Prince Edward Island
CANADA

Education and Early Years

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

Grade 4

Acknowledgments

The Department of Education and Early Years of Prince Edward Island gratefully acknowledges the contributions of the following groups and individuals toward the development of the *Prince Edward Island Grade 5 Mathematics Curriculum Guide*:

- Eric Arseneault,
  Sec. French Math/Science Specialist,
  Department of Education, Early Learning and Culture

  J. Blaine Bernard,
  Secondary Mathematics Specialist,
  Department of Education, Early Learning and Culture

  Brenda Larsen,
  Elementary Mathematics & Science Specialist
  Department of Education, Early Learning and Culture

  Jonathan Hayes,
  Secondary Science Specialist,
  Department of Education, Early Learning and Culture

  Bill MacIntyre,
  Elementary Mathematics Specialist, Department of
  Department of Education, Early Learning and Culture

  Lauren Gill
  K-9 Mathematics Curriculum Leader,
  Department of Education and Early Years

- The 2008-2009 grade four mathematics pilot teachers:

  Tammy Cann, Southern Kings Consolidated
  Cheryl Clark, Glen Stewart Elementary
  Bev Coles, West Royalty Elementary
  Elizabeth Crawford, West Royalty Elementary
  Nadine DeRoche, Englewood School
  Anne Doucette, Glen Stewart Elementary
  Erin Kilfoil Duffy, Glen Stewart Elementary
  Brandy Francis, Athena Consolidated
  Lisa Gallant, Greenfield Elementary
  Melissa Noonan Gallant, Athena Consolidated
  Kerri Hambly, Glen Stewart Elementary
  Marion Harris, Amherst Cove Consolidated
  Geraldine Keoughan, West Royalty Elementary
  Susan Kirkham, Tignish Elementary
  Angela MacLean, Morell Consolidated
  Elsie Moore, Southern Kings Consolidated
  Sherri Ramsay, Greenfield Elementary
  Mark Ronahan, Parkside Elementary

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education
- The Alberta Department of Education
- The New Brunswick Department of Education

Prince Edward Island
Department of Education and Early Years
Holman Centre
250 Water Street, Suite 101
Summerside, Prince Edward Island
Canada C1N 1B6
Implemented 2009
Revised 2023

Table of Contents

Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

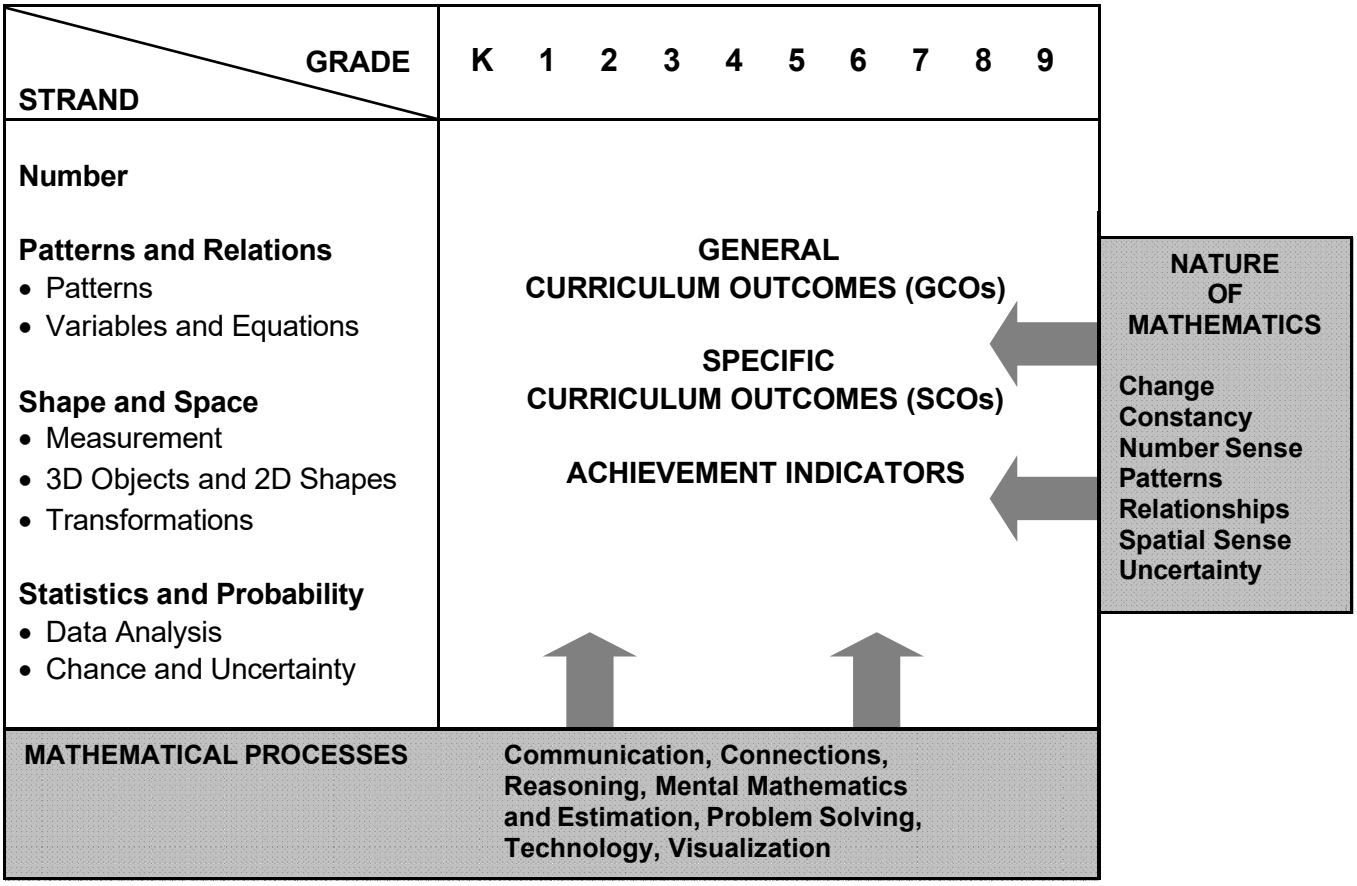
- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

Conceptual Framework for K-9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connections among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.
- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

