Prince Edward Island
CANADA

Education and Early Childhood Development
English Programs

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

MAT631A

Prince Edward Island
CANADA

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- Alberta Education

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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for Grades 10-12 Mathematics* (2008) has been adopted as the basis for a revised high school mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential Graduation Learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

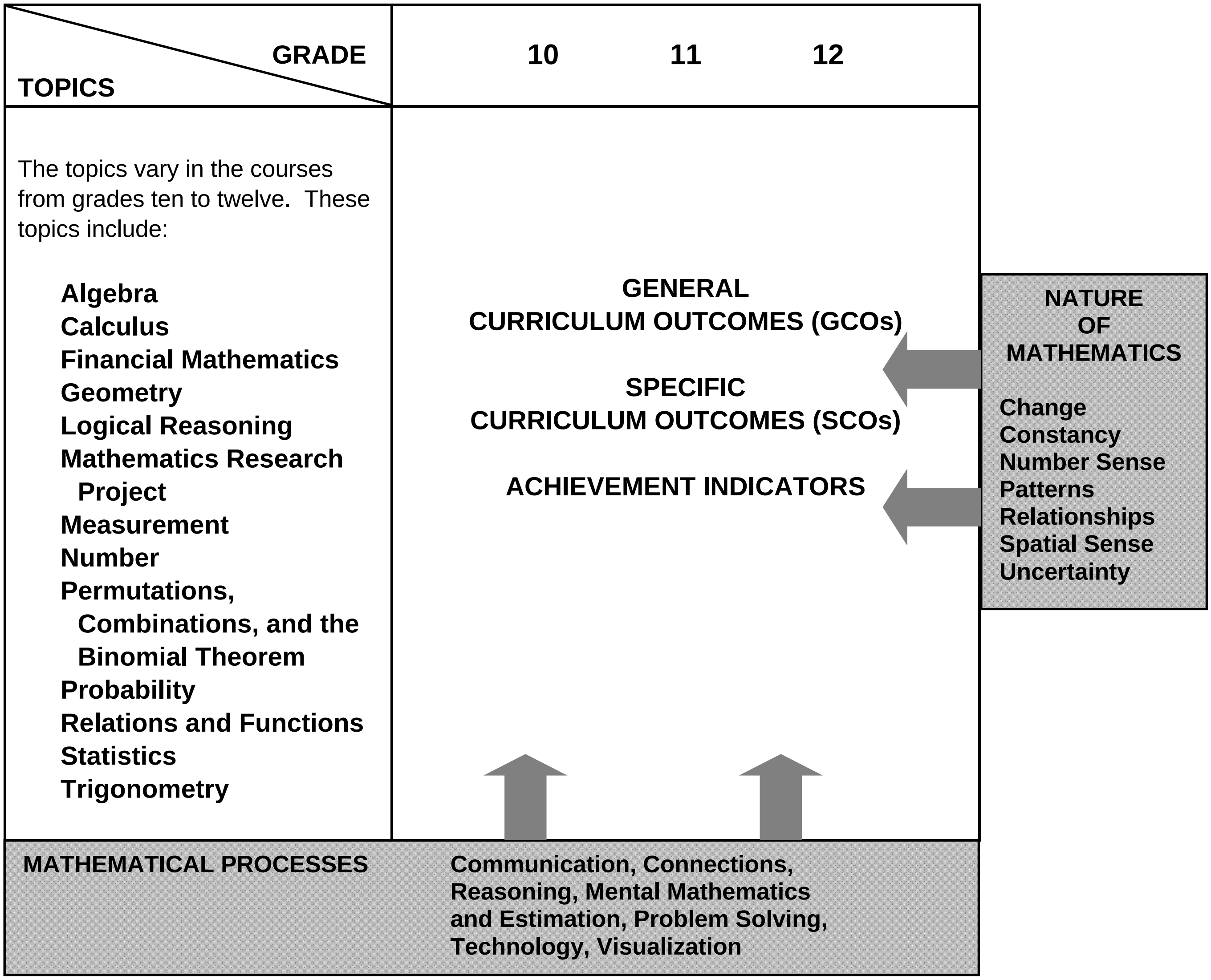
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for 10-12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



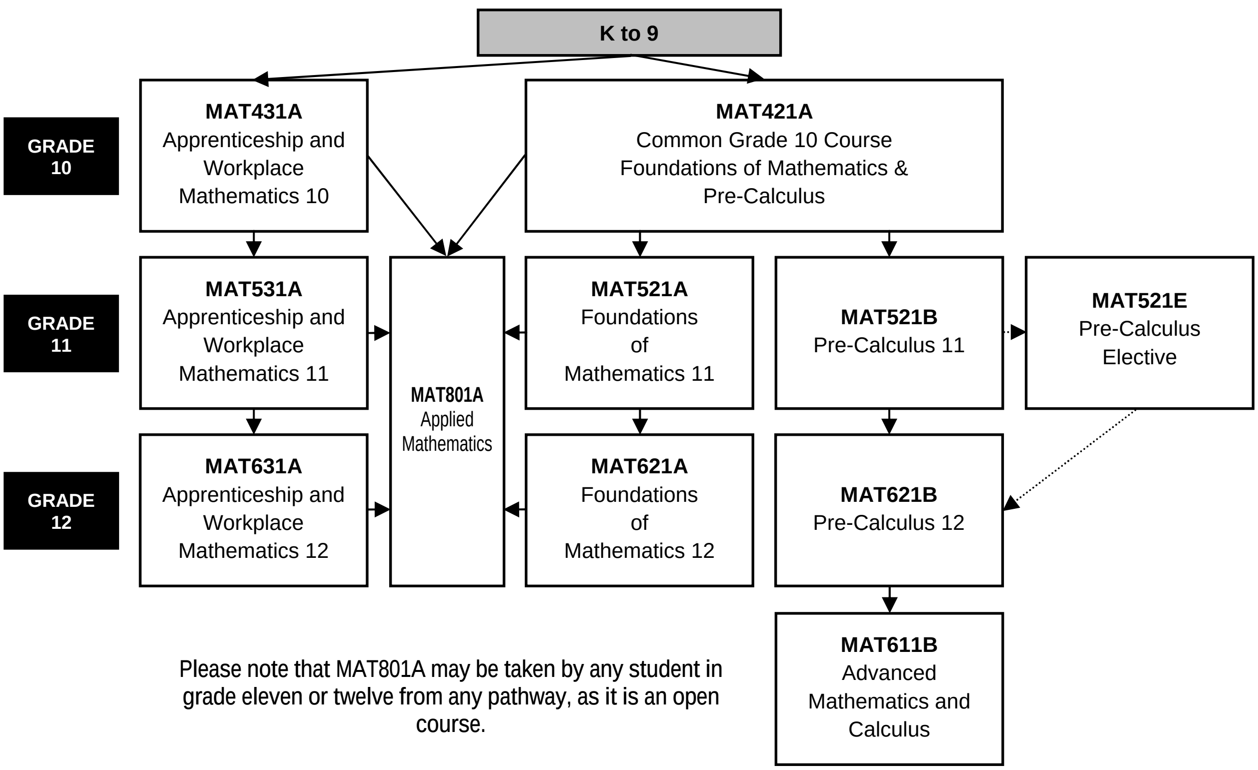
The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into a number of topics, as described above. These topics are not intended to be discrete units of instruction. The integration of outcomes across topics makes mathematical experiences meaningful. Students should make the connections among concepts both within and across topics. Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.

- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Pathways and Topics

The Prince Edward Island 10-12 mathematics curriculum includes pathways with corresponding topics rather than strands, which are found in the Prince Edward Island K-9 mathematics curriculum. Three pathways are available: **Apprenticeship and Workplace Mathematics**, **Foundations of Mathematics**, and **Pre-Calculus**. A common grade ten course (Foundations of Mathematics and Pre-Calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-Calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings. These pathways are illustrated in the diagram below:



The goals of all three pathways are to provide the prerequisite knowledge, skills, understandings, and attitudes for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study or for direct entry into the work force. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Apprenticeship and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics, and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, algebra and number, logical reasoning, relations and functions, statistics, probability, and a mathematics research project.

Pre-Calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, combinatorics, and introductory calculus.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

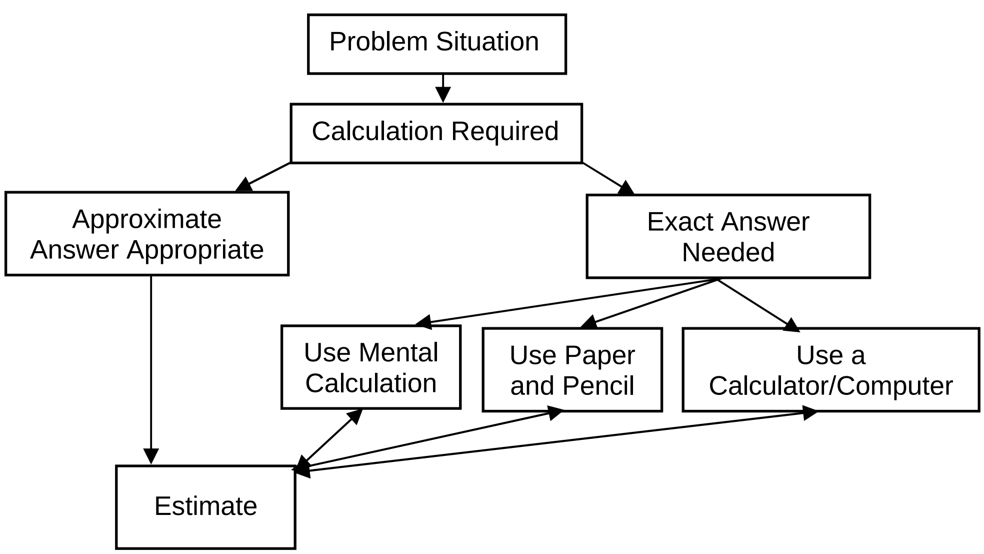
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?" or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.

- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all topics in mathematics and it is important that connections are made among topics. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical topics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate “communicating to learn mathematics and learning to communicate mathematically” (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

➢ Inquiry-Based Learning and Project Based Learning

Inquiry-based learning (IBL) allows students to explore, investigate, and construct new meaning from prior knowledge and from new information that is retrieved from other sources. It is not linear in nature, but promotes a continual looping back and forth throughout the process as students gather and process new information, redirect their inquiries, and continue through the process. Mathematical inquiry will require students to practise and refine their critical and creative-thinking skills. The terms *inquiry* and *research* are often used interchangeably within an educational context. While research often becomes the end-result of an inquiry process, it is the process itself that should be emphasized within an educational context. More information regarding the development of a mathematics research project is included in the appendix at the end of this document.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, teaching has been effective, or how best to address student learning needs. The quality of assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

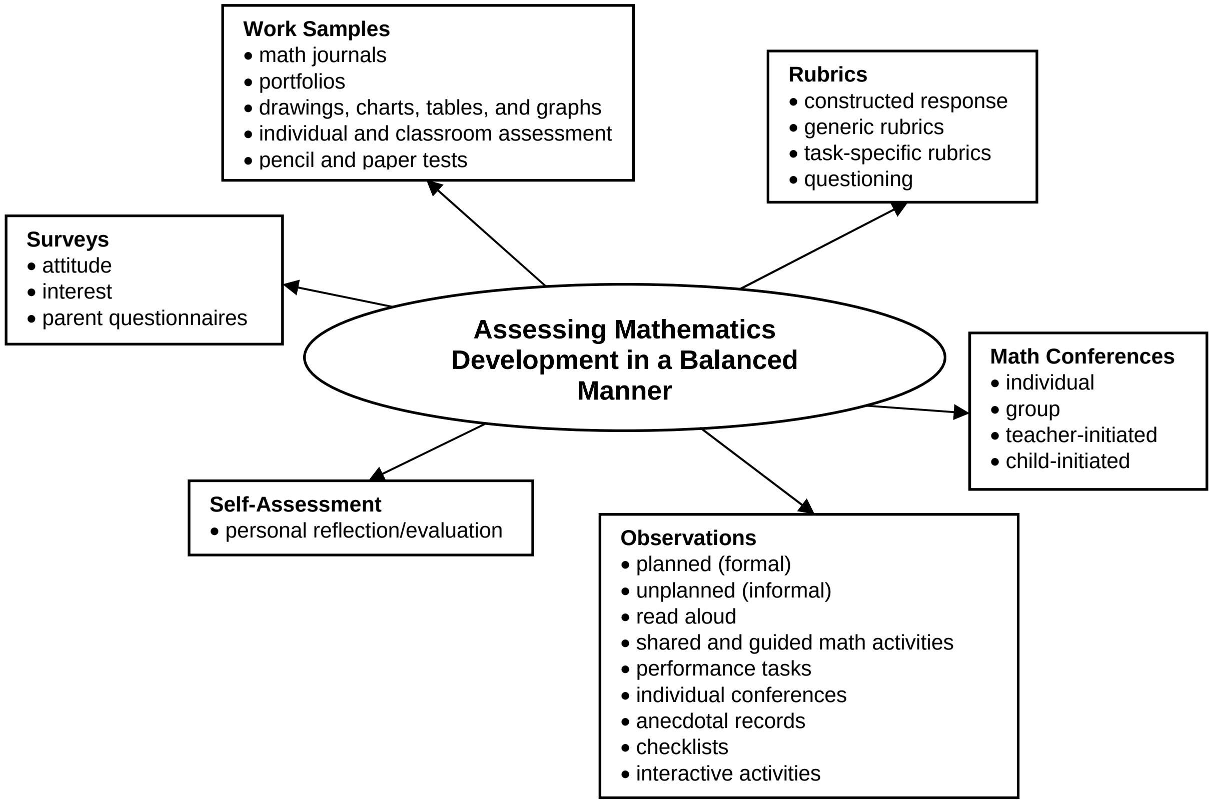
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;

- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, phone calls, and electronic methods.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;

- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he or she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island high school mathematics curriculum are organized into a number of topics across the grades from ten to twelve. Each topic has associated with it a general curriculum outcome (GCO). They are overarching statements about what students are expected to learn in each topic from grades ten to twelve.

Topic	General Curriculum Outcome (GCO)
Algebra (A)	Develop algebraic reasoning.
Algebra and Number (AN)	Develop algebraic reasoning and number sense.
Calculus (C)	Develop introductory calculus reasoning.
Financial Mathematics (FM)	Develop number sense in financial applications.
Geometry (G)	Develop spatial sense.
Logical Reasoning (LR)	Develop logical reasoning.
Mathematics Research Project (MRP)	Develop an appreciation of the role of mathematics in society.
Measurement (M)	Develop spatial sense and proportional reasoning. **(*Foundations of Mathematics and Pre-Calculus*)**
	Develop spatial sense through direct and indirect measurement. **(*Apprenticeship and Workplace Mathematics*)**
Number (N)	Develop number sense and critical thinking skills.
Permutations, Combinations and Binomial Theorem (PC)	Develop algebraic and numeric reasoning that involves combinatorics.
Probability (P)	Develop critical thinking skills related to uncertainty.
Relations and Functions (RF)	Develop algebraic and graphical reasoning through the study of relations.
Statistics (S)	Develop statistical reasoning.
Trigonometry (T)	Develop trigonometric reasoning.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific curriculum outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding topic and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades eleven to twelve which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;

- a list of the sections in *Math at Work 12* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

In the second half of this document, a curriculum guide supplement is presented which follows the primary resource, *Math at Work 12*. As well, an appendix is included which outlines the steps to follow in the development of an effective mathematics research project.

