis, then *b* is the *y*-intercept.

The equation of a non-vertical straight line graph can be written in slope-intercept form. The form of the equation is $y = mx + b$, where *m* represents the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ and *b* represents the *y*-intercept.

To determine an equation of a line that is parallel or perpendicular to a given line, use the properties of the slopes of parallel and perpendicular lines, along with the slope-intercept form of the line.

MAT421A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 9	GRADE 10 – MAT421A	GRADE 11
PR1 Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution. **PR2** Model and solve problems using linear equations of the form: • $ax = b$; • $\frac{x}{a} = b,\ a \neq 0$; • $ax + b = c$; • $\frac{x}{a} + b = c,\ a \neq 0$; • $ax = b + cx$; • $a(x + b) = c$; • $ax + b = cx + d$; • $a(bx + c) = d(ex + f)$; and • $\frac{a}{x} = b,\ x \neq 0$ where *a*, *b*, *c*, *d*, *e* and *f* are rational numbers.	**RF8** Represent a linear function using function notation.	**MAT521A** **MAT521B**

SCO: **RF8 – Represent a linear function using function notation. [CN, ME, V]**

Students who have achieved this outcome should be able to:

A. Express the equation of a linear function in two variables, using function notation.

B. Express an equation given in function notation as a linear function in two variables.

C. Determine the related range value, given a domain value, for a linear function; *e.g.*, if $f(x) = 3x - 2$, determine $f(-1)$.

D. Determine the related domain value, given a range value, for a linear function; *e.g.*, if $g(t) = 7 + t$, determine *t* so that $g(t) = 15$.

E. Sketch the graph of a linear function expressed in function notation.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.2 (A B C D)

5.5 (C D)

5.7 (E)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF8 – Represent a linear function using function notation. [CN, ME, V]

Elaboration

Functions can be written using function notation. For example, the function $y = 4x + 1$ can be written as $f(x) = 4x + 1$. The name of the function is *f*, with a variable name of *x*. In this example, $4x + 1$ is the rule that assigns a unique value to *y* for each value of *x*. Any letter may be used to name a function. Two other examples of functions are $v(t) = 9.8t^2$, for the velocity, in metres per second, of a dropped object after *t* seconds and $A(r) = \pi r^2$, for the area of a circle with radius *r*.

Function notation highlights the input-output aspect of a function. The function $f(x) = 4x + 1$ takes any input value for *x*, multiplies it by 4, and adds 1 to give the result. For example, if $x = 2$ is the input, then $f(2) = 9$ is the output, since $f(2) = 4(2) + 1 = 9$. Therefore, the point (2,9) is a on the graph of the function $f(x) = 4x + 1$.

MAT421A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

<table>
<tr><th>GRADE 9</th><th>GRADE 10 – MAT421A</th><th>GRADE 11</th></tr>
<tr><td rowspan="2">PR2 Model and solve problems using linear equations of the form:
• $ax = b$;
• $\frac{x}{a} = b,\ a \neq 0$;
• $ax + b = c$;
• $\frac{x}{a} + b = c,\ a \neq 0$;
• $ax = b + cx$;
• $a(x + b) = c$;
• $ax + b = cx + d$;
• $a(bx + c) = d(ex + f)$; and
• $\frac{a}{x} = b,\ x \neq 0$
where a, b, c, d, e and f are rational numbers.</td><td rowspan="2">RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.</td><td>MAT521A
RF1 Model and solve problems that involve systems of linear inequalities in two variables.</td></tr>
<tr><td>MAT521B
AN3 Solve radical equations (limited to square roots).
AN6 Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials).
RF5 Solve problems that involve quadratic equations.
RF6 Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.</td></tr>
</table>

SCO: **RF9 – Solve problems that involve systems of linear equations in two variables, graphically and algebraically.** **[CN, PS, R, T, V]**

Students who have achieved this outcome should be able to:

A. Model a situation, using a system of linear equations.

B. Relate a system of linear equations to the context of a problem.

C. Determine and verify the solution of a system of linear equations graphically, with and without technology.

D. Explain the meaning of the point of intersection of a system of linear equations.

E. Determine and verify the solution of a system of linear equations algebraically.

F. Explain, using examples, why a system of equations may have no solution, one solution or an infinite number of solutions.

G. Explain a strategy to solve a system of linear equations.

H. Solve a problem that involves a system of linear equations.

Section(s) in Foundations and Pre-Calculus Mathematics 10 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (A B)

7.2 (A C D H)

7.3 (C H)

7.4 (A B E G H)

7.5 (A B E G H)

7.6 (A F)

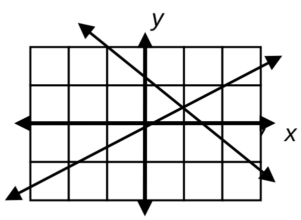
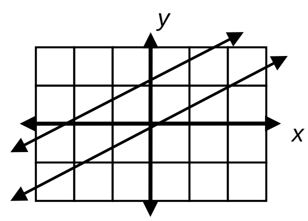
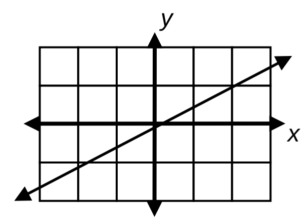
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF9 – Solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]

Elaboration

A pair of two linear equations is called a system of linear equations. It can be represented graphically in order to make comparisons or solve problems. The point(s) of intersection of the two lines on a graph represents the solution to the system of linear equations.

A system of linear equations can have one solution, no solution or an infinite number of solutions. Before solving, the number of solutions for a linear system can be predicted by comparing the slopes and *y*-intercepts of the equations.

INTERSECTING LINES	PARALLEL LINES	COINCIDENT LINES
One solution	**No solution**	**An infinite number of solutions**
		
different slopes *y*-intercepts can be the same or different	same slope different *y*-intercepts	same slope same *y*-intercepts

Systems of linear equations can be solved using three methods:

- **Graphically:** Graph both lines on the same coordinate plane. The solution will occur at the point of intersection.
- **Substitution Method:** Solve one equation for one variable, substitute that expression into the other equation, and then solve for the remaining variable.
- **Elimination Method:** Add or subtract the equations to eliminate one variable and then solve for the remaining variable.

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