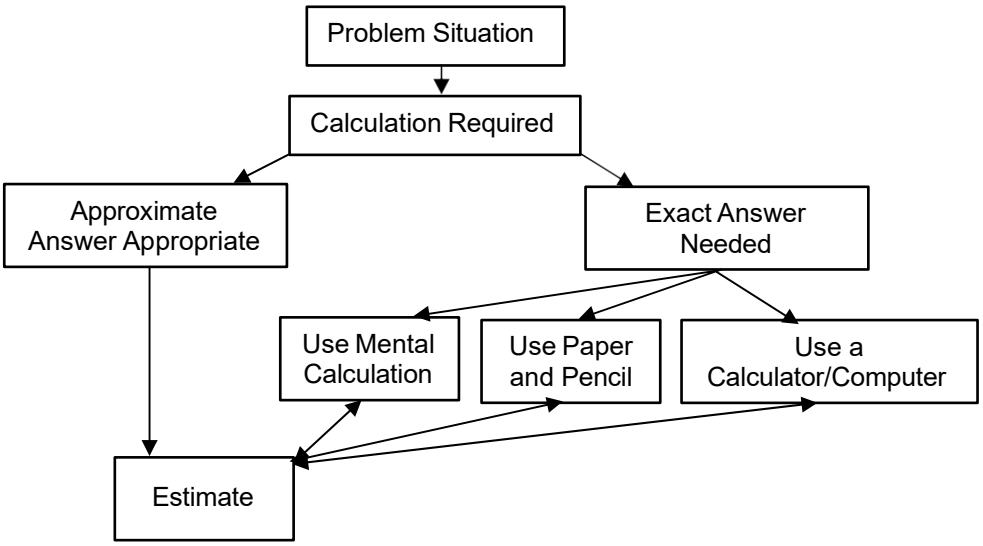
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001). Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you. . . ?" or "How could you. . . ?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not

a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3D objects and 2D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2D shapes and 3D objects, and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2D shapes and 3D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3D or 2D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of

probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Connections Across the Curriculum

There are many possibilities for connecting Grade 4 mathematical learning with the learning occurring in other subject areas. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education (p.60)." The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate "communicating to learn mathematics and learning to communicate mathematically" (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database

Resources for Rethinking, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, or teaching has been effective, or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

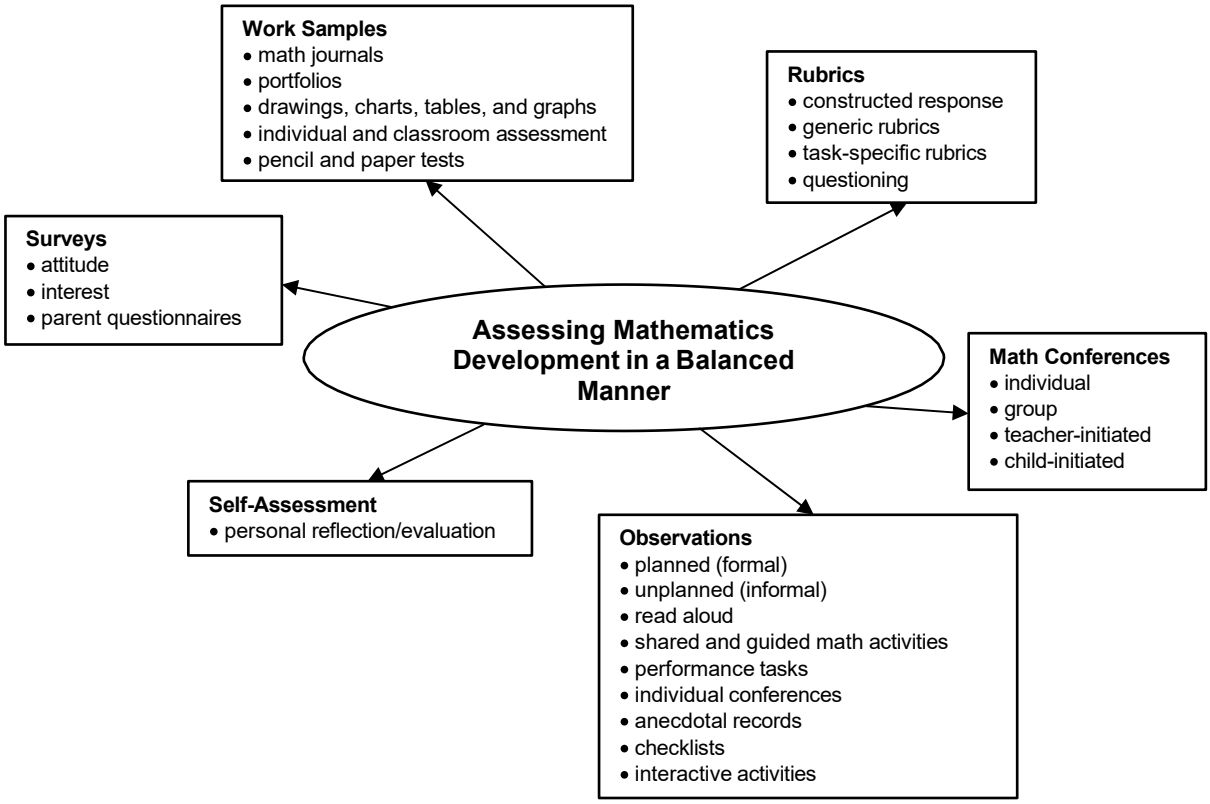
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram below.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

a. to engage students in their own learning and self-assessment;
b. to help students understand what is important in the mathematical concepts and particular tasks they encounter;
c. to develop effective habits of metacognition and self-coaching;
d. to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

a. to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
b. to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
c. to provide feedback to students about how they are doing and how they might improve;
d. to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

a. to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
b. to facilitate reporting;
c. to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, and phone calls.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense.
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems.
	3D Objects and 2D Shapes: Describe the characteristics of 3D objects and 2D shapes, and analyse the relationships among them.
	Transformations: Describe and analyse position and motion of objects and shapes.
Statistics and Probability (SP)	**Data Analysis**: Collect, display, and analyse data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

The first two pages for each outcome contain the following information:

- the corresponding **strand** and **general curriculum outcome**;
- the **Specific Curriculum Outcome**(s) and the mathematical **processes** which link this content to instructional methodology;
- the **scope and sequence** of concept development related to this outcome(s) from 3 - 5;
- a list of **achievement indicators**; and
  Students who have achieved a particular outcome should be able to demonstrate their understanding in the manner specified by the achievement indicators. It is important to remember, however, that these indicators are not intended to be an exhaustive list for each outcome. Teachers may choose to use additional indicators as evidence that the desired learning has been achieved.
- an **elaboration** of the outcome.

The last two pages for each outcome contain lists of **instructional strategies** and **strategies for assessment**.
The primary use of this section of the guide is as an ***assessment for learning*** (formative assessment) tool to assist teachers in planning instruction to improve learning. However, teachers may also find the ideas and suggestions useful in gathering ***assessment of learning*** (summative assessment) data to provide information on student achievement.

The **Mental Math Guide,** which outlines the **Fact Learning, Mental Computation and Estimation** strategies for this grade level, can be found at learn.edu.pe.ca. Included is an **Overview of the Thinking Strategies in Mental Math** for grades one to six complete with a description of each strategy as well as a scope and sequence table of the strategies for the elementary grades.

NUMBER

SPECIFIC CURRICULUM OUTCOMES

4.N1 – Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.

4.N2 – Compare and order numbers to 10 000.

4.N3 – Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: using personal strategies for adding and subtracting, estimating sums and differences, and solving problems involving addition and subtraction.