MEASUREMENT

SPECIFIC CURRICULUM OUTCOMES

M1 – Demonstrate an understanding of the limitations of measuring instruments, including:
- **precision;**
- **accuracy;**
- **uncertainty;**
- **tolerance**

and solve problems.

MAT631A – Topic: Measurement (M)

GCO: Develop spatial sense through direct and indirect measurement.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
	M1 Demonstrate an understanding of the limitations of measuring instruments, including: • precision; • accuracy; • uncertainty; • tolerance and solve problems.

SCO: M1 – Demonstrate an understanding of the limitations of measuring instruments, including:

- **precision;**
- **accuracy;**
- **uncertainty;**
- **tolerance**

and solve problems. [C, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Explain why, in a given context, a certain degree of precision is required.

B. Explain why, in a given context, a certain degree of accuracy is required.

C. Explain, using examples, the difference between precision and accuracy.

D. Compare the degree of accuracy of two given instruments used to measure the same attribute.

E. Relate the degree of accuracy to the uncertainty of a given measure.

F. Analyse precision and accuracy in a contextual problem.

G. Calculate maximum and minimum values, using a given degree of tolerance in context.

H. Describe, using examples, the limitations of measuring instruments used in a specific trade or industry, *e.g.*, tape measure versus Vernier caliper.

I. Solve a problem that requires precision, accuracy or tolerance.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A B C D E F G H I)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: M1 – Demonstrate an understanding of the limitations of measuring instruments, including:

- **precision;**
- **accuracy;**
- **uncertainty;**
- **tolerance**

and solve problems. [C, PS, R, T, V]

Elaboration

When measuring something, it is important that both accuracy and precision are taken into account. Accuracy refers to the degree to which the measurement is measured and reported correctly, and precision refers to the exactness to which a measurement is expressed. Different measuring instruments have different levels of precision, which must be taken into account when reporting measurements.

It is also important to understand that no measurement can be perfectly accurate, as there will always be some degree of error, regardless of the precision. This degree of error is called the tolerance of the measurement. When measuring, it is common to also report what level of tolerance is allowed.

GEOMETRY

SPECIFIC CURRICULUM OUTCOMES

G1 – Solve problems by using the sine law and cosine law, excluding the ambiguous case.

G2 – Solve problems that involve:
- **triangles;**
- **quadrilaterals;**
- **regular polygons.**

G3 – Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including:
- **translations;**
- **rotations;**
- **reflections;**
- **dilations.**

MAT631A – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
G1 Solve problems that involve two and three right triangles.	**G1** Solve problems by using the sine law and cosine law, excluding the ambiguous case.

SCO: **G1 – Solve problems by using the sine law and cosine law, excluding the ambiguous case.** [CN, PS, V]

Students who have achieved this outcome should be able to:

A. Identify and describe the use of the sine law and cosine law in construction, industrial, commercial and artistic applications.

B. Solve a problem, using the sine law or cosine law, when a diagram is given.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (A B)

7.2 (A B)

7.3 (A B)

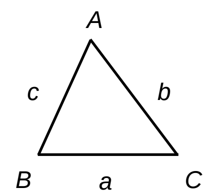
[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: G1 – Solve problems by using the sine law and cosine law, excluding the ambiguous case. [CN, PS, V]

Elaboration

The ratios of $\dfrac{\text{length of opposite side}}{\sin\left(\text{angle}\right)}$ are equivalent for all three side-angle pairs in a triangle. As a result, in any triangle, ΔABC,

$$\frac{a}{\sin\ A}=\frac{b}{\sin\ B}=\frac{c}{\sin\ C}\ \text{ or }\ \frac{\sin\ A}{a}=\frac{\sin\ B}{b}=\frac{\sin\ C}{c}$$



This relationship is known as the sine law.

The sine law can be used to solve a problem modelled by any triangle when the following information is known:

- two sides and an angle opposite a known side;
- two angles and any side.

If the measures of two angles are known, the third angle can be found by using the property that the three angles in a triangle must add up to 180^0.

The cosine law can be used to determine an unknown side length or angle measure in any triangle. The three versions of the cosine law for triangle *ABC* are:

$$a^2=b^2+c^2-2bc\ \cos A$$
$$b^2=a^2+c^2-2ac\ \cos B$$
$$c^2=a^2+b^2-2ab\ \cos C$$

The cosine law can be used to solve a problem modelled by a triangle when the following information is known:

- two sides and the contained angle;
- all three sides.

When solving any problem involving a triangle, it is important to draw a labelled diagram, as the diagram will help determine which strategy to use when solving the problem.

MAT631A – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
G1 Solve problems that involve two and three right triangles.	**G2** Solve problems that involve: • triangles; • quadrilaterals; • regular polygons;

SCO: **G2 – Solve problems that involve:**

- **triangles;**
- **quadrilaterals;**
- **regular polygons.**

[C, CN, PS, V]

Students who have achieved this outcome should be able to:

A. Describe and illustrate properties of triangles, including isosceles and equilateral.

B. Describe and illustrate properties of quadrilaterals in terms of angle measures, side lengths, diagonal lengths and angles of intersection.

C. Describe and illustrate properties of regular polygons.

D. Explain, using examples, why a given property does or does not apply to certain polygons.

E. Identify and explain an application of the properties of polygons in construction, industrial, commercial, domestic and artistic contexts.

F. Solve a contextual problem that involves the application of the properties of polygons.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.1 (A B C D E F)

5.2 (A B C D E F)

5.3 (A B C D E F)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

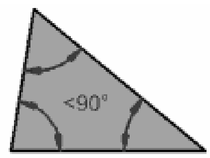
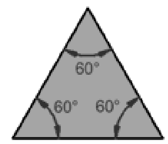
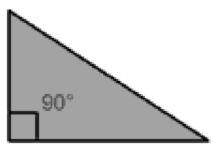
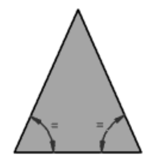
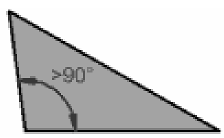
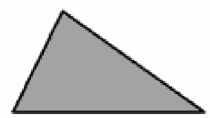
SCO: G2 – Solve problems that involve:

- **triangles;**
- **quadrilaterals;**
- **regular polygons.**

[C, CN, PS, V]

Elaboration

Triangles can be classified by their angles as acute, obtuse, or right, and by their sides as scalene, isosceles, or equilateral.

CLASSIFICATION BY ANGLES		CLASSIFICATION BY SIDES	
Acute Triangle All angles are less than 90^0		**Equilateral Triangle** Three equal sides and three equal angles that are always 60^0	
Right Triangle Has a right angle (90^0)		**Isosceles Triangle** Two equal sides and two equal angles	
Obtuse Triangle Has an angle more than 90^0		**Scalene Triangle** No equal sides and no equal angles	

A quadrilateral is a polygon with four sides. Some examples of quadrilaterals are squares, rectangles, parallelograms, and trapezoids. Regular polygons are polygons whose sides all have the same length and whose angles all have the same measure. The formula for the sum of the interior angles of a regular polygon is $180(n-2)$ degrees, and the measure of each interior angle of a regular polygon is $\frac{180(n-2)}{n}$ degrees.

MAT631A – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
G3 Model and draw 3-D objects and their views. **G4** Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.	**G3** Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including: • translations; • rotations; • reflections; • dilations.

SCO: **G3 – Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including:**

- **translations;**
- **rotations;**
- **reflections;**
- **dilations.**

[C, CN, R, T, V]

Students who have achieved this outcome should be able to:

A. Identify a single transformation that was performed, given the original 2-D shape or 3-D object and its image.

B. Draw the image of a 2-D shape that results from a given single transformation.

C. Draw the image of a 2-D shape that results from a given combination of successive transformations.

D. Create, analyse and describe designs, using translations, rotations and reflections in all four quadrants of a coordinate grid.

E. Identify and describe applications of transformations in construction, industrial, commercial, domestic and artistic contexts.

F. Explain the relationship between reflections, and lines or planes of symmetry.

G. Determine and explain whether a given image is a dilation of another given shape, using the concept of similarity.

H. Draw, with or without technology, a dilation image for a given 2-D shape or 3-D object, and explain how the original 2-D shape or 3-D object and its image are proportional.

I. Solve a contextual problem that involves transformations.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A B C E G H I)

6.2 (A B C D E I)

6.3 (A B C D E F I)

6.4 (A B C D E I)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: G3 – Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including:

- **translations;**
- **rotations;**
- **reflections;**
- **dilations.**

[C, CN, R, T, V]

Elaboration

Students have been exposed to transformational geometry in previous grades. An emphasis at this level should be the use of the formal language of transformations, such as *dilation*, *translation, reflection* and *rotation*, instead of *stretches*, *slides*, *flips* and *turns*. Students will be working with transformations and combinations of transformations in the Cartesian plane.

With respect to describing transformations, students should be able to recognize a given transformation as one of the following: a dilation, a reflection, a translation, a rotation or some combination of these. In addition, when given an image and its translation image, students should be able to describe:

- a dilation, by using the given scale factor;
- a translation, using words and notation describing the translation [*e.g.,* $\Delta A'B'C'$ is the translation image of ΔABC, or $D'(5,8)$ is the translation image of $D(-5,8)$];
- a reflection, by determining the location of the line of reflection;
- a rotation, using degree or fraction-of-turn measures, both clockwise and counterclockwise, and identify the location of the centre of a rotation. A centre of rotation may be located on the shape (such as a vertex of the original image) or off the shape.

When investigating properties of transformations, students should consider the concepts of congruence, which were developed informally in previous grades. In discussing the properties of transformations, students should consider if the transformation of the image:

- has side lengths and angle measures the same as the given image;
- is similar to or congruent to the given image;
- has the same orientation as the given image; or
- appears to have remained stationary with respect to the given image.

NUMBER

SPECIFIC CURRICULUM OUTCOMES

N1 – Analyse puzzles and games that involve logical reasoning, using problem-solving techniques.

N2 – Solve problems that involve the acquisition of a vehicle by:
- **buying;**
- **leasing;**
- **leasing to buy.**

N3 – Critique the viability of small business options by considering:
- **expenses;**
- **sales;**
- **profit or loss.**

MAT631A – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
N1 Analyse puzzles and games that involve numerical reasoning, using problem-solving techniques.	**N1** Analyse puzzles and games that involve logical reasoning, using problem-solving strategies.

SCO: **N1 – Analyse puzzles and games that involve logical reasoning, using problem-solving techniques.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Determine, explain and verify a strategy to solve a puzzle or to win a game; *e.g.*,
- guess and check;
- look for a pattern;
- make a systematic list;
- draw or model;
- eliminate possibilities;
- simplify the original problem;
- work backward;
- develop alternate approaches.

B. Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

C. Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Note: It is intended that this outcome be integrated throughout the course by using puzzles and games such as Sudoku, Mastermind, Nim and logic puzzles.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

Integrated throughout the course.

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: N1 – Analyse puzzles and games that involve logical reasoning, using problem-solving techniques.
[C, CN, PS, R]

Elaboration

This particular outcome is integrated through the course. Each chapter has a section called *Games and Puzzles* that can be used to meet this particular SCO. They are found on pages 59, 107, 165, 221, 263, 321, and 365 of the textbook.

MAT631A – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
N2 Solve problems that involve personal budgets. **N5** Demonstrate an understanding of credit options, including: • credit cards; • loans.	**N2** Solve problems that involve the acquisition of a vehicle by: • buying; • leasing; • planning to buy.

SCO: **N2 – Solve problems that involve the acquisition of a vehicle by:**

- **buying;**
- **leasing;**
- **leasing to buy.**

[C, CN, PS, R, T]

Students who have achieved this outcome should be able to:

A. Describe and explain various options for buying, leasing, and leasing to buy a vehicle.

B. Solve, with or without technology, problems that involve the purchase, lease, or lease to purchase of a vehicle.

C. Justify a decision related to buying, leasing, or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenance, warranties, mileage and insurance.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (A B)

4.2 (C)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: N2 – Solve problems that involve the acquisition of a vehicle by:
- **buying;**
- **leasing;**
- **leasing to buy.**

[C, CN, PS, R, T]

Elaboration

When deciding whether to buy or lease a car, there are a number of things that must be considered. Some of these considerations are listed below:

Buying Option:

- By far, the greatest benefit of buying a car is that you may actually own it one day. Implied in this benefit is that you will one day be free of car payments. The car will be yours to sell at any time, and you are not locked into any type of fixed ownership period.
- When you buy a car, the insurance limits on your policy are typically lower than if you lease. In addition, by owning a car, there are no mileage restrictions that typically exist when leasing.
- The most obvious downside of owning versus leasing is the monthly payment, which is usually higher on a purchased car than on a leased car. Additionally, the dealers usually require a reasonable down payment, so the initial out-of-pocket cost is higher when buying a car.
- Presumably, as you pay down your car loan, you have the ability to build equity in the vehicle. Unfortunately, however, this is not always the case. When you purchase a car, your payments reflect the whole cost of the car, usually amortized over a four- to six-year period. But depreciation can take a significant toll on the value of your car, especially in the first couple of years, so the car will lose much of its original value in a short amount of time.

Leasing Option:

- Perhaps the greatest benefit of leasing a car is the lower out-of-pocket costs when acquiring and maintaining the car. Leases require little or no down payment, and there are no upfront sales tax payments. Additionally, monthly payments are usually lower, and you get the pleasure of owning a new car every few years.
- With a lease, you are essentially renting the car for a fixed number of months (typically 36 to 48 months). Therefore, you pay only for the use, or depreciation, of the car for that period, and you are not forced to absorb the full depreciation cost of the vehicle.
- Leasing also provides an alternative when buying a car is not an option, due to not having the required down payment or having a higher monthly payment.
- For business owners, leasing a car may offer tax advantages if the vehicle is used for business purposes.
- By leasing a car, you always have a car payment. As long as you lease, you will never really own it. However, depending on your type of lease, when your lease term is up, you either hand the keys over to the car dealership and look for another vehicle, or finance the remaining value of the vehicle and go from making lease payments to loan payments, thereby leasing to own.
- The mileage restrictions of leasing pose another drawback. If you drive a great deal during the year, this can cost a significant amount of money. If you go over your allotted annual kilometres, you pay extra per kilometre.
- Insurers usually charge higher coverage costs for leased vehicles. However, depending on your age, driving record and place of residence, that additional cost may be nominal.

MAT631A – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
	N3 Critique the viability of small business options by considering: • expenses; • sales; • profit or loss.

N3 – Critique the viability of small business options by considering:

- **expenses;**
- **sales;**
- **profit or loss.**

[C, CN, R]

Students who have achieved this outcome should be able to:

A. Identify expenses in operating a small business, such as a hot dog stand.

B. Identify feasible small business options for a given community,

C. Generate options that might improve the profitability of a small business.

D. Determine the break-even point for a small business.

E. Explain factors, such as seasonal variations and hours of operation, that might impact the profitability of a small business.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.3 (A B C D E)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

N3 – Critique the viability of small business options by considering:
- **expenses;**
- **sales;**
- **profit or loss.**

[C, CN, R]

Elaboration

Before opening a small business, it is important to do a feasibility study on starting such a venture. This feasibility study includes researching the possible expenses and sources of revenue for the business. After this study, if a profit is predicted, then the business could be a viable one.

ALGEBRA

SPECIFIC CURRICULUM OUTCOMES

A1 – Describe an understanding of linear relations by:

- **recognizing patterns and trends;**
- **graphing;**
- **creating tables of values;**
- **writing equations;**
- **interpolating and extrapolating;**
- **solving problems.**

MAT631A – Topic: Algebra (A)

GCO: Develop algebraic reasoning.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
A2 Demonstrate an understanding of slope: • as rise over run; • as a rate of change; • by solving problems.	**A1** Demonstrate an understanding of linear relations by: • recognizing patterns and trends; • graphing; • creating tables of values; • writing equations; • interpolating and extrapolating; • solving problems.

SCO: A1 – Describe an understanding of linear relations by:

- **recognizing patterns and trends;**
- **graphing;**
- **creating tables of values;**
- **writing equations;**
- **interpolating and extrapolating;**
- **solving problems.**

[CN, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Identify and describe the characteristics of a linear relation represented in a graph, table of values, number pattern or equation.

B. Sort a set of graphs, tables of values, number patterns and/or equations into linear and nonlinear relations.

C. Write an equation for a given context, including direct or partial variation.

D. Create a table of values for a given equation of a linear relation.

E. Sketch the graph for a given table of values.

F. Explain why the points should or should not be connected on the graph for a context.

G. Create, with or without technology, a graph to represent a data set, including scatter plots.

H. Describe the trends in the graph of a data set, including scatter plots.

I. Sort a set of scatter plots according to the trends represented (linear, nonlinear or no trend).

J. Solve a contextual problem that requires interpolation or extrapolation of information.

K. Relate slope and rate of change to linear functions.

L. Match given contexts with their corresponding graphs, and explain the reasoning.

M. Solve a contextual problem that involves the application of a formula for a linear relation.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.3 (G H)

3.1 (A B E G H I)

3.2 (A B C D E F J K L)

3.3 (A B C D E F J K L)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: A1 – Describe an understanding of linear relations by:

- **recognizing patterns and trends;**
- **graphing;**
- **creating tables of values;**
- **writing equations;**
- **interpolating and extrapolating;**
- **solving problems.**

[CN, PS, R, T, V]

Elaboration

Students have been exposed to patterns through the interpretation of graphs of linear relations. From a pictorial pattern, students should be able to identify and write the pattern rule and create a table of values in order to write an expression to represent the situation. When an oral or written pattern is given, students should be able to write an expression directly from that pattern.

When students are looking at a table of values, such as the following,

Term Number (*n*)	1	2	3	4	5
Term (*t*)	2	8	14	20	26

they should look at the pattern and recognize a constant increase or decrease (here an increase of 6) between the values. Students should recognize that multiplying the term number, n, by 6 always results in four more than the associated term, t. Therefore, they will need to subtract 4 from $6n$. As an equation, the pattern is represented by $t = 6n - 4$. Students should verify their equation by substituting values from the table (for example, $n = 5$, $t = 26$). Students should use their equation to solve for any value of n or t.

Statistical studies often find linear correlations between two variables. A scatter plot can often reveal the relationship between two variables. The independent variable is usually plotted on the horizontal axis and the dependent variable on the vertical axis. A linear correlation can be positive or negative, and its magnitude can vary in strength from zero (no correlation) to one (perfect correlation).

Students will be asked to solve problems involving direct and partial variation. Both types of variations are examples of linear relations. A direct variation is a linear relation of the form $y = mx$, and a partial variation is a linear relation of the form $y = mx + b$, where $b \neq 0$.

Students will also be asked to interpolate and extrapolate graphs in order to solve problems. Interpolation consists of estimating a value between two given values, while extrapolation consists of estimating a value beyond a given set of values. In order to extrapolate, students must extend the pattern beyond the given data.

STATISTICS

SPECIFIC CURRICULUM OUTCOMES

S1 – Solve problems that involve measures of central tendency, including:

- **mean;**
- **median;**
- **mode;**
- **weighted mean;**
- **trimmed mean.**

S2 – Analyse and describe percentiles.

MAT631A – Topic: Statistics (S)

GCO: Develop statistical reasoning.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
S1 Solve problems that involve creating and interpreting graphs, including: • bar graphs; • histograms; • line graphs; • circle graphs.	**S1** Solve problems that involve measures of central tendency, including: • mean; • median; • mode; • weighted mean; • trimmed mean.

SCO: S1 – Solve problems that involve measures of central tendency, including:

- **mean;**
- **median;**
- **mode;**
- **weighted mean;**
- **trimmed mean.**

[C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Explain, using examples, the advantages and disadvantages of each measure of central tendency.

B. Determine the mean, median and mode for a set of data.

C. Identify and correct errors in a calculation of a measure of central tendency.

D. Identify the outlier(s) in a set of data.

E. Explain the effect of outliers on mean, median and mode.

F. Calculate the trimmed mean for a set of data, and justify the removal of the outliers.

G. Explain, using examples such as course marks, why some data in the set would be given a greater weighting in determining the mean.

H. Calculate the mean of a set of numbers after allowing the data to have different weightings (weighted mean).

I. Explain, using examples, from print and other media, how measures of central tendency and outliers are used to provide different interpretations of data.

J. Solve a contextual problem that involves measures of central tendency.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B C)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: S1 – Solve problems that involve measures of central tendency, including:
- **mean;**
- **median;**
- **mode;**
- **weighted mean;**
- **trimmed mean.**

[C, CN, PS, R]

Elaboration

The three principal measures of central tendency are the mean, median, and mode. These measures for a sample can differ from those of the whole population.

The mean is the sum of the values in a set of data divided by the number of data values in the set. Outliers can have a dramatic effect on the mean if the sample size is small. The median is the middle value when the values are ranked in order. If there are two middle values, then the median is the mean of these two middle values. The mode is the most frequency occurring value.

A weighted mean can be a useful measure when all of the data are not all of equal significance. In this case, each data item is multiplied by a number which corresponds to its importance in the data set. A trimmed mean is the mean of all of the numbers in the data set excluding the same number of values from the top and bottom of the data set.

MAT631A – Topic: Statistics (S)

GCO: Develop statistical reasoning.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
	S2 Analyse and describe percentiles.

SCO: **S2 – Analyse and describe percentiles.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Explain, using examples, percentile ranks in a context.

B. Explain decisions based on a given percentile rank.

C. Explain, using examples, the difference between percent and percentile rank.

D. Explain the relationship between median and percentile.

E. Solve a contextual problem that involves percentiles.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.2 (D E F G H I J)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: S2 – Analyse and describe percentiles. [C, CN, PS, R]

Elaboration

Percentiles are numbers that divide a set of ordered data into 100 intervals with equal numbers of data in each set. They have the advantage of being resistant to outliers in the data set. Percentiles are particularly useful statistics when working with a very large data set. They make it very easy to categorize such data based on rank. As a result, they can help make decisions based on rank.

PROBABILITY

SPECIFIC CURRICULUM OUTCOMES

P1 – Analyse and interpret problems that involve probability.

MAT631A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT531A	GRADE 12 – MAT631A
	P1 Analyse and interpret problems that involve probability.

SCO: **P1 – Analyse and interpret problems that involve probability.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Describe and explain the applications of probability, *e.g.*, medication, warranties, insurance, lotteries, weather prediction, 100-year flood, failure of a design, failure of a product, vehicle recalls, approximation of area.

B. Calculate the probability of an event based on a data set, *e.g.*, determine the probability of a randomly chosen light bulb being defective.

C. Express a given probability as a fraction, decimal and percent and in a statement.

D. Explain the difference between odds and probability.

E. Determine the probability of an event, given the odds for or odds against.

F. Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, experimental results and subjective judgments.

G. Solve a contextual problem that involves a given probability.

Section(s) in Math at Work 12 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.2 (A B C D E)

1.3 (F)

1.4 (G)

[C] Communication	**[ME]** Mental Mathematics	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections	and Estimation	**[R]** Reasoning	**[V]** Visualization

SCO: P1 – Analyse and interpret problems that involve probability. [C, CN, PS, R]

Elaboration

A probability experiment is a well-defined process in which clearly identifiable outcomes are measured for each trial. An event is a collection of outcomes satisfying a particular condition. The probability of an event can range from 0 or 0% (impossible), to 1 or 100% (certain).

The theoretical probability of an event is the ratio of the number of favourable outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely. It can only be used to predict what will happen in the long run, when events represented are equally likely to occur. Students should realize that the probability in many situations cannot be characterized as equally likely, such as tossing a thumb tack to see if it lands with the point up or down, and therefore theoretical probability is more difficult to determine. In such cases, experiments should be limited to determining the relative frequency of a particular event. The theoretical probability of event is:

$$P = \frac{\text{number of ways that event } P \text{ can successfully occur}}{\text{sample space (total number of possible outcomes)}}$$

Curriculum Guide Supplement

This supplement to the *Prince Edward Island MAT631A Mathematics Curriculum Guide* is designed to parallel the primary resource, *Math at Work 12*.

For each of the chapters in the text, an approximate timeframe is suggested to aid teachers with planning. The timeframe is based on a total of 80 classes, each with an average length of 75 minutes:

CHAPTER	SUGGESTED TIME
Chapter 1 – Measurement and Probability	13 classes
Chapter 2 – Working with Data	10 classes
Chapter 3 – Linear Relationships	13 classes
Chapter 4 – Real-Life Decisions	12 classes
Chapter 5 – Properties of Figures	9 classes
Chapter 6 – Transformations	13 classes
Chapter 7 – Trigonometry	10 classes

Each chapter of the text is divided into a number of sections. In this document, each section is supported by a one-page presentation, which includes the following information:

- the name and pages of the section in *Math at Work 12*;
- the specific curriculum outcome(s) and achievement indicator(s) addressed in the section (see the first half of the curriculum guide for an explanation of symbols);
- the student expectations for the section, which are associated with the SCO(s);
- the new concepts introduced in the section;
- other key ideas developed in the section;
- suggested problems in *Math at Work 12*;
- possible instructional and assessment strategies for the section.

CHAPTER 1
MEASUREMENT AND PROBABILITY

SUGGESTED TIME

13 classes

Section 1.1 – Accuracy and Precision (pp. 6-19)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **M1 (A B C D E F G H I)**

After this lesson, students will be expected to:

- understand the difference between accuracy and precision
- determine the importance of degrees of accuracy and precision
- calculate maximum and minimum values, using a given degree of tolerance

After this lesson, students should understand the following concepts:

- **accuracy –** the degree to which a measurement is measured and reported correctly
- **precision –** the degree of exactness to which a measurement is expressed; the precision of a measurement depends on the scale of the instrument used
- **tolerance –** the total amount that a measurement is allowed to vary

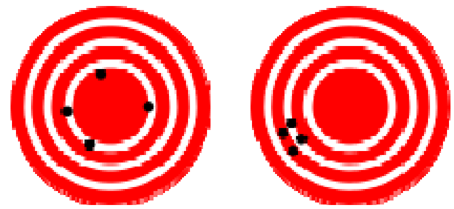
Suggested Problems in *Math at Work 12*:

- **pp. 9-10:** #1-9
- **pp.13-14:** #1-7
- **pp. 16-17:** #1-8
- **pp. 18-19:** #1-10

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- A good way to help explain the difference between accuracy and precision is to use the following example:

  In the target above left, all of the shots are close to the bulls eye, but they are not close together. We would say that this shows high accuracy but low precision. In the target above right, the shots miss the bulls eye, but they are clustered together. We would say that this shows high precision, but low accuracy
- When describing the tolerance of a measure, some students may not be familiar with $\pm$ notation. Ensure that all students understand what an expression like $8\text{ cm} \pm 0.5\text{ cm}$ means.

Possible Assessment Strategies:

- Determine the maximum and minimum allowable measurements.

  a. $34'' \pm \frac{1}{8}''$

  b. $3\text{ kg} \pm 5\text{ g}$
- Calculate the volume of concrete needed for a sidewalk block that measures 5 ft by 5 ft by 9 in.
- a. The living room in a house has dimensions that are 16 ft 3 in by 20 ft 8 in. The ceiling is 7 ft 6 in high. John wants to paint the walls of the living room. Without taking the doorway into account, what is the total surface area of the walls of the living room?

  b. A doorway in the living room is 78 inches high and 24 inches wide. A door casing will be installed so that about $\frac{1}{8}$ of an inch of the door frame is exposed. How long should the pieces of casing be cut?
- The mass of a Canadian toonie produced by the Royal Canadian Mint is 6.92 g with a tolerance of $\pm 0.01\text{ g}$. Otherwise, they are rejected for circulation. What are the maximum and minimum allowable masses for a toonie?

Section 1.2 – Probability and Odds (pp. 20-29)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **P1 (A B C D E)**

After this lesson, students will be expected to:

- determine the probability of an event
- express a probability as a fraction, a decimal, a percent, and in words
- determine the odds for and odds against an event

After this lesson, students should understand the following concepts:

- **probability –** the mathematical likelihood of something happening; a ratio that compares the number of possible successful outcomes to the total number of possible outcomes
- **odds –** a ratio that compares the number of possible successful outcomes to the number of possible unsuccessful outcomes
- **tree diagram –** a type of organizer for displaying outcomes of an event; each branch represents a different possible outcome

Suggested Problems in *Math at Work 12*:

- **pp. 24-25:** #1-7
- **pp. 27-28:** #1-7
- **pp. 28-29:** #1-8

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Review benchmarks and how they relate to probability with students:

← less often →← more often→

0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
0.0	0.25	0.50	0.75	1.0
0%	25%	50%	75%	100%
impossible	unlikely	half of the time	likely	certain

- Ensure that students clearly understand the difference between odds and probability.

Possible Assessment Strategies:

- You are asked to pick a card from a standard deck of 52 cards. What is the probability that the card drawn is
  a. a face card?
  b. a diamond?
  c. a black card?
- A drawer has 3 pairs of white socks, 4 pairs of black socks, and 2 pairs of grey socks. A pair of socks is selected at random. What are the odds of
  a. selecting a pair of white socks?
  b. selecting a pair of grey socks?
  c. not selecting a pair of black socks?
- a. When rolling a die, a person is considered to be a winner if he rolls a 6. John rolls a die three times. What is the probability of rolling three sixes in a row?
  b. What are the odds of rolling three sixes?

Section 1.3 – Theoretical and Experimental Probability (pp. 30-41)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P1 (F)** **After this lesson, students will be expected to:** • compare theoretical probability and experimental results • express a probability as a fraction, a decimal, a percent, and in words • determine the probability of an event **After this lesson, students should understand the following concepts:** • **theoretical probability –** a ratio that compares the number of possible successful outcomes to the total number of possible outcomes; determined by reason or calculation • **experimental probability –** a ratio that compares the number of times an event occurs to the total number of trials or events; determined by experiment **Suggested Problems in *Math at Work 12*:** • **pp. 33-34:** #1-10 • **pp. 36-39:** #1-9 • **pp. 39-41:** #1-7	**Possible Instructional Strategies:** • Ensure that students understand the difference between experimental probability and theoretical probability. • Discuss how experimental probability and theoretical probability might approach each other if an experiment is conducted enough times. Some students may have difficulty with this concept and may need to see it played out many times. **Possible Assessment Strategies:** • A boy flips a coin 9 times and gets 9 heads in a row. If the boy flips the coin again, what is the probability of getting heads on the tenth flip? • Two dice are rolled. What is the probability of getting a. a double? b. a sum of 7? c. a product of 12? • In order to attract more customers to a store, they run a contest where the probability of winning a prize is $\frac{1}{5}$. a. Express the probability of winning a prize as a percent. b. Calculate the probability of not winning a prize. Express the answer as a percent. c. What are the odds of winning a prize in this contest? d. If John enters the contest 10 times, is there any guarantee that he will win at least once? • In a local lottery, the odds of winning a prize are $1:100$. What is the probability of winning a prize?