4.N4 – Explain the properties of 0 and 1 for multiplication and the property of 1 for division.

4.N5 – Describe and apply mental mathematics strategies, such as: skip counting from a known fact, using doubling or halving, using doubling or halving and adding or subtracting one more group, and using patterns in the 9s facts to determine basic multiplication facts to 9 x 9 and related division facts.

4.N6 – Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by using personal strategies for multiplication (2- or 3-digit by 1-digit) to solve problems by using personal strategies for multiplication with and without concrete materials, using arrays to represent multiplication, connecting concrete representation to symbolic representations and estimating products.

4.N7 – Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by using personal strategies for dividing with and without concrete materials, estimating quotients and relating division to multiplication.

4.N8 – Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to name and record fractions for the parts of a whole or a set, compare and order fractions, model and explain that for different wholes, two identical fractions may not represent the same quantity and to provide examples of where fractions are used.

4.N9 – Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.

4.N10 – Relate decimals to fractions (to hundredths).

4.N11 – Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by using compatible numbers, estimating sums and differences and using mental math strategies to solve problems.

SCO: **4.N1 Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.**
[C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.N2 Represent and describe numbers to 1000, concretely, pictorially and symbolically. **3.N4** Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.	**4.N1** Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.	**5.N1** Represent and describe whole numbers to 1 000 000.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. write a given numeral 0 – 10 000 in words;

B. represent a given numeral using a place value chart or diagrams;

C. describe the meaning of each digit in a given numeral;

D. express a given numeral in expanded notation, e.g., 4301 = 4000 + 300 + 1;

E. write the numeral represented by a given expanded notation. e.g. 2000 + 400 + 60 = 2460;

F. explain and show the meaning of each digit in a given 4-digit numeral with all digits the same, e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens and the fourth digit two ones;

G. write a given numeral using proper spacing without commas, e.g., 4567 or 4 567, 10 000; and

H. read numerals up to four-digits without using the word “and,” e.g., 365 is read as “three hundred, sixty-five; 5321 is read as “five thousand, three hundred, twenty-one.” The word “and” is reserved for reading decimal numbers, e.g., 3.8 is “three *and* eight tenths”.

SCO: **4.N1 Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.**
[C, CN, V]

Elaboration

Students should recognize the value represented by each digit in a number, as well as what the number means as a whole. The “2” in 2300 means 2 thousands whereas the “2” in 3200 means 2 hundreds.

Students should have many opportunities to:

- **model** numbers containing zeroes. For example, 1003 means 1 thousand and 3 ones but is read as “one thousand, three.”
- **read** numbers several ways. For example, 9347 is read 9 thousand, three hundred forty-seven but might also be expressed as 93 hundred, 47 (other examples may include:9 thousands, 34 tens, 7 ones; 9 thousands, 33 tens, 17 ones);
- **record** numbers. For example, ask students to **write** twenty-eight hundred sixty; a number which is eighty less than ninety thousand; as well as write numbers in standard form and **expanded notation** (7453 = 7000 + 400 + 50 + 3).

Through these experiences, students will develop flexibility in identifying, modeling, and representing numbers up to 10 000. It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real life contexts that are personally meaningful. Use numbers from student’s experiences, such as capacity for local arenas, or population of the school or community. Students can use these **personal referents** to think of other large numbers. They can also use **benchmarks** that students may find helpful are multiples of 100 and 1000, as well as 250, 500, 750, 2500, 5000, and 7500.

Include situations in which students use a variety of models, such as:

- base-ten blocks (e.g., to model 10 000 have the class make a long rod with 10 big cubes. It will be a 10 thousand rod. Students should recognize that this also models 10 000 unit cubes.)
- money (e.g., How many $100 bills are there in $9347?)
- place value charts.

Thousands			Ones		
H	T	O	H	T	O

The focus of instruction should be on ensuring students develop a strong sense of number.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 2, Lesson 1, pp. 34-37

SCO: **4.N1 Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.**
[C, CN, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Invite students to investigate the length of a line comprising 10 000 pennies. Encourage students to share the various strategies they used to investigate this problem. It is also important to have them share strategies that they considered, but rejected.
- Use base ten blocks or have students draw pictorial representations of the blocks. Have students use them to explore what numbers might be represented by using exactly 10 base ten blocks. (Note: It is important to use the correct vocabulary when referring to the blocks; “flat”, not “hundred flat” and “rod”, not “ten rod”, etc., so students are flexible in their thinking of the models when working with decimals.)

Suggested Activities

- Provide a stack of 4 sets of shuffled cards numbered 0 - 9. Ask the students to select 5 cards and arrange them to make the greatest possible number. Ask them to record and read the number and to rearrange the cards to make the least possible number. Have this number recorded under the larger number. As an extension, have the students estimate the difference between the two numbers. This activity is an ideal opportunity for students to practise front-end subtraction (left-to-right calculations).
- Have students, as a class, create a “ten thousands” chart. Provide each small group of students with hundred grids (or other pictorial representations such as arrays of dots) and have them create a model to represent 1000. Combine these models to create a class representation of 10 000.
- Ask students to create a four-digit number using 9, 2, 7, and 5. The digit in the hundreds place must be two more than the ones place. List all of the possible numbers.
- Ask questions about the reasonableness of numbers such as, “Would it be reasonable for an elementary school to have 9600 students?” or “Would it be reasonable for an elevator to hold 20 people?” “Would someone be able to drive 26 hundred kilometres in a day?” “Would it be reasonable to pay $5 000 for a boat/book/computer?” Investigate and discuss possible answers. Have students create their own “reasonable” questions about a variety of topics.
- Have students find large numbers from newspapers and magazines. Ask them to share and discuss the numbers within their group. Have students read, write, and model the numbers in different ways.

SCO: **4.N1 Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.**
[C, CN, V]

Assessment Strategies

- Ask the student to use base-ten materials to model 2016 in three different ways. Have him/her explain the models.
- Ask the students to record a series of numbers that have been read to them (such as eight thousand eighty-two, sixteen hundred five). Include examples such as the greatest 4-digit number or a number one hundred less than the greatest 4-digit number.
- Ask: How are 903 and 9003 different? Similar?
- Tell the student that a boat costs $6135. Ask: If one were to pay for it in $100 bills, how many would be needed? Extend by asking how many $10 bills would be needed.

SCO: **4.N2 Compare and order numbers to 10 000.**
[C, CN]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.N3 Compare and order numbers to 1000.	**4.N2** Compare and order numbers to 10 000.	

Achievement Indicators

Students who have achieved this outcome should be able to:

A. order a given set of numbers in ascending or descending order and explain the order by making references to place value;

B. create and order three different 4-digit numerals;

C. identify the missing numbers in an ordered sequence or on a number line; and

D. identify incorrectly placed numbers in an ordered sequence or on a number line.

SCO: **4.N2 Compare and order numbers to 10 000.**
[C, CN]

Elaboration

Comparing and **ordering** numbers is fundamental to understanding numbers. Students should investigate meaningful contexts to compare and order two or more numbers, both with and without models. For example, ask them to compare/order populations of communities or capacities of arenas. The capacity of the Charlottetown Civic Center arena is 3400 fixed seats. At the Bell Centre in Montreal, it's 21 273.