Section 1.4 – Working with Probability (pp. 42-53)

<table>
<tr><th>ELABORATIONS &
SUGGESTED PROBLEMS</th><th>POSSIBLE INSTRUCTIONAL &
ASSESSMENT STRATEGIES</th></tr>
<tr><td>
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:
P1 (G)
After this lesson, students will be expected to:

work with everyday events involving probability
solve problems that involve probability
solve problems that involve the need for precision

Suggested Problems in Math at Work 12:

pp. 46-48: #1-10
pp. 50-52: #1-7
pp. 52-53: #1-6

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Possible Instructional Strategies:
Discuss with the class examples where probability is used in everyday life. For example, explain what meant by the probability of precipitation in weather forecasts.
Possible Assessment Strategies:

What is the probability of obtaining success in each of the following experiments?
a. flipping a coin and it lands tails up
b. rolling two dice and getting a sum of 12
c. selecting a card from a standard deck and getting a black card
What results would you expect from each experiment?
a. You flip a coin 30 times. How many tails would you expect?
b. You roll a die 90 times. How many times would you expect a 4?
Write each probability as a decimal, to three decimal places, and as a percent, to one decimal place.
a. $\frac{1}{8}$
b. $\frac{3}{13}$
Calculate the success rate of each of the following results. Express the answer as a fraction in lowest terms.
a. You want diamonds. You select a card 40 times and get 9 diamonds.
b. You want even numbers. You roll a die 90 times and get 60 even numbers.
In a recent survey of students in Canada, 1 in 9 students responded that they were left-handed.
a. What percent of students are left-handed? Round off the answer to one decimal place.
b. A particular high school has 774 students. Using the survey results, how many students would you expect to be left-handed in that high school?
Contest A indicates that the probability of winning a prize is 1 in 5 and Contest B indicates that the probability of winning a prize is 2 in 11. Which contest offers the better chance of winning? Explain.

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CHAPTER 2
WORKING WITH DATA

SUGGESTED TIME

10 classes

Section 2.1 – Measures of Central Tendency (pp. 64-79)

<table>
<tr><th>ELABORATIONS &
SUGGESTED PROBLEMS</th><th>POSSIBLE INSTRUCTIONAL &
ASSESSMENT STRATEGIES</th></tr>
<tr><td>

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S1 (A B C)**

After this lesson, students will be expected to:

- determine the three measures of central tendency
- determine weighted means
- solve a problem involving measures of central tendency

After this lesson, students should understand the following concepts:

- **median –** the middle number in a set of data after the data have been arranged in order; for example, the median of 2, 4, 6, 8, 11 is 6; when there is an even number of data, average the two middle numbers to find the median; for example, the median of 1, 5, 9, 13, 16, 20 is 11, because $\frac{9+13}{2}=11$
- **mode –** the number(s) the occur(s) most frequently in a set of data; a data set can have no mode, one mode, or more than one mode
- **data set –** a collection of related information
- **mean –** the average of the data values; add the data values and divide by the total number of data values; for example, the mean of 4, 7, 9, 10, 11, 16 is $\frac{4+7+9+10+11+16}{6}=9.5$
- **measure of central tendency –** a value that represents the centre of a set of data; can be the mean, median, or mode
- **stem-and-leaf plot –** a way to organize numerical data in order of place value; the “tens digit and greater” is the stem and the “ones digit” is the leaf; to plot decimal numbers, the “ones digit and greater” is the stem and the “tenths and less” is the leaf
- **weighted mean –** the average of mean of a data set in which each data point does not contribute an equal amount to the final average

Suggested Problems in *Math at Work 12*:

- **pp. 68-69:** #1-6
- **pp. 71-72:** #1-5
- **pp. 76-77:** #1-5
- **pp. 77-79:** #1-7

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Possible Instructional Strategies:

- Ensure that the students understand when it is appropriate to use the mean, median, or mode. Also ensure that students understand the limitations of each of these measures of central tendency.

Possible Assessment Strategies:

- The mode of this set of data is 75. Determine the missing value, *n*.

  81 75 90 68 81 68 75 *n*

- For the following set of quiz marks out of 10, calculate the mean, median, and mode. Round off to one decimal place, where necessary.

  8 7 9 10 8 6 5 6
  8 9 8 10 7 7 7 8
  6 10 10 4 7 9 7 9

- Suppose the mean of a set of test scores is 89. One of the scores is erased from the report card, and the other four scores are 90, 95, 85 and 100. Determine the value of the missing test score.
- When Mr. Brown gave a science test, he found the following:
  - The mean for the test was 72%.
  - The mode for the test was 65%.
  - The median for the test was 65%.

  When he gave back the test, it was determined that his answer key was wrong, and all of the students had a certain question correct which was valued at 5%. He was then compelled to increase all the marks by 5%. How did this affect the mean, median and mode?
- In a particular math course, the final examination is worth 40%, the class tests are worth 50%, and the class assignments are worth 10%. Jack received the following marks:

  Final exam: 82%
  Class tests: 89%, 92%, 78%, 84%, 97%
  Class assignments: 97%, 99%, 92%, 91%, 100%

  Use a weighted mean to determine Jack’s final grade for the course. Round off the answer to the nearest whole number.

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Section 2.2 – Using Other Statistical Measures (pp. 80-93)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S2 (D E F G H I J)**

After this lesson, students will be expected to:

- find the range for a data set
- identify outliers in a data set
- calculate the trimmed mean for a data set
- work with percentiles and percentile ranks
- solve problems involving trimmed means and percentiles

After this lesson, students should understand the following concepts:

- **range –** the difference between the largest value and the smallest value of a data set
- **outlier –** a value that is much smaller or larger than the other data values; a data set may have no outliers, one outlier, or more than one outlier
- **trimmed mean –** a calculation of the mean found removing the highest and lowest values; you must remove the same number of values from the top and bottom of the data set; removing outliers can result in a more accurate mean
- **percentile –** a value below which a certain percent of the data set falls; the median is also called the 50th percentile, because 50% of the values in the data set are below the median value
- **percentile rank –** a number between 0 and 100 that indicates the percent of cases that fall at or below that score

Suggested Problems in *Math at Work 12*:

- **pp. 85-86:** #1-6
- **pp. 90-91:** #1-7
- **pp. 92-93:** #1-7

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students understand that range is calculated using the highest and lowest values in a data set. Also ensure that students understand that range is always a positive number.
- Use activities where the outlier is an obvious error to illustrate situations where the outlier would not be used in calculating the averages. If the outlier is not an error it should still be used in calculations but students should recognize that, in this case, the median is a better measure of central tendency.

Possible Assessment Strategies:

- The following data set shows the number of birds at a feeder from Monday to Sunday. What is the range?

M	T	W	T	F	S	S
3	4	6	9	2	11	13

- a. Identify the outlier(s) in this data set:

  24, 30, 26, 54, 28, 19

  b. Remove the lowest and highest scores, then calculate the trimmed mean.

- Twenty students participated in a basketball shooting contest. Each student made 30 attempts. The results were as follows:

15	17	19	23	19
23	9	21	20	19
12	22	26	17	21
19	16	13	8	7

  a. Organize the data in a stem-and-leaf plot.

  b. What result is at the 50th percentile?

  c. Mary got 20 baskets. At what percentile rank is her result?

  d. What results are in the 85th percentile or above?

Section 2.3 – Scatter Plots (pp. 94-101)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

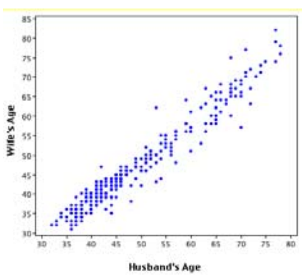
- **A1 (G H)**

After this lesson, students will be expected to:

- interpret scatter plots
- determine whether a trend exists

After this lesson, students should understand the following concepts:

- **scatter plot –** a graph of plotted points that shows the relationship between two data sets



- **trend –** the general direction in which values in a data set tend to move
- **independent variable –** the variable being changed; graphed on the *x*-axis
- **dependent variable –** the result when the independent variable is changed; graphed on the *y*-axis

Suggested Problems in *Math at Work 12*:

- **pp. 98-99:** #1-6
- **pp. 100-101:** #1-6

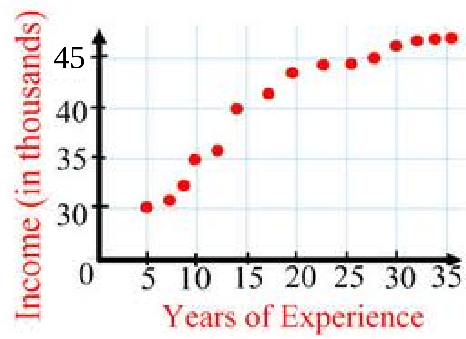
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- If necessary, review with the class how to read points on a coordinate grid.

Possible Assessment Strategies:

- Use the given scatter plot to answer the following questions. It shows the relationship between years of experience and income (in thousands of dollars).



a. What is the approximate income for a person with 15 years of experience?

b. Approximately how many years of experience does a person who earns $42,500 have?

CHAPTER 3
LINEAR RELATIONSHIPS

SUGGESTED TIME

13 classes

Section 3.1 – Understanding Linear Trends and Relationships (pp. 112-126)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

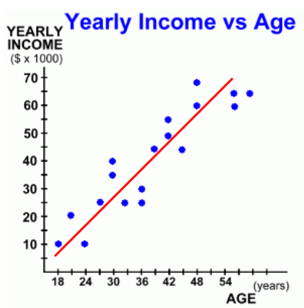
- **A1 (A B E G H I)**

After this lesson, students will be expected to:

- create tables of values and graphs, including scatter plots with lines of best fit
- identify and describe linear trends
- determine whether a relationship is linear or non-linear
- solve problems involving linear trends and relationships

After this lesson, students should understand the following concepts:

- **line of best fit –** a straight line that represents a trend in a scatter plot that follows a linear pattern



- **linear trend –** a trend in which the relationship between two variables follows a linear pattern:
  - ➢ the trend is positive when one variable increases as the other variable also increases
  - ➢ the trend is negative when one variable increases as the other variable decreases
- **linear relationship –** a direct relationship between the *y*-coordinate and the *x*-coordinate; all the points on the graph of a linear relationship lie along a straight line
- **non-linear relationship –** no direct relationship between the *y*-coordinate and the *x*-coordinate; the points on the graph of a non-linear relationship do not lie along a straight line

Suggested Problems in *Math at Work 12*:

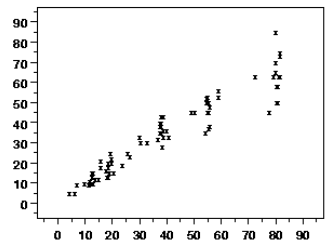
- **pp. 117-119:** #1-5
- **pp. 123-124:** #1-6
- **pp. 125-126:** #1-7

Possible Instructional Strategies:

- Ensure that students are able to interpret different types of linear trends.

Possible Assessment Strategies:

- Determine whether the relationship in the following scatter plot is a linear relationship or a non-linear relationship. Explain your reasoning.



- The table shows the MAT531A and the MAT631A marks for ten randomly chosen students.

MAT531A	MAT631A
78	82
89	87
97	98
66	63
80	81
98	99
75	75
86	84
78	86
52	54

a. Create a scatter plot of the data. What do you notice about the pattern in the points?

b. Describe the trend in the relationship between the variables.

c. Draw the line of best fit. Describe how well the line represents the trend in the relationship.

d. Use the graph to predict the mark of a student in MAT631A who got a mark of 85 in MAT531A.

e. Use the graph to predict the mark of a student in MAT531A who got a mark of 73 in MAT631A.

Section 3.2 – Direct Variation (pp. 127-142)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

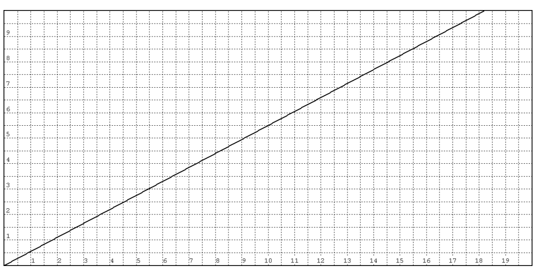
- **A1 (A B C D E F J K L)**

After this lesson, students will be expected to:

- understand why a relationship is a direct variation
- model a direct variation relationship with a table of values, a graph, or an equation
- solve problems involving direct variation relationships

After this lesson, students should understand the following concepts:

- **direct variation –** a linear relationship in which one variable is always a fixed multiple of the other variable, for example, the *y*-value is always 3 times the *x*-value; in the graph of a direct variation relationship, the slope of the line is the fixed multiple, and the *y*-intercept is always zero



- **initial value –** the value of the dependent variable when the independent variable is zero; in a direct variation relationship, the initial value is always zero
- **rate of change –** the amount by which the dependent variable changes when the independent variable increases by 1 unit; in a direct variation relationship, the rate of change is constant

Suggested Problems in *Math at Work 12*:

- **pp. 131-132:** #1-7
- **pp. 135-136:** #1-6
- **pp. 139-140:** #1-9
- **pp. 141-142:** #1-8

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

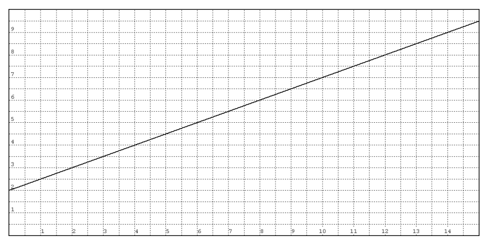
- Ensure that students understand the characteristics of a direct variation relationship.

Possible Assessment Strategies:

- Determine whether each graph represents a direct variation. Explain your reasoning.

  a.

  b.



- Jerry works at a grocery store and earns $12 per hour.
  a. If he works for 7 hours per day, create a table of values to show how much money he has earned after each hour on a given day.
  b. Use this table of values to draw the graph of this relationship. Use the graph to determine whether this relationship is a direct variation.
  c. What is the rate of change in his total earnings?
  d. After how many hours will he earn $156?
- A gas station raises its gas prices from $1.17/L to $1.22/L.
  a. Write an equation to show the relationship between the number of litres bought and the total price in each case.
  b. The gas tank in Beth's car holds 45 L. How much more money will it cost her to fill her tank?

Section 3.3 – Partial Variation (pp. 143-159)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

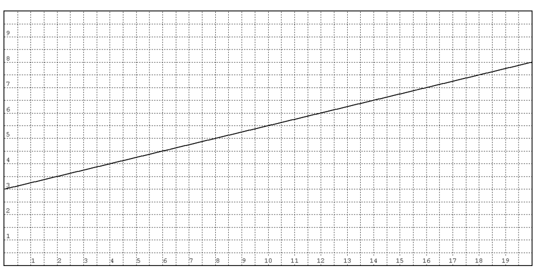
- **A1 (A B C D E F J K L)**

After this lesson, students will be expected to:

- understand why a relationship is a partial variation
- model a partial variation relationship with a table of values, a graph, or an equation
- solve problems involving partial variation relationships
- compare direct and partial variation relationships

After this lesson, students should understand the following concept:

- **partial variation –** one variable in a linear relationship is a fixed multiple of the independent variable plus a constant amount; for example, the *y*-value is always 3 times the *x*-value plus 2; in the graph of a partial variation relationship, the constant amount is the *y*-intercept and the fixed multiple is the slope of the line



Suggested Problems in *Math at Work 12*:

- **pp. 147-148:** #1-7
- **pp. 152-153:** #1-6
- **pp. 156-157:** #1-8
- **pp. 158-159:** #1-8

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students understand the characteristics of a partial variation relationship.

Possible Assessment Strategies:

- Determine whether each table represents a partial variation. Explain your reasoning.

  a.

x	*y*
0	1
1	3
2	5
3	7

  b.

x	*y*
0	0
5	2.5
10	5
15	7.5

- Determine which of the following equations represent a direct variation and which represent a partial variation.

  a. $y = \frac{1}{2}x$

  b. $y = 3x - 2$

  c. $s = 45 - 3t$

  d. $d = \frac{r}{2}$

- Taxi companies in Toronto charge a $5.50 fixed fee plus a variable charge of $0.25 per kilometre.
  a. State the initial value and the rate of change.
  b. Create a table of values for the first 6 km of a taxi ride.
  c. Use the table of values to sketch of graph of this relationship.
  d. What is the total cost of a 20-km ride?

CHAPTER 4
REAL-LIFE DECISIONS

SUGGESTED TIME
12 classes

Section 4.1 – Owning a Vehicle (pp. 170-189)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **N2 (A B)**

After this lesson, students will be expected to:

- explain the difference between buying, leasing, and leasing-to-own a vehicle
- calculate the costs of buying, leasing, and leasing-to-own a vehicle
- identify situations in which leasing or buying a vehicle is the better option

After this lesson, students should understand the following concepts:

- **lease –** a type of financing in which you pay for a vehicle for a specified amount of time; at the end of the term you can return the vehicle to the dealer or buy the vehicle for a previously set price
- **depreciation –** the value that an item loses over time; for example, the average car depreciates about 15% to 20% per year; car depreciation usually slows down after year five
- **lessee –** the customer leasing the vehicle from the car dealership
- **residual value –** the estimated value of the car at the end of the lease; determined by the car dealership when the lease is signed

Suggested Problems in *Math at Work 12*:

- **pp. 175-176:** #1-7
- **pp. 180-182:** #1-8
- **pp. 186-187:** #1-8
- **pp. 188-189:** #1-7

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students are able to explain the advantages and disadvantages of buying, leasing, and leasing-to-own a vehicle.

Possible Assessment Strategies:

- Determine the monthly payment for each of the following cars.

	COST	INTEREST RATE	TERM
a.	$18,000	2.5%	5 years
b.	$10,000	0.9%	3 years
c.	$12,500	1.5%	4 years

- Sarah is buying a new van for $25,000 at a car dealership.
  a. What is the cost of the van, including 14% HST?
  b. The dealership is offering 0.9% financing annually for up to 60 months. Sarah decides to finance the van for 60 months. What will it cost her, in total, for the van?
  c. Sarah has $5000 for a down payment. How much money will she have to finance?
  d. What will Sarah's monthly payment be?
- John wants to lease a new car with a price of $15,500.
  a. His monthly payment is $254.17 plus 14% HST. What will be his total monthly payment
  b. John has to pay a license fee of $155 plus the first month's payment. How much must he pay before he can take the car?
  c. John leases the car for 36 months. At the end of the lease, he decides to buy the car. The interest rate is 3%, and he will take out the loan for 2 years. How much will John pay in total for the car if the residual value of the car is $10,000?
  d. If John had bought the car initially, he could have had an interest rate of 0.9% for 5 years. How much would he have saved if he had bought the car initially?

Section 4.2 – Operating a Vehicle (pp. 190-201)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **N2 (C)**

After this lesson, students will be expected to:

- calculate annual fixed ownership costs of buying or leasing a vehicle
- identify and calculate variable operating costs associated with owning a vehicle
- calculate the average monthly costs of owning and operating a vehicle

After this lesson, students should understand the following concepts:

- **fixed costs –** costs that do not change from month to month; have to be paid regardless of how much the vehicle is used; examples are license fees and insurance
- **extended warranty –** a service contract between the vehicle owner and warranty provider; covers specific maintenance and repairs for the vehicle after the manufacturer's warranty expenses
- **variable costs –** costs that change in amount or in how frequently they are paid; examples are gas, tires, and maintenance; the distance you drive, climate, your driving style, and maintenance affect the variable costs

Suggested Problems in *Math at Work 12*:

- **pp. 194-195:** #1-8
- **p. 199:** #1-6
- **pp. 200-201:** #1-7

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Discuss with the class the various fixed and variable costs that are associated with operating a vehicle.

Possible Assessment Strategies:

- Jane wants to buy a used car that she sees on a dealer's lot. She has $2500 for a down payment. The details are:
  - ➢ payments are $4200 per year for three years
  - ➢ annual insurance premium is $1955
  - ➢ two year/100,000 km extended warranty is available for $1000

  a. Calculate Jane's monthly fixed cost.

  b. If Jane keeps her car for two more years after it is paid off, calculate her monthly fixed cost during this two-year period.

  c. She decides to buy the extended warranty. How much money, in total, will she have paid on the car during the first five years?

- Zack has just bought a car, and expects that he will drive about 3000 km per month. The car's fuel consumption is 5.5 L/100 km. The table shows common maintenance and repair costs:

MAINTENENCE/ REPAIR	SCHEDULE	COST
Change oil and filter	Every 4 months	$35
Rotate tires	Every 4 months	$25
Replace air filter	Every 12 months	$30
Replace windshield wipers	Every 6 months	$40
Replace front brakes	Every 50,000 km	$400
Replace back brakes	Every 100,000 km	$400
Replace tires	Every 4 years	$1200
Replace timing belt	Every 150,000 km	$1050

  a. How much will Zack spend on gas each month? Assume that gas is $1.20/L.

  b. How much will Zack spend on maintenance in the first year?

  c. Zack intends to keep the car for 6 years. How much can he expect to spend over that time in repairs and maintenance?

Section 4.3 – Operating a Small Business (pp. 202-215)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **N3 (A B C D E)**

After this lesson, students will be expected to:

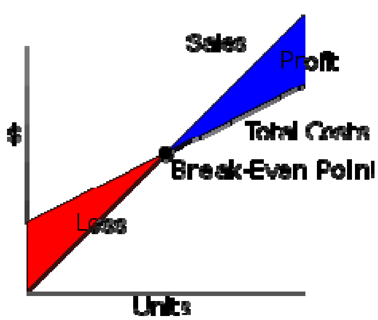
- calculate annual and average monthly revenue and expenses for a business
- determine whether a business has experienced a profit, loss, or break-even point
- identify ways in which business can improve its profitability
- identify some viable small business operations

After this lesson, students should understand the following concepts:

- **expenses –** the money spent running a business; examples are wages, advertising, and rent
- **revenue –** income from normal business activities, usually the sale of goods and/or services
- **profit –** when a company's revenues are more than its expenses
- **loss –** when a company's expenses are more than its revenues
- **net income –** a company's total profit or loss after subtracting expenses from revenue

$$\text{Net income} = \text{Revenue} - \text{Expenses}$$

- **break-even point –** when expenses are revenue are equal



Suggested Problems in *Math at Work 12*:

- **pp. 207-208:** #1-6
- **pp. 212-214:** #1-10
- **pp. 214-215:** #1-8

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Have a local business person present to the class, highlighting the different aspects of operating a small business.

Possible Assessment Strategies:

- Ralph is a painter. He is self-employed and runs a small painting business. He works for 10 months and goes on vacation in October and November. Last year's revenue and expenses for his business are shown in the table.

MONTH	REVENUE	EXPENSES
January	$3000	$250
February	$5250	$1000
March	$1250	$1250
April	$3250	$850
May	$8000	$750
June	$7500	$850
July	$12,000	$3000
August	$12,000	$2000
September	$2000	$750
October	$0	$0
November	$0	$0
December	$10,000	$2250

a. During which month did his business show the greatest profit? How much was the profit?

b. Did his business show an overall loss or profit for the year? How much?

c. In June, Zack spent $350 on supplies. What percent of his expenses in June were spent on supplies? Round off the answer to one decimal place.

CHAPTER 5
PROPERTIES OF FIGURES

SUGGESTED TIME

9 classes

Section 5.1 – Angle Properties of Polygons (pp. 226-237)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G2 (A B C D E F)**

After this lesson, students will be expected to:

- determine the sum and measure of the angles in a polygon
- understand the types of angles in a polygon
- explore tessellations of polygons

After this lesson, students should understand the following concepts:

- **regular polygon –** a closed figure with three or more straight sides and equal side and angle measurements
- **equilateral triangle –** a triangle with equal side lengths
- **diagonal –** a line segment connecting two non-adjacent vertices in a polygon
- **isosceles triangle –** a triangle with two equal side lengths
- **obtuse angle –** an angle that is greater than 90^0 and less than 180^0
- **acute angle –** an angle that is greater than 0^0 and less than 90^0
- **isosceles trapezoid –** a four-sided figure with one set of sides parallel and the other set of sides equal in length
- **parallelogram –** a four-sided figure with two pairs of parallel sides
- **scalene triangle –** a triangle with no equal sides
- **tessellate –** to cover an area using the repetition of geometric shapes, with no overlaps and no gaps

Suggested Problems in *Math at Work 12*:

- **pp. 230-232:** #1-10
- **pp. 235-236:** #1-6
- **pp. 236-237:** #1-9

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

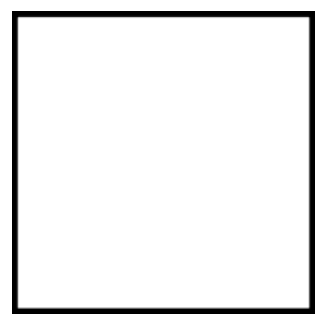
Possible Instructional Strategies:

- Review with the class the terms that are highlighted in this section, as much of this material has not been seen by the students since grade nine.

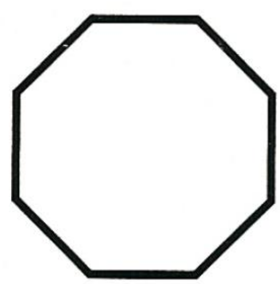
Possible Assessment Strategies:

- What is the measure for each interior angle in a regular pentagon?
- What is the sum of the interior angles of a hexagon?
- Determine if each of the following shapes will tessellate.

  a.



  b .



- Identify each of the following shapes. Be as specific as possible.
  - a. It is a four-sided figure with four right angles, and having all sides of equal length.
  - b. It is a three-sided figure having three angles with different measures.

Section 5.2 – Side Lengths and Diagonal Properties of Polygons (pp. 238-247)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G2 (A B C D E F)**

After this lesson, students will be expected to:

- determine the properties of quadrilaterals related to side lengths and diagonals
- determine the angle properties of quadrilaterals and triangles
- determine the properties of triangles related to side lengths
- relate the side lengths and angle measures of triangles

Suggested Problems in *Math at Work 12*:

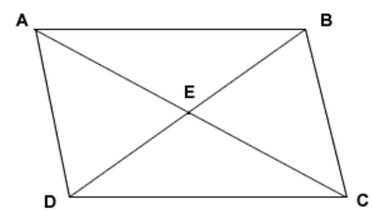
- **pp. 241-243:** #1-9
- **pp. 245-246:** #1-6
- **pp. 246-247:** #1-8

Possible Instructional Strategies:

- Ensure that students see the connection between the side lengths and the diagonals of a quadrilateral.
- Ensure that students see the connection between the side lengths and the angle measures of a triangle.

Possible Assessment Strategies:

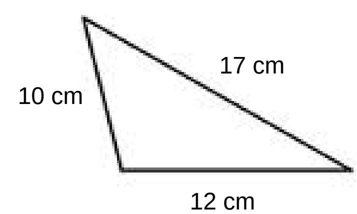
- Determine the missing measurements.



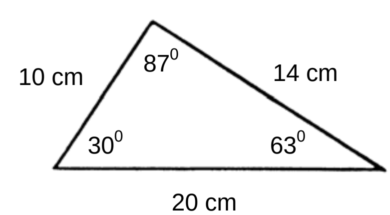
$AD = 4$	$AB = 6$
$BD = 5.2$	$AC = 8.8$
$AE = \square$	$BC = \square$
$BE = \square$	$CD = \square$
$CE = \square$	$DE = \square$

- Determine whether each of the following triangles is possible.

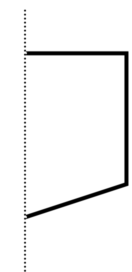
  a.



  b.



Section 5.3 – Symmetry (pp. 248-257)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G2 (A B C D E F)** **After this lesson, students will be expected to:** • relate the property of symmetry to the classification of triangles, quadrilaterals, and other polygons • determine which polygons have lines of symmetry • determine the number of lines of symmetry in polygons **After this lesson, students should understand the following concepts:** • **line symmetry –** a type of symmetry in which an image or object can be divided into two identical halves by a line of symmetry • **line of symmetry –** a line that divides a figure into two identical halves; sometimes called a line of reflection or axis of symmetry **Suggested Problems in *Math at Work 12*:** • **pp. 251-252:** #1-6 • **pp. 255-256:** #1-7 • **p. 257:** #1-7	**Possible Instructional Strategies:** • Use materials such as a mirror or tracing paper to help show the concept of symmetry. **Possible Assessment Strategies:** • The following diagram shows one half of a polygon. The dotted line is a line of symmetry. Draw and identify the type of polygon.  • Which type of regular polygon has 6 lines of symmetry? • Draw each of the following, if possible. a. a quadrilateral with exactly one line of symmetry b a quadrilateral with exactly two lines of symmetry c. a quadrilateral with exactly three lines of symmetry d. a quadrilateral with exactly four lines of symmetry

CHAPTER 6

TRANSFORMATIONS

SUGGESTED TIME

13 classes

Section 6.1 – Dilations (pp. 268-281)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G3 (A B C E G H I)**

After this lesson, students will be expected to:

- identify and draw a dilation of a 2-D shape and a 3-D object
- use similarity to determine and explain dilations

After this lesson, students should understand the following concepts:

- **transformation –** a change in a figure that results in a new position or size; for example, dilations, translations, reflections and rotations
- **image –** the final shape and/or position of a figure after transformation
- **dilation –** a transformation in which a figure is enlarged or reduced by a constant factor
- **scale factor –** the constant factor by which all dimensions of an object are enlarged or reduced; for example, if the dimensions of a rectangle are multiplied by 3, then the scale factor is 3

Suggested Problems in *Math at Work 12*:

- **pp. 272-274:** #1-8
- **pp. 278-279:** #1-7
- **pp. 280-281:** #1-6

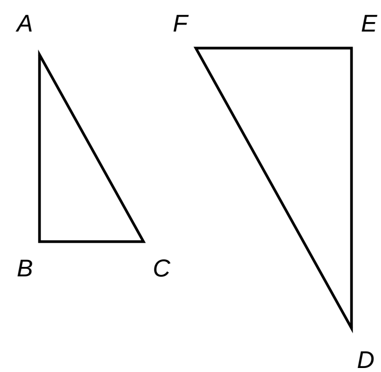
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

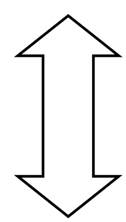
- Students have previously studied dilations in grade nine. Ensure that they understand that a dilation can either be an expansion or a reduction of a figure.

Possible Assessment Strategies:

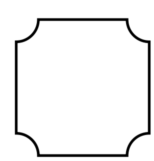
- List the corresponding angles and corresponding sides for $\triangle ABC$ and $\triangle DEF$.