Students must realize that when comparing two numbers with the same number of digits, the digit with the greatest value needs to be addressed first. For example, when asked to explain why one number is greater or less than another, they might say that 2542 < 3653 because 2542 is less than 3 thousands while 3653 is more than 3 thousands. When comparing 6456 and 6546, students will begin comparing the thousands and move to the right.

Students must recognize that when comparing the size of a number, the 4 in 4289 has a greater value than the 9 and they should be able to provide an explanation.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

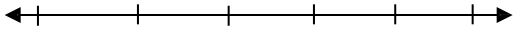
- Unit 2, Lesson 2, pp. 38-41
- Unit 2, Lesson 3, pp. 42-45

SCO: **4.N2 Compare and order numbers to 10 000.**
[C, CN]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide the students with opportunities to practise comparing numbers such as 12 098 and 12 210 and ask them to explain their reasoning.
- Have students discuss what would be the most appropriate benchmarks for various number lines, such as 0 to 50, 90 to 150, 200 to 1000, or 243 to 2448.
- Provide situations in which students name numbers which are greater than or less than a given number (Note: In some cases the amount greater or less could be specified, such as 29 more or 3000 less, etc) and name numbers which are between given numbers.
- Use a variety of number lines, including open number lines, in which students can place numbers and/or correct placed numbers. Using benchmarks encourage students to consider the magnitude of various numbers in relation to appropriate benchmarks.



Suggested Activities

- Display a 4-digit number on an overhead calculator (or on a card, or on the board). Have students enter on their calculators a number which differs by 1 digit. Have them read their numbers and ask others to determine if they are greater than or less than the number on the overhead. Collect five, or more, of their numbers and ask the students to order them on a number line. Explain.
- Assign pairs of students the task of making challenging number card for their classmates to order.
- Provide a list of populations of communities within your area ranging from a few hundred to about ten thousand. Ask students to order them from least to greatest. As an extension they could cluster the populations and graph them.
- Provide the following riddle: I am thinking of a number. It is between 8000 and 10 000. All the digits are even and the sum of the digits is 16. What are some possibilities? Use an open number line to display their numbers. Challenge students to write their own riddles.
- Tape numbers on students' backs and ask them to order themselves, without seeing their own number, from least to greatest without talking.
- When asked to find which of 9199 and 9210 is greater, Sadie said, "That is easy!" How did Sadie determine the greater one?
- Have the students find large numbers from newspapers and magazines. Ask them to create a collage that would illustrate the order of the numbers from least to greatest.
- Prepare cards for students to order from least to greatest. For example: 6183, 9104, 9080, 7102, 6604, 1999, 6540.
- Ask students to decide which is worth more: 4356 quarters, 8462 dimes or 9999 pennies. Have the students predict first, then use calculators to help solve the problem.
- Have students place numbers on the line using several benchmarks numbers to guide them.

SCO: **4.N2 Compare and order numbers to 10 000.**
[C, CN]

Assessment Strategies

- Give the students some number cards and ask them to order them from greatest to least.
- As a writing assignment, ask the student how he/she might advise a younger student to determine which of 2 numbers is greater.
- Ask the student to record two numbers to meet these requirements: the first has 3 in the thousands place but is less than the second which has 3 in the hundreds place.
- Ask the students to write a number that has 980 tens.
- Ask the students to write a number that would fall about halfway between 9490 and 10 000.
- Tell the students that you are thinking of a 5-digit number that has 2 thousands, a greater number of tens, and an even greater number of ones. Ask them to give three possibilities.
- Have the students create all of the possible numbers using the digits: 8, 9, 7, 6. Have students place their answers on a number line.
- Tell the student that Jodi's number had 9 hundreds, but Fran's had only 6 hundreds. Fran's number was greater. Ask: How was this possible?
- Ask: Which four-digit number below must be greater? Explain why.
  □4□□2 or 9□□3
- Ask the student how many whole numbers are greater than 8000, but less than 8750.

SCO: **4.N3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**
- **using personal strategies for adding and subtracting**
- **estimating sums and differences**
- **solving problems involving addition and subtraction.**

[C, CN, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.N7 Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context. **3.N8** Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by: using personal strategies for adding and subtracting with and without the support of manipulatives; creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.	**4.N3** Demonstrate an understanding of addition of numbers with sums to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: • *using personal strategies for adding and subtracting* • *estimating sums and differences* • *solving problems involving addition and subtraction*.	**5.N2** Use estimation strategies, including: • *front-end rounding* • *compensation* • *compatible numbers* *in problem-solving contexts*.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. explain how to keep track of digits that have the same place value when adding numbers, limited to 3- and 4-digit numerals;

B. explain how to keep track of digits that have the same place value when subtracting numbers, limited to 3- and 4-digit numerals;

C. represent concretely, pictorially, symbolically the addition and subtraction of whole numbers up 4-digit by 4-digit;

D. describe a situation in which an estimate rather than an exact answer is sufficient;

E. estimate sums and differences using different strategies, i.e., front-end estimation and compensation;

F. solve problems that involve addition and subtraction of whole numbers (one or more steps/where some numbers may be irrelevant). Explain solutions to problems;

G. create a problem given a number sentence for addition or subtraction; and

H. solve problems that involve addition and subtraction in more than one way, limited to 3- and 4-digit numerals. For example, 385 + ☐ = 500 or 500 – 385 = ☐.

SCO: **4.N3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**
- **using personal strategies for adding and subtracting**
- **estimating sums and differences**
- **solving problems involving addition and subtraction.**

[C, CN, ME, PS, R]

Elaboration

Personal strategies need to make sense to students and be accurate, as well as efficient. These alternatives are as valid as a traditional algorithm, which is the ultimate goal once their understanding of the operations has been developed (note: When introducing addition and subtraction with 3-and 4-digit numbers, use base-ten materials to model the operations).

Students need to recognize that **estimation** is a useful skill in their lives. To be efficient when estimating sums and differences mentally, students must be able to access a strategy quickly and they need a variety from which to choose. Some strategies to consider using **benchmarks**, **rounding**, **front-end addition** and **subtraction (left-to-right calculations)**, and clustering of **compatible numbers**.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

Computational fluency is a balance between conceptual understanding (thinking about the structure of numbers and the relationship among numbers and the operations) and computational proficiency (includes both efficiency and accuracy). (NCTM, 2000, p.35)

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:
- Unit 2, Lesson 4, pp. 46-48
- Unit 2, Lesson 5, pp. 49, 50
- Unit 2, Lesson 6, pp. 51-54
- Unit 2, Lesson 7, pp. 55-58
- Unit 2, Lesson 8, pp. 59-61
- Unit 2, Lesson 9, pp. 62, 63
- Unit 2, Lesson 10, pp. 64-67
- Unit 2, Lesson 11, pp. 68, 69
- Unit 2, Lesson 12, pp. 70, 71
- Unit 2, Lesson 13, pp. 73-75
- Unit 2, Unit Problem, pp. 78, 79

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to the Grade 4 Mathematics page at learn.edu.pe.ca for the Mental Math Guide.)