- Enlarge the size of the figure by a scale factor of 3.



- Reduce the size of the figure by a scale factor of $\frac{1}{2}$.



Section 6.2 – Translations (pp. 282-291)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G3 (A B C D E I)**

After this lesson, students will be expected to:

- identify and draw a vertical or horizontal translation
- draw successive translations
- create and analyse designs made with tessellations

After this lesson, students should understand the following concepts:

- **translation –** a transformation that slides an object in a straight line without changing its size or orientation
- **successive translation –** a pattern created by translating a figure multiple times using the same translation

Suggested Problems in *Math at Work 12*:

- **pp. 285-286:** #1-8
- **pp. 289-290:** #1-7
- **p. 291:** #1-4

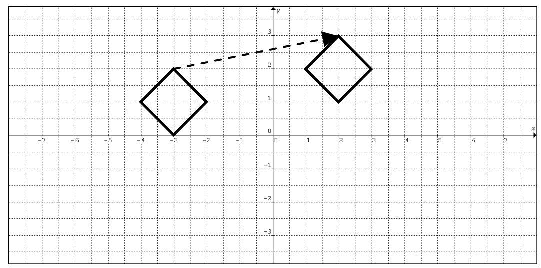
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Students have previously studied translations in grade seven. Ensure that they understand that a translation does not change the size or the orientation of a figure.

Possible Assessment Strategies:

- Describe the translation shown in the diagram.



- Point *A* is at $(1,2)$ on a coordinate grid. What are the coordinates of point *A* after it is translated
  a. 4 units left
  b. 6 units up
  c. 7 units right and 2 units down
  d. 3 units left and 9 units down
- Plot the point $(1,1)$ on a coordinate grid. If this point is translated 2 units left and 4 units up three successive times, what will be the new coordinates of the point?

Section 6.3 – Reflections (pp. 292-301)

<table>
<tr><th>ELABORATIONS &
SUGGESTED PROBLEMS</th><th>POSSIBLE INSTRUCTIONAL &
ASSESSMENT STRATEGIES</th></tr>
<tr><td>

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G3 (A B C D E F I)**

After this lesson, students will be expected to:

- identify a reflection and draw the image of a shape that is reflected
- draw shapes that are reflected and translated
- create and analyse designs made with reflections

After this lesson, students should understand the following concepts:

- **reflection –** a transformation in which an object is shown as its mirror image over a line of reflection
- **line of reflection –** a line that an object is reflected over; the corresponding points on both sides of the line are the same distance away from the line

Suggested Problems in *Math at Work 12*:

- **pp. 295-296:** #1-7
- **pp. 299-300:** #1-8
- **p. 301:** #1-5

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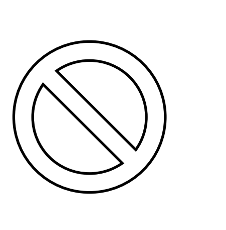
Possible Instructional Strategies:

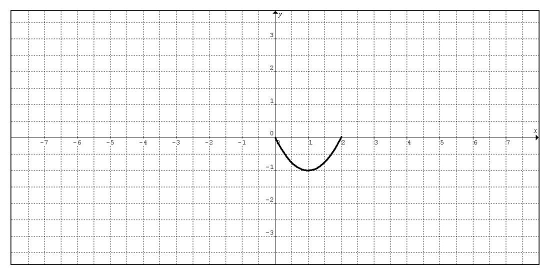
- Students have previously studied reflections in grade seven. Ensure that they understand that a translation does not change the size but may change the orientation of a figure.

Possible Assessment Strategies:

- Reflect the given shape across the given line of reflection.



- Reflect the given shape across the line of reflection at $x = -1$.



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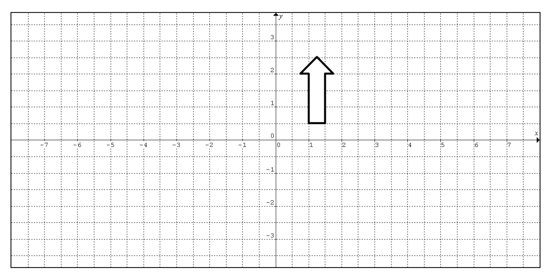
Section 6.4 – Rotations (pp. 302-315)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G3 (A B C D E I)** **After this lesson, students will be expected to:** • identify a rotation and draw the image of a shape that is rotated • draw shapes that are rotated, reflected, and translated • create and analyse designs made with rotations **After this lesson, students should understand the following concepts:** • **rotation –** a transformation that moves an object around a fixed point that is called the centre of rotation • **centre of rotation –** the point about which an object is rotated **Suggested Problems in *Math at Work 12*:** • **pp. 308-310:** #1-9 • **pp. 313-315:** #1-10 • **p. 315:** #1-4	**Possible Instructional Strategies:** • Students have previously studied rotations in grade seven. Ensure that they understand that a rotation does not change the size or the orientation of a figure. **Possible Assessment Strategies:** • Rotate each of the following points according to the instructions. a. $(4,6)$; 90^0 counterclockwise about the origin b. $(3,-1)$; 180^0 about the origin c. $(-1,-3)$; 90^0 clockwise about the center of rotation $(0,1)$ • Rotate the given shape 90^0 clockwise about the origin.  • On a coordinate grid, transform each point as indicated. a. $(3,1)$; rotate 90^0 counterclockwise about the origin, and reflect over the *x*-axis b. $(2,-1)$; rotate 180^0 about the origin, reflect over the line $x=1$, and translate 2 units down

CHAPTER 7
TRIGONOMETRY

SUGGESTED TIME
10 classes

Section 7.1 – The Sine Law (pp. 326-337)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G1 (A B)**

After this lesson, students will be expected to:

- recognize and describe an oblique triangle
- apply the sine law
- explain how to use the sine law
- describe how the sine law is used in problem situations

After this lesson, students should understand the following concepts:

- **oblique triangle –** a triangle that does not contain a right triangle; it can be acute or obtuse
- **acute triangle –** a triangle with three acute interior angles (between 0^0 and 90^0)
- **obtuse triangle –** a triangle with one obtuse interior angle (between 90^0 and 180^0)
- **sine law –** the sides of a triangle are proportional to the sines of the opposite angles

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Suggested Problems in *Math at Work 12*:

- **pp. 330-331:** #1-7
- **pp. 334-335:** #1-8
- **pp. 336-337:** #1-7

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

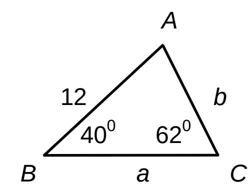
Possible Instructional Strategies:

- Ensure that students are able to recognize when the sine law can be used to solve a triangle.

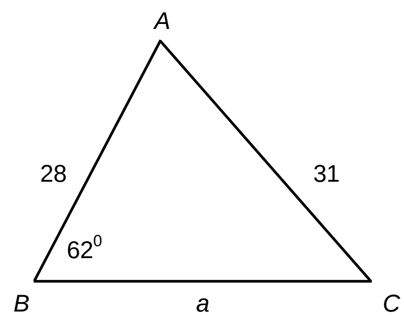
Possible Assessment Strategies:

- Solve each of the following triangles. Round off all answers to one decimal place.

  a.

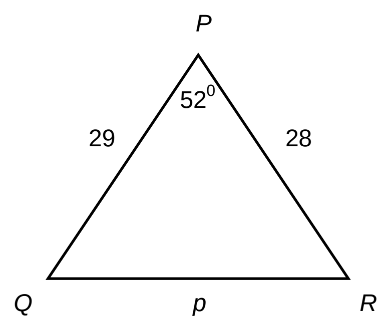
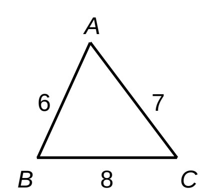


  b.



- In ΔABC, $\angle A = 57^0$, $\angle B = 73^0$, and $c = 24$. Solve the triangle. Round off all answers to one decimal place.
- John wants to measure the length of the trunk of a tree. He walks exactly 35 m from the base of the tree and looks up. The angle from the ground to the top of the tree is 33^0. This particular tree grows at an angle of 83^0 with respect to the ground rather than vertically. What is the length of the trunk of the tree? Round off the answer to one decimal place.
- A chandelier is suspended from a horizontal beam by two support chains. One of the chains is 3.6 m long and forms an angle of 62^0 with the beam. The second chain is 4.8 m long. What angle does the second chain make with the beam? Round off the answer to one decimal place.

Section 7.2 – The Cosine Law (pp. 338-351)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G1 (A B)** **After this lesson, students will be expected to:** • apply the cosine law • explain how to use the cosine law • describe how the cosine law is used in problem situations **After this lesson, students should understand the following concept:** • **cosine law –** a law that relates all three side lengths of a triangle with the cosine of one of the angles $c^2 = a^2 + b^2 - 2ab\ \cos\ C$ $b^2 = a^2 + c^2 - 2ac\ \cos\ B$ $a^2 = b^2 + c^2 - 2bc\ \cos\ A$ **Suggested Problems in *Math at Work 12*:** • **pp. 342-344:** #1-9 • **pp. 347-348:** #1-7 • **pp. 349-351:** #1-11	**Possible Instructional Strategies:** • Ensure that students are able to recognize when the cosine law can be used to solve a triangle. • Students should be familiar with all three versions of the cosine law. **Possible Assessment Strategies:** • Solve each of the following triangles. Round off all answers to one decimal place. a.  b.  • In ΔABC, $\angle C = 85^0$, $a = 22$, and $b = 20$. Solve the triangle. Round off all answers to one decimal place. • A surveyor needs to find the length of a swampy area near a lake. The surveyor sets up her transit at point *A*. She measures the distance to one end of the swamp as 468.2 m, the distance to the other end of the swamp as 692.6 m, and the angle of sight between the two ends of the swamp as 78.6^0. Determine the length of the swampy area, to the nearest tenth of a metre. • A triangular lawn has side lengths of 25 m, 20 m, and 22 m. Determine the measure of the smallest angle formed by two of the sides, to the nearest tenth of a degree.

Section 7.3 – Solving Trigonometric Problems (pp. 352-359)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G1 (A B)**

After this lesson, students will be expected to:

- decide whether to use the sine law or the cosine law to solve a problem
- solve a problem using the sine law or the cosine law

Suggested Problems in *Math at Work 12*:

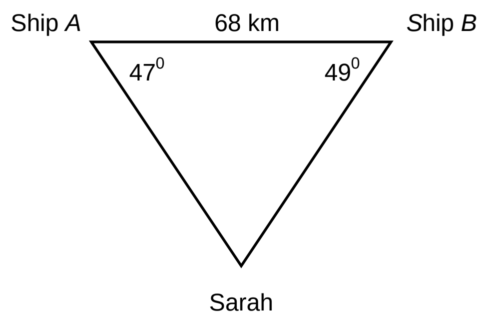
- **pp. 355-357:** #1-5
- **pp. 358-359:** #1-8

Possible Instructional Strategies:

- Ensure that students know when it is appropriate to use the primary trigonometric ratios, the sine law, and the cosine law.

Possible Assessment Strategies:

- Sarah runs a deep-sea fishing charter. On one of her expeditions, she has travelled 40 km from her port when engine trouble occurs. There are two Search and Rescue ships, as shown below.



  Which ship is closer to Sarah? How far is that ship from Sarah? Round off the answer to one decimal place.

- Two boats leave a dock at the same time. Each travels in a straight line but in different directions. The angle between their courses measures 54^0. One boat travels at 48 km/h and the other travels at 54 km/h. How far apart are the two boats after 4 hours? Round off the answer to one decimal place.

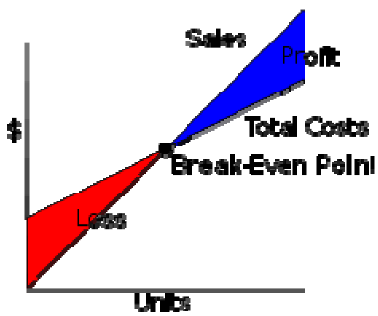
GLOSSARY OF MATHEMATICAL TERMS

A

- **accuracy –** the degree to which a measurement is measured and reported correctly
- **acute angle –** an angle that is greater than 0^0 and less than 90^0
- **acute triangle –** a triangle with three acute interior angles (between 0^0 and 90^0)

B

- **break-even point –** when expenses are revenue are equal



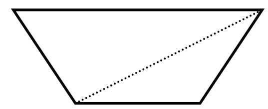
C

- **centre of rotation –** the point about which an object is rotated
- **cosine law –** a law that relates all three side lengths of a triangle with the cosine of one of the angles

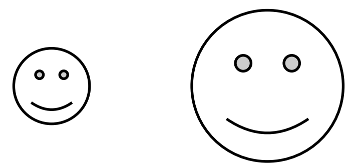
$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$

D

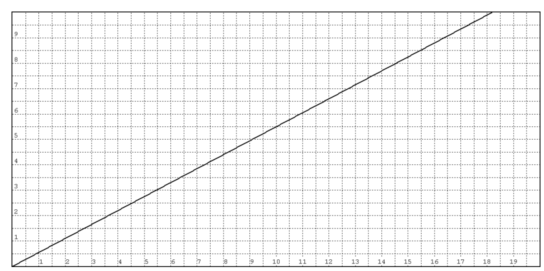
- **data set –** a collection of related information
- **dependent variable –** the result when the independent variable is changed; graphed on the *y*-axis
- **depreciation –** the value that an item loses over time; for example, the average car depreciates about 15% to 20% per year; car depreciation usually slows down after year five
- **diagonal –** a line segment connecting two non-adjacent vertices in a polygon



- **dilation –** a transformation in which a figure is enlarged or reduced by a constant factor



- **direct variation –** a linear relationship in which one variable is always a fixed multiple of the other variable, for example, the *y*-value is always 3 times the *x*-value; in the graph of a direct variation relationship, the slope of the line is the fixed multiple, and the *y*-intercept is always zero



E

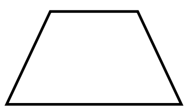
- **equilateral triangle –** a triangle with equal side lengths
- **expenses –** the money spent running a business; examples are wages, advertising, and rent
- **experimental probability –** a ratio that compares the number of times an event occurs to the total number of trials or events; determined by experiment
- **extended warranty –** a service contract between the vehicle owner and warranty provider; covers specific maintenance and repairs for the vehicle after the manufacturer's warranty expenses

F

- **fixed costs –** costs that do not change from month to month; have to be paid regardless of how much the vehicle is used; examples are license fees and insurance

I

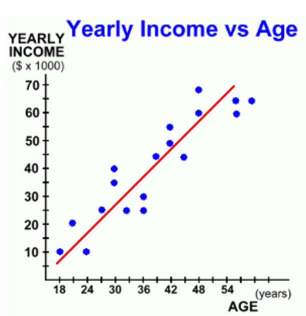
- **image –** the final shape and/or position of a figure after transformation
- **independent variable –** the variable being changed; graphed on the *x*-axis
- **initial value –** the value of the dependent variable when the independent variable is zero; in a direct variation relationship, the initial value is always zero
- **isosceles trapezoid –** a four-sided figure with one set of sides parallel and the other set of sides equal in length



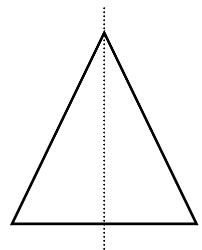
- **isosceles triangle –** a triangle with two equal side lengths

L

- **lease –** a type of financing in which you pay for a vehicle for a specified amount of time; at the end of the term you can return the vehicle to the dealer or buy the vehicle for a previously set price
- **lessee –** the customer leasing the vehicle from the car dealership
- **line of best fit –** a straight line that represents a trend in a scatter plot that follows a linear pattern



- **line of reflection –** a line that an object is reflected over; the corresponding points on both sides of the line are the same distance away from the line
- **line of symmetry –** a line that divides a figure into two identical halves; sometimes called a line of reflection or axis of symmetry
- **line symmetry –** a type of symmetry in which an image or object can be divided into two identical halves by a line of symmetry



- **linear relationship –** a direct relationship between the *y*-coordinate and the *x*-coordinate; all the points on the graph of a linear relationship lie along a straight line
- **linear trend –** a trend in which the relationship between two variables follows a linear pattern:
  - ➢ the trend is positive when one variable increases as the other variable also increases
  - ➢ the trend is negative when one variable increases as the other variable decreases
- **loss –** when a company's expenses are more than its revenues

M

- **mean –** the average of the data values; add the data values and divide by the total number of data values; for example, the mean of 4, 7, 9, 10, 11, 16 is $\frac{4+7+9+10+11+16}{6}=9.5$
- **measure of central tendency –** a value that represents the centre of a set of data; can be the mean, median, or mode
- **median –** the middle number in a set of data after the data have been arranged in order; for example, the median of 2, 4, 6, 8, 11 is 6; when there is an even number of data, average the two middle numbers to find the median; for example, the median of 1, 5, 9, 13, 16, 20 is 11, because $\frac{9+13}{2}=11$

- **mode –** the number(s) the occur(s) most frequently in a set of data; a data set can have no mode, one mode, or more than one mode

N

- **net income –** a company's total profit or loss after subtracting expenses from revenue

  $$\text{Net income} = \text{Revenue} - \text{Expenses}$$

- **non-linear relationship –** no direct relationship between the *y*-coordinate and the *x*-coordinate; the points on the graph of a non-linear relationship do not lie along a straight line

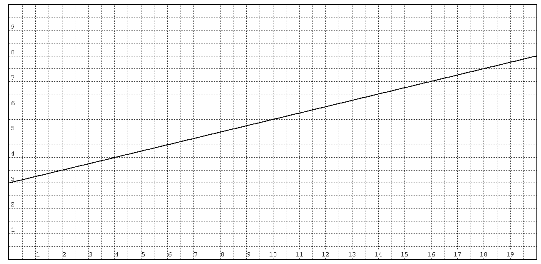
O

- **oblique triangle –** a triangle that does not contain a right triangle; it can be acute or obtuse
- **obtuse angle –** an angle that is greater than 90^0 and less than 180^0
- **obtuse triangle –** a triangle with one obtuse interior angle (between 90^0 and 180^0)
- **odds –** a ratio that compares the number of possible successful outcomes to the number of possible unsuccessful outcomes
- **outlier –** a value that is much smaller or larger than the other data values; a data set may have no outliers, one outlier, or more than one outlier

P

- **parallelogram –** a four-sided figure with two pairs of parallel sides

- **partial variation –** one variable in a linear relationship is a fixed multiple of the independent variable plus a constant amount; for example, the *y*-value is always 3 times the *x*-value plus 2; in the graph of a partial variation relationship, the constant amount is the *y*-intercept and the fixed multiple is the slope of the line

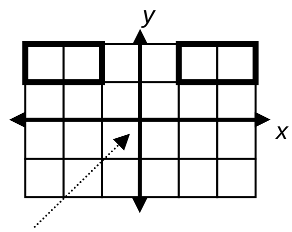


- **percentile –** a value below which a certain percent of the data set falls; the median is also called the 50th percentile, because 50% of the values in the data set are below the median value
- **percentile rank –** a number between 0 and 100 that indicates the percent of cases that fall at or below that score
- **precision –** the degree of exactness to which a measurement is expressed; the precision of a measurement depends on the scale of the instrument used
- **probability –** the mathematical likelihood of something happening; a ratio that compares the number of possible successful outcomes to the total number of possible outcomes
- **profit –** when a company's revenues are more than its expenses

R

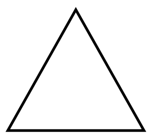
- **range –** the difference between the largest value and the smallest value of a data set
- **rate of change –** the amount by which the dependent variable changes when the independent variable increases by 1 unit; in a direct variation relationship, the rate of change is constant

- **reflection –** a transformation in which an object is shown as its mirror image over a line of reflection

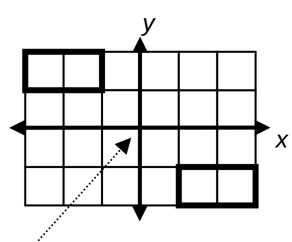


line of reflection

- **regular polygon –** a closed figure with three or more straight sides and equal side and angle measurements



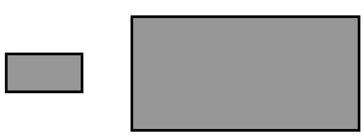
- **residual value –** the estimated value of the car at the end of the lease; determined by the car dealership when the lease is signed
- **revenue –** income from normal business activities, usually the sale of goods and/or services
- **rotation –** a transformation that moves an object around a fixed point that is called the centre of rotation



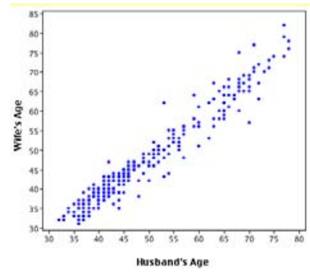
centre of rotation

S

- **scale factor –** the constant factor by which all dimensions of an object are enlarged or reduced; for example, if the dimensions of a rectangle are multiplied by 3, then the scale factor is 3



- **scalene triangle –** a triangle with no equal sides
- **scatter plot –** a graph of plotted points that shows the relationship between two data sets



- **sine law –** the sides of a triangle are proportional to the sines of the opposite angles

$$\frac{a}{\sin\ A} = \frac{b}{\sin\ B} = \frac{c}{\sin\ C}$$

- **stem-and-leaf plot –** a way to organize numerical data in order of place value; the “tens digit and greater” is the stem and the “ones digit” is the leaf; to plot decimal numbers, the “ones digit and greater” is the stem and the “tenths and less” is the leaf

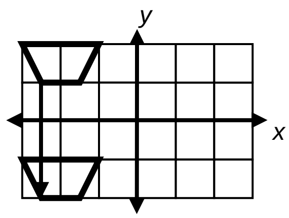
stem	leaf
9	0 4
8	3 4 5 7 9
7	0 2 2 9
6	5 8 9
5	3

- **successive translation –** a pattern created by translating a figure multiple times using the same translation

T

- **tessellate –** to cover an area using the repetition of geometric shapes, with no overlaps and no gaps
- **theoretical probability –** a ratio that compares the number of possible successful outcomes to the total number of possible outcomes; determined by reason or calculation
- **tolerance –** the total amount that a measurement is allowed to vary
- **transformation –** a change in a figure that results in a new position or size; for example, dilations, translations, reflections and rotations

- **translation –** a transformation that slides an object in a straight line without changing its size or orientation



- **tree diagram –** a type of organizer for displaying outcomes of an event; each branch represents a different possible outcome
- **trend –** the general direction in which values in a data set tend to move
- **trimmed mean –** a calculation of the mean found removing the highest and lowest values; you must remove the same number of values from the top and bottom of the data set; removing outliers can result in a more accurate mean

V

- **variable costs –** costs that change in amount or in how frequently they are paid; examples are gas, tires, and maintenance; the distance you drive, climate, your driving style, and maintenance affect the variable costs

W

- **weighted mean –** the average of mean of a data set in which each data point does not contribute an equal amount to the final average

SOLUTIONS TO POSSIBLE ASSESSMENT STRATEGIES

SECTION 1.1

- a. $33\frac{7}{8}''$, $34\frac{1}{8}''$

  b. 2.995 kg, 3.005 kg
- $18\frac{3}{4}$ ft^3
- a. $553\frac{3}{4}$ ft^2

  b. Vertical pieces: $78\frac{1}{8}$ inches each;

  Horizontal piece: $24\frac{1}{4}$ inches
- 6.91 g, 6.93 g

SECTION 1.2

- a. $\frac{4}{13}$

  b. $\frac{1}{4}$

  c. $\frac{1}{2}$
- a. $\frac{1}{3}$

  b. $\frac{2}{9}$

  c. $\frac{5}{9}$
- a. $\frac{1}{216}$

  b. $1:215$

SECTION 1.3

- $\frac{1}{2}$
- a. $\frac{1}{6}$

  b. $\frac{1}{6}$

  c. $\frac{1}{9}$
- a. 20%

  b. 80%

  c. $1:4$

  d. If he enters the contest 10 times, John would expect to win 2 times, but there is no guarantee that he will win at least once.
- $\frac{1}{101}$

SECTION 1.4

- a. $\frac{1}{2}$

  b. $\frac{1}{36}$

  c. $\frac{1}{2}$
- a. 15 tails

  b. 15 fours
- a. 0.125, 12.5%

  b. 0.231, 23.1%
- a. $\frac{9}{40}$

  b. $\frac{2}{3}$
- a. 11.1%

  b. 86 students
- The probability of winning Contest A is 20% and the probability of winning Contest B is approximately 18.2%, so Contest A offers the better chance of winning.

SECTION 2.1

- $n = 75$
- mean: 7.7; median: 8; mode: 7
- 75
- The mean, median and mode all increased by 5%.
- 86%

SECTION 2.2

- 11 birds
- a. 54

b. 27

- a.

0	7 8 9
1	2 3 5 6 7 7 9 9 9 9
2	0 1 1 2 3 3 6

b. 19 baskets

c. 70th percentile

d. 22 baskets or more

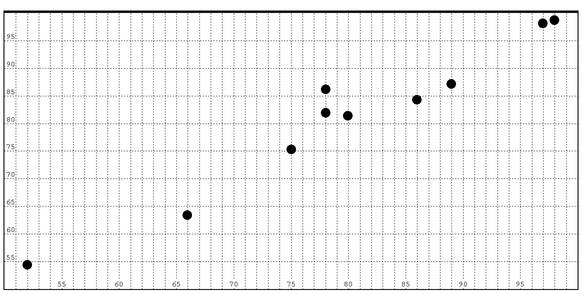
SECTION 2.3

- a. Answers may vary slightly. The approximate answer is $38,000.

  b. Answers may vary slightly. The approximate answer is 24 years.

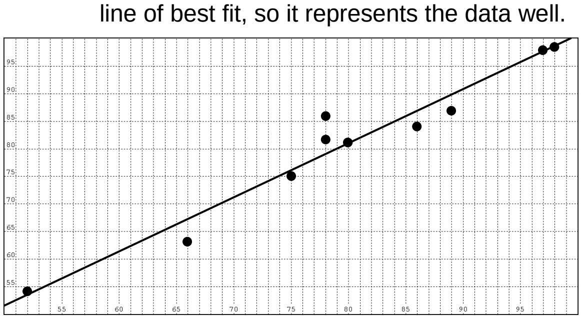
SECTION 3.1

- The scatter plot generally shows a positive linear relationship, as the points show a trend where one variable increases as the other variable increases.
- a. The points generally follow a linear relationship.



  b. The linear relationship is generally positive.

  c. The points are closely scattered around the line of best fit, so it represents the data well.



  d. Answers may vary slightly. The approximate answer is 86.

  e. Answers may vary slightly. The approximate answer is 72.

SECTION 3.2

- a. direct variation; the *y*-intercept of the line is zero

  b. not a direct variation; the *y*-intercept of the line is not zero
- a.

TIME (h)	WAGES ($)
1	12
2	24
3	36
4	48
5	60
6	72
7	84

  b. direct variation

  c. $12/h

  d. 13 h
- a. $y = 1.17x$; $y = 1.22x$

  b. $2.25

SECTION 3.3

- a. partial variation; the *y*-intercept is not zero

  b. not a partial variation; the *y*-intercept is zero
- a. direct variation

  b. partial variation

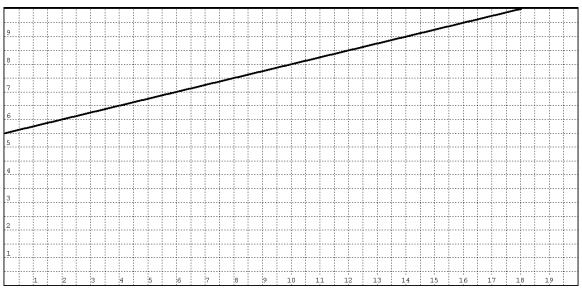
  c. partial variation

  d. direct variation
- a. initial value: $5.50; rate of change: $0.25/km

b.

DISTANCE (km)	COST ($)
0	5.50
1	5.75
2	6.00
3	6.25
4	6.50
5	6.75
6	7.00

c.



d. $10.50

SECTION 4.1

- a. $339.42
  b. $285.35
  c. $276.40
- a. $28,500
  b. $29,805.79
  c. $24,805.79
  d. $413.43
- a. $289.75
  b. $444.75
  c. $21,195.14
  d. $4984.97

SECTION 4.2

- a. $512.92
  b. $162.92
  c. $25,875
- a. $198
  b. $290
  c. $6390

SECTION 4.3

- a. August; $10,000
  b. Profit; $51,300
  c. 41.2%

SECTION 5.1

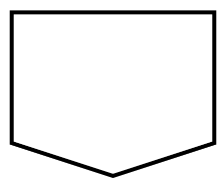
- 108^0
- 720^0
- a. yes
  b. no
- a. square
  b. scalene triangle

SECTION 5.2

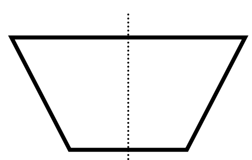
- $AE = 4.4$; $BC = 4$; $BE = 2.6$; $CD = 6$; $CE = 4.4$; $DE = 2.6$
- a. yes
  b. no

SECTION 5.3

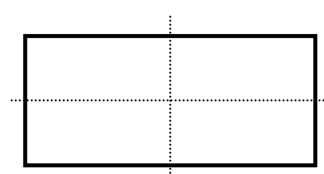
- pentagon



- regular hexagon
- a. Answers may vary. Here is one possible answer.

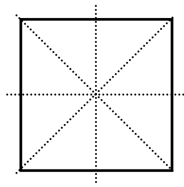


  b. Answers may vary. Here is one possible answer.



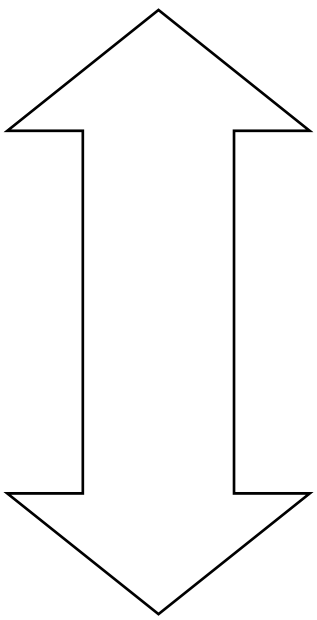
c. Not possible

d. Answers may vary. Here is one possible answer.