SCO: **4.N3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**
- **using personal strategies for adding and subtracting**
- **estimating sums and differences**
- **solving problems involving addition and subtraction.**

[C, CN, ME, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Encourage students to estimate prior to calculating the answer.
- Use a variety of models, such as base ten blocks and number lines to assist in estimation.
- Help students to develop and apply a variety of estimation strategies, including:
  rounding: 439 + 52 is approximately 440 + 50.
  front-end for addition and subtraction: 138 + 24; 100 + 200 = 300. 476 – 348; 400 – 300 = 100
  ***adjusted* front-end for addition and subtraction:** 138 + 24; 100 + 200 = 300 and 30 + 40 is 70 for an estimate of 370. Depending on their *number sense*, some students may also consider the ones in their estimate and refine their answer to about 380. 476 – 348; 400 – 300 is 100, 70 – 40 is 30, 6 and 8 are about the same so I'll ignore them; my estimate is 130.

- Use computation strategies for solving problems, such as the open number line.
- Explore personal strategies such as those that can be found in *Teaching Student-Centered Mathematics, Grades 3-5,* Van de Walle and Lovin, p. 109 -111.
  Reinforce proper math vocabulary. "Regrouping" or "trading" is preferred to the terms of "borrowing" or "carrying".

Suggested Activities

- Have students paraphrase various story problems to enhance understanding and to recognize which numbers in a problem refer to a part or to a whole. Share solutions.
- Provide the students with an addition number sentence, such as the following: 328 + 462 = 330 + 460. Have them decide if the number sentence is true or false and explain how they know. Encourage the students to think of the equal sign as "the same as" so that they are deciding whether the two sides balance each other.
- Ask the students to find two numbers with a difference of about 150 and a sum of about 500.
- Present the students with problems and have them decide which problems can be answered with an estimate only and which problems require calculation as well as an estimate. For example:

Will a container that holds 2000 mL be large enough to hold 1350 mL of water from another container as well as 1015 mL of water from a different container?

SCO: **4.N3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**
- **using personal strategies for adding and subtracting**
- **estimating sums and differences**
- **solving problems involving addition and subtraction.**

[C, CN, ME, PS, R]

Assessment Strategies

- Model the addition of 1273 and 485 using concrete and/or visual representations and record the process symbolically. Students should be able to explain their method.
- Subtract 248 from 5073 and model the subtraction using concrete or visual representations. Record the process symbolically. Students should be able to explain their method.
- Create an addition or subtraction story problem for the number sentence:
  330 – 185 = ☐ or 185 + ☐ = 330.
- Determine the sum/difference of 3185 and 628 using a personal strategy and explain how the strategy works.
- Present the students with the following problem:
  You drink 250 mL of milk on the first day, 375 mL of milk the second day and 450 mL of milk on the third day. About how many millilitres of milk did you drink during these three days? Stimulate the students' thinking by asking whether 900 mL would be a good estimate for the answer.
- Tell the student that, to estimate 583 - 165, Jeff said, “575 subtract 175.” Ask him/her if the estimate will be high or low, and to explain why Jeff might have chosen to estimate in this way.

SCO: **4.N4 Explain the properties of 0 and 1 for multiplication and the property of 1 for division.** [C, CN, R]

4.N5 Describe and apply mental mathematics strategies, such as:

- **skip counting from a known fact**
- **using doubling or halving**
- **using doubling or halving and adding or subtracting one more group**
- **using patterns in the 9s facts**

to determine basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.N9 Apply mental mathematics strategies and number properties to determine answers for basic addition facts and related subtraction facts (to 18). **3.N10** Demonstrate an understanding of multiplication to products of 36 with single digit factors. **3.N11** Demonstrate an understanding of division limited to division related to multiplication facts up to products of 36 with single digit factors.	**4.N4** Explain the properties of 0 and 1 for multiplication and the property of 1 for division. **4.N5** Describe and apply mental mathematics strategies, such as: • skip counting from a known fact • using doubling or halving • using doubling or halving and adding or subtracting one more group • using patterns in the 9s facts • using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts.	**5.N3** Apply mental mathematics strategies and number properties, such as: skip counting from a known fact, using doubling or halving, using patterns in the 9s facts, using repeated doubling or halving, to determine answers for basic multiplication facts to 81 and related division facts. **5.N4** Apply mental mathematics strategies for multiplication, such as: annexing then adding zero, halving and doubling, using the distributive property.

Achievement Indicators

Students who have achieved these outcomes should be able to:

4.N4

A. explain the property for determining the answer when multiplying numbers by one;

B. explain the property for determining the answer when multiplying numbers by zero; and

C. explain the property for determining the answer when dividing numbers by one.

4.N5

A. provide examples for applying mental mathematics strategies:

doubling (for 4×3, think $2 \times 3 = 6$, so $4 \times 3 = 6 + 6$)

doubling and adding one more group (for 3×7, think $2 \times 7 = 14$, and $14 + 7 = 21$

using known facts (for example, when multiplying 9×6, think $10 \times 6 = 60$, and $60 - 6 = 54$)

halving (if 4×6 is equal to 24, then 2×6 is equal to 12)

think division for multiplication facts (for $64 \div 8$, think $8 \times \square = 64$).

SCO: **4.N4 Explain the properties of 0 and 1 for multiplication and the property of 1 for division.** [C, CN, R]

4.N5 Describe and apply mental mathematics strategies, such as:

- **skip counting from a known fact**
- **using doubling or halving**
- **using doubling or halving and adding or subtracting one more group**
- **using patterns in the 9s facts**

to determine basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R]

Elaboration

It is important to address zero and one in multiplication. A number line can be used to illustrate both of these properties. To explore that the **product** is 0 when multiplying by 0, 3×0 can be shown by making 3 hops of 0 or making 0 hops of 3. The property of multiplying and dividing by 1 can similarly be explored on the number line. Provide opportunities for students to not only solve multiplication and division problems, but create their own problems requiring the use of these operations.

Developing basic **multiplication facts** to 9 × 9 and related division facts requires that the students have a strong foundation in patterns, number relationships, place value, and the meaning, relationships and properties of operations as described below:

- patterns are used in developing mental strategies, such as skip counting from a known fact and using the constant sum of the digits in products with the 9s facts;
- number relationships are evident when using the properties of operations or other strategies, such as repeated doubling; e.g., $4 \times 6 = (2 \times 6) \times 2 = 24$;
- place value is used extensively in various strategies, such as doubling and adding or subtracting one more group; e.g., $3 \times 7 = 2 \times 7 + 7 = 14 + 7 = 21$; $9 \times 9 = 10 \times 9 – 9 = 81$;
- the meaning of multiplication and division and the connection between the operations is crucial as the students develop understanding of multiplication and division facts. Students who have learned their multiplication facts have automatically learned their division facts.

4.N4: This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 3, Lesson 2, pp. 86-89
- Unit 3, Lesson 7, pp. 104-106

4.N5: This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 3, Lesson 1, pp. 82-85
- Unit 3, Lesson 3, pp. 90-93
- Unit 3, Lesson 4, pp. 94-97
- Unit 3, Lesson 5, pp. 98-100
- Unit 3, Lesson 7, pp. 104-106
- Unit 3, Lesson 8, pp. 107-109
- Unit 3, Lesson 9, pp. 110-112
- Unit 3, Lesson 10, pp. 113-115
- Unit 3, Unit Problem, pp. 118, 119

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to the Grade 4 Mathematics page at learn.edu.pe.ca for the Mental Math Guide.)

SCO: **4.N4 Explain the properties of 0 and 1 for multiplication and the property of 1 for division.** [C, CN, R]

4.N5 Describe and apply mental mathematics strategies, such as:

- **skip counting from a known fact**
- **using doubling or halving**
- **using doubling or halving and adding or subtracting one more group**
- **using patterns in the 9s facts**

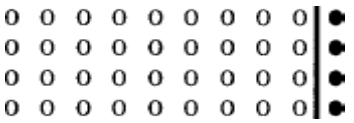
to determine basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use various concrete materials and pictorial representations to demonstrate the multiplication and division of zero. For example, use paper plates for the concept of multiplying by zero. Show six plates with zero counters on each. Ask: *How many plates are there?* (six) *How many counters are there on each plate?* (zero). *Six groups of zero are how many?* $6 \times 0 = 0$.
- Introduce a strategy with the use of materials, practice the strategy, and continue to introduce and practice new strategies. When students have two or more strategies, it is important to focus on strategy selection; choosing the strategy that will be most efficient to determine a particular fact.
- Encourage students to visualize the process for the strategy they are using.
  For example, using an array model.

$4 \times 9 = 4 \times 10$, or 40, subtract 4 is 36.



- Have students begin with what they know. For example, to figure out 6×8, one student might think, "I know $5 \times 8 = 40$ and one more 8 is 48." Another might think, "I know 3×8 is 24 and twice 24 is 48."
- Use the properties of multiplication in developing mental strategies: the associative property; e.g., $(2 \times 2) \times 6 = 2 \times (2 \times 6)$; the commutative property: 3×4 is read 3 sets, or groups of 4; the product however, is the same if the factors are reversed (4×3); distributive property: $4 \times 8 = (4 \times 5) + (4 \times 3) = 20 + 12 = 32$
- Address the misconception that multiplication always makes the product greater: any number multiplied or divided by 1 remains unchanged.

Suggested Activities

- Place students in pairs to practise the "double and double again" strategy for facts such as 4×7. (For example, 4×7 is double 2×7 which is 7 + 7. Since 2 sevens is 14, 4×7 is 28.) Students should take turns asking facts and providing answers by repeated doubling.
- Have students play the "Target Game."3 multiplied by what number is closest to the target number, without going over? Place individual items like the following on flashcards or on the overhead.
  $5 \times \square \longrightarrow$ 43 (Target) $\square$ are left over
- Tell the students that the "6" button on the calculator is not working. Have them suggest ways to solve "6×64" without using this button.

SCO: **4.N4 Explain the properties of 0 and 1 for multiplication and the property of 1 for division.** [C, CN, R]

4.N5 Describe and apply mental mathematics strategies, such as:

- **skip counting from a known fact**
- **using doubling or halving**
- **using doubling or halving and adding or subtracting one more group**
- **using patterns in the 9s facts**

to determine basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R]

Assessment Strategies

- Provide opportunities for students to demonstrate their understanding of these properties by posing questions and tasks such as:

  What general statement can you make about multiplying any number by one?
  What general statement can you make about dividing any number by one?
  Create a problem in which you are dividing a number by one.
  Create a problem in which you are multiplying a number by zero.
  What general statement can you make about multiplying any number by zero?

- To assess understanding of various thinking strategies for the multiplication facts, consider tasks and questions such as:

  Explain how would you find the answer to 30 ÷ 5 = by relating it to multiplication?
  What are two different strategies that we could use to figure out the answer to 6 × 7 = ?
  How does knowing 6 × 5 = 30 help you with 12 × 5 = ?
  How could 8 × 10 help you with 8 × 9 = ?
  Use counters or other materials to show how 5 × 8 is the same as (3 × 8) + (2 × 8).
  Jasmine said that she was supposed to divide 75 by 5, but found it easier to divide 150 by 10. Explain Jasmine's method. When is this strategy most useful?

SCO: **4.N6 Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**
- **using personal strategies for multiplication with and without concrete materials**
- **using arrays to represent multiplication**
- **connecting concrete representations to symbolic representations**
- **estimating products.**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.N10 Demonstrate an understanding of multiplication to products of 36 with single digit factors.	**4.N6** Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products.	**5.N2** Use estimation strategies, including: • front-end rounding • compensation • compatible numbers in problem-solving contexts. **5.N5** Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. model a given multiplication problem using the distributive property, e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5);

B. use concrete materials, such as base ten blocks or their pictorial representations, to represent multiplication and record the process symbolically;

C. create and solve a multiplication problem that is limited to 2- or 3-digits by 1-digit;

D. estimate a product using a personal strategy, e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250; and

E. model and solve a given multiplication problem with and without an array and record the process.

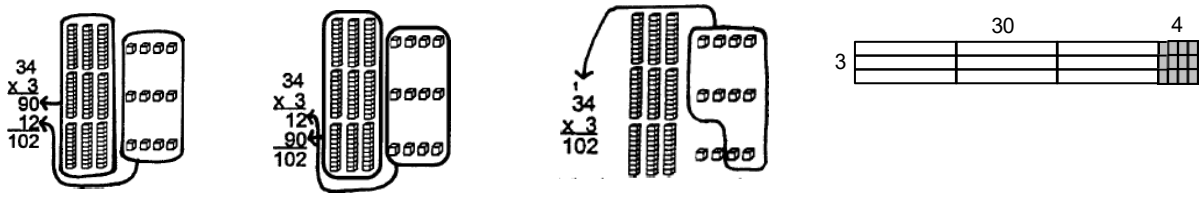
SCO: **4.N6 Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**
- **using personal strategies for multiplication with and without concrete materials**
- **using arrays to represent multiplication**
- **connecting concrete representations to symbolic representations**
- **estimating products.**

[C, CN, ME, PS, R, V]

Elaboration

Encourage the students to **estimate** products and explore their own methods prior to learning the traditional **algorithm** or procedure for finding the **product**. These "personal algorithms" often serve as the procedure of choice, but students need to strive to use the most efficient strategy to solve a particular problem.