SECTION 6.1

- Angles: $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$; Sides: *AB* and *DE*, *BC* and *EF*, *AC* and *DF*
-



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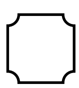


SECTION 6.2

- 5 units right and 1 unit up
- a. $(-3, 2)$

  b. $(1, 8)$

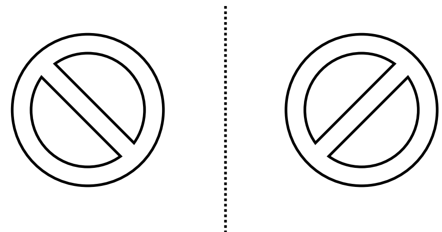
  c. $(8, 0)$

  d. $(-2, -7)$

- $(-5, 13)$

SECTION 6.3

-



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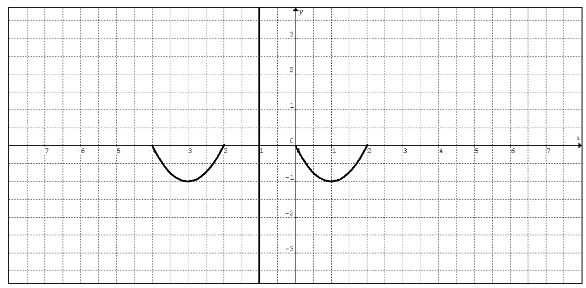


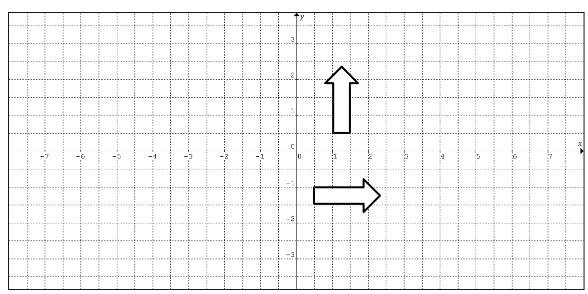
SECTION 6.4

- a. $(-6, 4)$

  b. $(-3, 1)$

  c. $(-4, 2)$

-



- a. $(-1, -3)$

  b. $(5, -1)$

SECTION 7.1

- a. $\angle A = 78^0$, $a = 13.3$, $b = 8.7$

  b. $\angle A = 65.1^0$, $\angle C = 52.9^0$, $a = 31.8$

- $\angle C = 50^0$, $a = 26.3$, $b = 30.0$
- 21.2 m

- 41.5^0

SECTION 7.2

- a. $\angle Q = 61.9^0,\ \angle R = 66.1^0,\ p = 25.0$

  b. $\angle A = 75.5^0,\ \angle B = 57.9^0,\ \angle C = 46.6^0$
- $\angle A = 50.5^0,\ \angle B = 44.5^0,\ c = 28.4$
- 755.5 m
- 49.9^0

SECTION 7.3

- Sarah is closer to Ship B, which is 50.0 km away.
- 186.5 km

APPENDIX

MATHEMATICS RESEARCH PROJECT

➢ Introduction

A research project can be a very important part of a mathematics education. Besides the greatly increased learning intensity that comes from personal involvement with a project, and the chance to show universities, colleges, and potential employers the ability to initiate and carry out a complex task, it gives the student an introduction to mathematics as it is – a living and developing intellectual discipline where progress is achieved by the interplay of individual creativity and collective knowledge. A major research project must successfully pass through several stages. Over the next few pages, these stages are highlighted, together with some ideas that students may find useful when working through such a project.

➢ Creating an Action Plan

As previously mentioned, a major research project must successfully pass though several stages. The following is an outline for an action plan, with a list of these stages, a suggested time, and space for students to include a probable time to complete each stage.

STAGE	SUGGESTED TIME	PROBABLE TIME
Select the topic to explore.	1 – 3 days	
Create the research question to be answered.	1 – 3 days	
Collect the data.	5 – 10 days	
Analyse the data.	5 – 10 days	
Create an outline for the presentation.	2 – 4 days	
Prepare a first draft.	3 – 10 days	
Revise, edit and proofread.	3 – 5 days	
Prepare and practise the presentation.	3 – 5 days	

Completing this action plan will help students organize their time and give them goals and deadlines that they can manage. The times that are suggested for each stage are only a guide. Students can adjust the time that they will spend on each stage to match the scope of their projects. For example, a project based on primary data (data that they collect) will usually require more time than a project based on secondary data (data that other people have collected and published). A student will also need to consider his or her personal situation – the issues that each student deals with that may interfere with the completion of his or her project. Examples of these issues may include:

- a part-time job;
- after-school sports and activities;
- regular homework;
- assignments for other courses;
- tests in other courses;
- time they spend with friends;
- family commitments;
- access to research sources and technology.

➢ Selecting the Research Topic

To decide what to research, a student can start by thinking about a subject and then consider specific topics. Some examples of subjects and topics may be:

SUBJECT	TOPIC
Entertainment	• effects of new electronic devices • file sharing
Health care	• doctor and/or nurse shortages • funding
Post-secondary education	• entry requirements • graduate success
History of Western and Northern Canada	• relations among First Nations • immigration

It is important to take the time to consider several topics carefully before selecting a topic for a research project. The following questions will help a student determine if a topic that is being considered is suitable.

- **Does the topic interest the student?**
  Students will be more successful if they choose a topic that interests them. They will be more motivated to do research, and they will be more attentive while doing the research. As well, they will care more about the conclusions they make.

- **Is the topic practical to research?**
  If a student decides to use first-hand data, can the data be generated in the time available, with the resources available? If a student decides to use second-hand data, are there multiple sources of data? Are the sources reliable, and can they be accessed in a timely manner?

- **Is there an important issue related to the topic?**
  A student should think about the issues related to the topic that he or she has chosen. If there are many viewpoints on an issue, they may be able to gather data that support some viewpoints but not others. The data they collect from all viewpoints should enable them to come to a reasoned conclusion.

- **Will the audience appreciate the presentation?**
  The topic should be interesting to the intended audience. Students should avoid topics that may offend some members of their audience.

➢ Creating the Research Question or Statement

A well-written research question or statement clarifies exactly what the project is designed to do. It should have the following characteristics:

- The research topic is easily identifiable.

- The purpose of the research is clear.
- The question or statement is focused. The people who are listening to or reading the question or statement will know what the student is going to be researching.

A good question or statement requires thought and planning. Below are three examples of initial questions or statements and how they were improved.

UNACCEPTABLE QUESTION OR STATEMENT	WHY?	ACCEPTABLE QUESTION OR STATEMENT
Is mathematics used in computer technology?	Too general	What role has mathematics played in the development of computer animation?
Water is a shared resource.	Too general	Homes, farms, ranches, and businesses east of the Rockies all use runoff water. When there is a shortage, that water must be shared.
Do driver's education programs help teenagers parallel park?	Too specific, unless the student is generating his or her own data	Do driver's education programs reduce the incidence of parking accidents?

The following checklist can be used to determine if the research question or statement is effective.

- Does the question or statement clearly identify the main objective of the research? After the question or statement is read to someone, can they tell what the student will be researching?
- Is the student confident that the question or statement will lead him or her to sufficient data to reach a conclusion?
- Is the question or statement interesting? Does it make the student want to learn more?
- Is the topic that the student chose purely factual, or is that student likely to encounter an issue, with different points of view?

➢ Carrying Out the Research

As students continue with their projects, they will need to conduct research and collect data. The strategies that follow will help them in their data collection.

There are two types of data that students will need to consider – primary and secondary. Primary data is data that the student collects himself or herself using surveys, interviews and direct observations. Secondary data is data that the student obtains through other sources, such as online publications, journals, magazines, and newspapers.

Both primary and secondary data have their advantages and disadvantages. Primary data provide specific information about the research question or statement, but may take time to collect and process. Secondary data is usually easier to obtain and can be analysed in less time. However, because the data was originally gathered for other purposes, a student may need to sift through it to find exactly what he or she is looking for.

The type of data chosen can depend on many factors, including the research question, the skills of the student, and available time and resources. Based on these and other factors, the student may chose to use primary data, secondary data, or both.

When collecting primary data, the student must ensure the following:

- For surveys, the sample size must be reasonably large and the random sampling technique must be well designed.
- For surveys and interviews, the questionnaires must be designed to avoid bias.
- For experiments and studies, the data must be free from measurement bias.
- The data must be compiled accurately.

When collecting secondary data, the student should explore a variety of resources, such as:

- textbooks, and other reference books;
- scientific and historical journals, and other expert publications;
- the Internet;
- library databases.

After collecting the secondary data, the student must ensure that the source of the data is reliable:

- If the data is from a report, determine what the author's credentials are, how recent the data is, and whether other researchers have cited the same data.
- Be aware that data collection is often funded by an organization with an interest in the outcome or with an agenda that it is trying to further. Knowing which organization has funded the data collection may help the student decide how reliable the data is, or what type of bias may have influenced the collection or presentation of the data.
- If the data is from the Internet, check it against the following criteria:
    - authority – the credentials of the author should be provided;
    - accuracy – the domain of the web address may help the student determine the accuracy;
    - currency – the information is probably being accurately managed if pages on a site are updated regularly and links are valid.

➢ Analysing the Data

Statistical tools can help a student analyse and interpret the data that is collected. Students need to think carefully about which statistical tools to use and when to use them, because other people will be scrutinizing the data. A summary of relevant tools follows.

Measures of central tendency will give information about which values are representative of the entire set of data. Selecting which measure of central tendency (mean, median, or mode) to use depends on the distribution of the data. As the researcher, the student must decide which measure most accurately describes the tendencies of the population. The following criteria should be considered when deciding upon which measure of central tendency best describes a set of data.

- Outliers affect the mean the most. If the data includes outliers, the student should use the median to avoid misrepresenting the data. If the student chooses to use the mean, the outliers should be removed before calculating the mean.

- If the distribution of the data is not symmetrical, but instead strongly skewed, the median may best represent the set of data.
- If the distribution of the data is roughly symmetrical, the mean and median will be close, so either may be appropriate to use.
- If the data is not numeric (for example, colour), or if the frequency of the data is more important than the values, use the mode.

Both the range and the standard deviation will give the student information about the distribution of data in a set. The range of a set of data changes considerably because of outliers. The disadvantage of using the range is that it does not show where most of the data in a set lies – it only shows the spread between the highest and the lowest values. The range is an informative tool that can be used to supplement other measures, such as standard deviation, but it is rarely used as the only measure of dispersion.

Standard deviation is the measure of dispersion that is most commonly used in statistical analysis when the mean is used to calculate central tendency. It measures the spread relative to the mean for most of the data in the set. Outliers can affect standard deviation significantly. Standard deviation is a very useful measure of dispersion for symmetrical distributions with no outliers. Standard deviation helps with comparing the spread of two sets of data that have approximately the same mean. For example, the set of data with the smaller standard deviation has a narrower spread of measurement around the mean, and therefore has comparatively fewer high or low scores, than a set of data with a higher standard deviation.

When working with several sets of data that approximate normal distributions, you can use *z*-scores to compare the data values. A *z*-score table enables a student to find the area under a normal distribution curve with a mean of zero and a standard deviation of one. To determine the *z*-score for any data value in a set that is normally distributed, the formula $z = \frac{x - \bar{x}}{s}$ can be used where x is any observed data value in the set, $\bar{x}$ is the mean of the set, and is s is the standard deviation of the set.

When analysing the results of a survey, a student may need to interpret and explain the significance of some additional statistics. Most surveys and polls draw their conclusions from a sample of a larger group. The margin of error and the confidence level indicate how well a sample represents a larger group. For example, a survey may have a margin of error of plus or minus 3% at a 95% level of confidence. This means that if the survey were conducted 100 times, the data would be within 3 percent points above or below the reported results in 95 of the 100 surveys.

The size of the sample that is used for a poll affects the margin of error. If a student is collecting data, he or she must consider the size of the sample that is needed for a desired margin of error.

➢ Identifying Controversial Issues

While working on a research project, a student may uncover some issues on which people disagree. To decide on how to present an issue fairly, he or she should consider some questions to ask as the research proceeds.

- **What is the issue about?**

  The student should identify which type of controversy has been uncovered. Almost all controversy revolves around one or more of the following:
  - Values – What should be? What is best?
  - Information – What is the truth? What is a reasonable interpretation?

- Concepts – What does this mean? What are the implications?

- **What positions are being taken on the issue?**

  The student should determine what is being said and whether there is reasonable support for the claims being made. Some questions to ask are:
  - Would you like that done to you?
  - Is the claim based on a value that is generally shared?
  - Is there adequate information?
  - Are the claims in the information accurate?
  - Are those taking various positions on the issue all using the same term definitions?

- **What is being assumed?**

  Faulty assumptions reduce legitimacy. The student can ask:
  - What are the assumptions behind an argument?
  - Is the position based on prejudice or an attitude contrary to universally held human values, such as those set out in the United Nations Declaration of Human Rights?
  - Is the person who is presenting a position or an opinion an insider or an outsider?

- **What are the interests of those taking positions?**

  The student should try to determine the motivations of those taking positions on the issue. What are their reasons for taking their positions? The degree to which the parties involved are acting in self-interest could affect the legitimacy of their opinions.

➢ The Final Product and Presentation

The final presentation should be more than just a factual written report of the information gathered from the research. To make the most of the student's hard work, he or she should select a format for the final presentation that suits his or her strengths, as well as the topic.

To make the presentation interesting, a format should be chosen that suits the student's style. Some examples are:

- a report on an experiment or an investigation;
- a short story, musical performance, or play;
- a web page;
- a slide show, multimedia presentation, or video;
- a debate;
- an advertising campaign or pamphlet;
- a demonstration or the teaching of a lesson.

Sometimes, it is also effective to give the audience an executive summary of the presentation. This is a one-page summary of the presentation that includes the research question and the conclusions that were made.

Before giving the presentation, the student can use these questions to decide if the presentation will be effective.

- Did I define my topic well? What is the best way to define my topic?
- Is my presentation focused? Will my classmates find it focused?
- Did I organize my information effectively? Is it obvious that I am following a plan in my presentation?
- Am I satisfied with my presentation? What might make it more effective?
- What unanswered questions might my audience have?

➢ Peer Critiquing of Research Projects

After the student has completed his or her research for the question or statement being studied, and the report and presentation have been delivered, it is time to see and hear the research projects developed by other students. However, rather than being a passive observer, the student should have an important role – to provide feedback to his or her peers about their projects and presentations.

Critiquing a project does not involve commenting on what might have been or should have been. It involves reacting to what is seen and heard. In particular, the student should pay attention to:

- strengths and weaknesses of the presentation;
- problems or concerns with the presentation.

While observing each presentation, students should consider the content, the organization, and the delivery. They should take notes during the presentation, using the following rating scales as a guide. These rating scales can also be used to assess the presentation.

Content

Shows a clear sense of audience and purpose.	1	2	3	4	5
Demonstrates a thorough understanding of the topic.	1	2	3	4	5
Clearly and concisely explains ideas.	1	2	3	4	5
Applies knowledge and skills developed in this course.	1	2	3	4	5
Justifies conclusions with sound reasoning.	1	2	3	4	5
Uses vocabulary, symbols and diagrams correctly.	1	2	3	4	5

Organization

Presentation is clearly focused.	1	2	3	4	5
Engaging introduction includes the research question, clearly stated.	1	2	3	4	5
Key ideas and information are logically presented.	1	2	3	4	5
There are effective transitions between ideas and information.	1	2	3	4	5
Conclusion follows logically from the analysis and relates to the question.	1	2	3	4	5

Delivery

Speaking voice is clear, relaxed, and audible.	1	2	3	4	5
Pacing is appropriate and effective for the allotted time.	1	2	3	4	5
Technology is used effectively.	1	2	3	4	5
Visuals and handouts are easily understood.	1	2	3	4	5
Responses to audience’s questions show a thorough understanding of the topic.	1	2	3	4	5

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