Students should use a variety of models to investigate multiplication problems to help develop an understanding of the connection between the model and the symbols. It is important to start with a word problem and then have students use materials to determine the product. Base-ten blocks serve as a tool for understanding the multiplication operation. It is important that the students use language as they manipulate the materials and record the corresponding symbols for the product. It is not expected that students would be explicitly taught all possible algorithms, but provide opportunities to discover which is most efficient for the numbers included in a given problem. Some examples of possible models for multiplication include:



Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:
- Unit 8, Lesson 1, pp. 278-281
- Unit 8, Lesson 2, pp. 282-284
- Unit 8, Lesson 3, pp. 285-287
- Unit 8, Lesson 5, pp. 290-292
- Unit 8, Lesson 6, pp. 293, 294
- Unit 8, Lesson 7, pp. 295-298
- Unit 8, Unit Problem, pp. 314, 315

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to the Grade 4 Mathematics page at learn.edu.pe.ca for the Mental Math Guide.)

SCO: **4.N6 Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**
- **using personal strategies for multiplication with and without concrete materials**
- **using arrays to represent multiplication**
- **connecting concrete representations to symbolic representations**
- **estimating products.**

[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide regular practice in estimation, accompanied by the sharing of strategies. When assessing estimation, the amount of time provided must be controlled in order to determine whether students are proficient in this skill. The goal is for students to automatically estimate in problem-solving situations, not only when instructed to do so.
- Have the students estimate the product to the problem before calculating so that they are better able to determine the reasonableness of their answers.
- Provide a variety of problems representing the different multiplication situations with varying degrees of difficulty to differentiate instruction.
- Provide time for students to create their personal strategies to solve the problem and share these strategies with members of their group or with the entire class.
- Challenge the students to solve the problem another way, do a similar problem without models or clarify the explanation of their personal strategies.

Suggested Activities

- Ask students how they would use the front-end mental multiplication strategy for questions such as $3 \times 125 = 375$ $(3 \times 100 + 3 \times 20 + 3 \times 5)$ and encourage strategies such as $(3 \times 100 + 3 \times 25)$.
- Ask the students to fill in the blanks with 3, 4, and 5 in three different ways and find all the possible products. $\square\square \times \square =$
- Provide students with problems to solve:
  You travel 375 km each day for 3 days. Will you reach the cabin that is 1200 km away by the end of the third day?
  You set up 6 rows of chairs with 28 chairs in each row in the gym. Are there enough chairs to seat 180 people? How many chairs did you set up?
  A toad jumps 135 cm on the first jump and twice as far on the second jump. About how far does it jump in all?
  You jog for 175 minutes each week. How many minutes do you jog in 28 days?
- Have students use a supermarket flyer. Ask them to select 6 of one item, 4 of another and 10 of a third item and give an estimate for the total.

SCO: **4.N6 Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**
- **using personal strategies for multiplication with and without concrete materials**
- **using arrays to represent multiplication**
- **connecting concrete representations to symbolic representations**
- **estimating products.**

[C, CN, ME, PS, R, V]

Assessment Strategies

- Tell the student that he/she has $60. Ask, “Do you have enough money to buy 3 CDs if each costs $17? How do you know?”.
- Ask the students if they can reach the cottage that is 1200 km away if they travel 375 km each day for 3 days. Explain your thinking.
- Write all the possible number sentences that are represented in the following array. Explain how each number sentence relates to the array.

  ********** ********** ********** * *
  ********** ********** ********** * *
  ********** ********** ********** * *

- Ask students to model 24×6. Have them explain the model.
- Tell students that for a school assembly, 9 rows of 38 chairs have been placed in the gym. Are there enough chairs for 370 students? Explain your thinking.
- Ask students to create and solve a realistic problem that includes the factors 6 and 329.
- Have students solve this problem. *You save 6 times as much money this year as you saved last year. If you saved $125 last year, how much money did you save this year?*

SCO: **4.N7 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**
- **using personal strategies for dividing with and without concrete materials**
- **estimating quotients**
- **relating division to multiplication.**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.N11 Demonstrate an understanding of division (limited to division related to multiplication facts up to products of 36 with single digit factors).	**4.N7** Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: • using personal strategies for dividing with and without concrete materials • estimating quotients • relating division to multiplication.	**5.N6** Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. solve a given division problem without a remainder using arrays or base ten materials;
B. solve a given division problem with a remainder using arrays or base ten materials;
C. solve a given division problem using a personal strategy and record the process;
D. create and solve a word problem involving a 1- or 2-digit dividend; and
E. estimate a quotient using a personal strategy, e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5.

(It is <u>not</u> intended that remainders be expressed as decimals or fractions.)

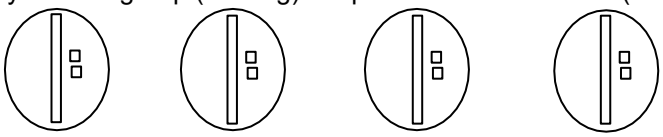
SCO: **4.N7 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**
- **using personal strategies for dividing with and without concrete materials**
- **estimating quotients**
- **relating division to multiplication.**

[C, CN, ME, PS, R, V]

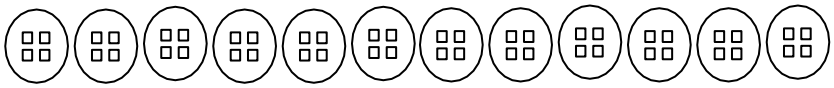
Elaboration

The concept of division needs to be taught in conjunction with multiplication. Students need to know the two meanings for division. These are as follows:

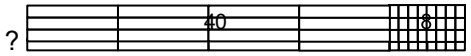
- identifying how many in each group (sharing): 48 pencils in 4 containers (base ten blocks as models).



- identifying how many groups (repeated subtraction): 48 pencils, 4 in each container.



Students can also use the area model with a missing dimension to show the relationship with multiplication.



Students should understand that the **remainder** (the number of units left over) must be less than the **divisor**. Models help to clarify this idea. In grade four, students are expected to express remainders as a digit and not as a fraction or decimal (e.g., a remainder of 7 is written as R7). Students also need to know that the answer for a division sentence is the **quotient** and the number to be divided is the **dividend**.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions of personal interest. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 8, Lesson 8, pp. 299-301
- Unit 8, Lesson 9, pp. 302-304
- Unit 8, Lesson 10, pp. 305-307
- Unit 8, Lesson 11, pp. 308-310
- Unit 8, Game, p. 311
- Unit 8, Unit Problem, pp. 314, 315

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to the Grade 4 Mathematics page at learn.edu.pe.ca for the Mental Math Guide.)

SCO: **4.N7 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**
- **using personal strategies for dividing with and without concrete materials**
- **estimating quotients**
- **relating division to multiplication.**

[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Present division questions in context to identify either the sharing (how many in each group: partitioning) or the repeated subtraction (how many groups) meaning.
- Provide a variety of problem structures that include both of the meanings of division used in a real-life context.
- Provide regular practice in estimation, accompanied by the sharing of strategies.
- Have students create and share problems that include both of the meanings of division. It is helpful for many students to model this for them.

Suggested Activities

- Ask students to use a model to explain to a classmate how to share 86 marbles among five people. Discuss the different strategies used.
- Ask students to use a model to explain to a classmate how to find the number of groups of 6 in 28.
- Ask students to make up division problems about situations in the classroom and post them. Encourage them to give examples of both the sharing and the measurement meanings of division. Invite others to try to guess what the division situations are. For example, 25 ÷ 6 (classmates divided into groups of 6. How many groups?).
- Provide a list of division questions to pairs of students and ask them to estimate a quotient and explain their strategy to their partner and tell whether the estimate is too high or too low and why
- Present the students with a problem and have them choose which of the number sentences provided could be used to solve the problem and why they chose it. Example: Diego saved $96 this month by doing odd jobs for the neighbours. Last month, he saved $8. How many times as much money did he save this month as last month?

  $96 \times 8 = \square$ $\square = 8 \times 96$ $8 \times \square = 96$
  $96 \times \square = 8$ $96 \div 8 = \square$ $8 \div 96 = \square$
  $\square \div 8 = 96$ $96 \div \square = 8$ $8 \div \square = 96$
- Present students with a variety of problems to solve, such as:
- Tell students that there are 77 baseball cards to be shared between 2 students. Ask them how they know that there will be a remainder. What about sharing them among 5 students? 7 students?
- Tina rode her bicycle every day for 8 days. She cycled 68 km. About how far did she ride each day?
- Use base ten models to solve: If the area of a rectangular field is $182m^2$ and the length is 14m, how wide is the field?
- Show students on an overhead projector a number of counters. Have the students count them. Get them to close their eyes while you change the amount by one or two. Ask the students to open their eyes and tell you how the group of counters has changed.

SCO: **4.N7 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**
- **using personal strategies for dividing with and without concrete materials**
- **estimating quotients**
- **relating division to multiplication.**

[C, CN, ME, PS, R, V]

Assessment Strategies

- Have the students use/draw models to show 83 ÷ 3.
- Ask students to explain why the answer to 69 ÷ 3 has to be 10 more than the answer to 39 ÷ 3.
- Ask: How many digits are there in the quotient of $4\overline{)57}$; explain how you know.
- Present the student with the following problem:
  You have 72 marbles to share equally among 4 friends. How many marbles will each friend receive? Explain how you know.
- Ask the student to explain the connection between multiplication and division by using counters or base ten materials. If necessary, coach the student to make an array and show how the array shows both multiplication and division.
- Have the student estimate 93 ÷ 5 and tell whether the estimate is probably too high or too low and why. Ask the student to suggest another division question for which the same estimate would be appropriate.
- Provide a set of base ten blocks. Ask the student to model 3 different division questions of his/her choice and to write the division sentence for each

SCO: **4.N8 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

- **name and record fractions for the parts of a whole or a set**
- **compare and order fractions**
- **model and explain that for different wholes, two identical fractions may not represent the same quantity**
- **provide examples of where fractions are used.**

[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.N12 Demonstrate an understanding of fractions by: explaining that a fraction represents a part of a whole; describing situations in which fractions are used; comparing fractions of the same whole with like denominators.	**4.N8** Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: • name and record fractions for the parts of a whole or a set • compare and order fractions • model and explain that for different wholes, two identical fractions may not represent the same quantity • provide examples of where fractions are used.	**5.N7** Demonstrate an understanding of fractions by using concrete and pictorial representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. represent a given fraction using concrete materials;
B. identify a fraction from its given concrete representation;
C. name and record the shaded and non-shaded parts of a given set;
D. name and record the shaded and non-shaded parts of a given whole;
E. represent a given fraction pictorially by shading parts of a given set;
F. represent a given fraction pictorially by shading parts of a given whole;
G. explain how denominators can be used to compare two given unit fractions with numerator 1;
H. order a given set of fractions that have the same numerator and explain the ordering;
I. order a given set of fractions that have the same denominator and explain the ordering;
J. identify which of the benchmarks: 0, $\frac{1}{2}$, or 1 is closer to a given fraction;
K. name fractions between two given benchmarks on a number line;
L. order a given set of fractions by placing them on a number line with given benchmarks;
M. provide examples of when two identical fractions may not represent the same quantity, e.g., half of a large apple is not equivalent to half of a small apple; half of ten oranges is not equivalent to half of sixteen oranges; and
N. provide an example of a fraction that represents part of a set and, a fraction that represents part of a whole from everyday contexts.

SCO: **4.N8 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

- **name and record fractions for the parts of a whole or a set**
- **compare and order fractions**
- **model and explain that for different wholes, two identical fractions may not represent the same quantity**
- **provide examples of where fractions are used.**

[C, CN, PS, R, V]

Elaboration

In order for students to construct a firm foundation for fraction concepts, they need to experience and discuss activities that promote the following understandings:

- Fractional parts are equal shares or equal-sized portions of a whole or unit. Equal-sized portions of a whole reflect a part of a region or area.
- A unit can be an object or a collection of things/set of items. More abstractly, the unit is counted as 1. On the number line, the distance from 0 to 1 is the unit.
- Fractional parts have special names that tell how many parts of that size are needed to make the whole. For example, thirds require three parts to make a whole.
- The more fractional parts used to make a whole, the smaller the parts. For example, eighths are smaller than fifths.
- The **denominator** of a fraction indicates by what number the whole has been divided in order to produce the type of part under consideration. Thus, the denominator is a divisor. In practical terms, the denominator names the kind of fractional part that is under consideration. The **numerator** of a fraction counts or tells how many of the fraction parts (or the type indicated by the denominator) are under consideration. Therefore, the numerator is a multiplier—it indicates a multiple of the given fractional part (Van de Walle and Lovin, vol. 1, 2006, p. 251).

Presenting fractions in context will make them much more meaningful to students. It is important that students develop visual images for fractions and be able to tell “about how much” a particular fraction represents. By estimating the size of the fraction in relation to the benchmarks of ¼, ½ and ¾ will promote better understanding of fractions. Therefore, students should model fractions using a variety of materials. To strengthen their fraction number sense, it is also recommended that the size of the whole be changed regularly. In grade four, the focus is on students initially developing a firm understanding of fractions less than one.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 5, Lesson 1, pp. 174-177
- Unit 5, Lesson, pp. 178, 179
- Unit 5, Lesson 3, pp. 180-182
- Unit 5, Lesson 4, pp. 183-185
- Unit 5, Lesson 5, pp. 186, 187
- Unit 5, Lesson 6, pp. 188, 189
- Unit 5, Lesson 7, pp. 190-192
- Unit 5, Lesson 8, pp. 193-196
- Unit 5, Unit Problem, pp. 218, 219

SCO: **4.N8 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

- **name and record fractions for the parts of a whole or a set**
- **compare and order fractions**
- **model and explain that for different wholes, two identical fractions may not represent the same quantity**
- **provide examples of where fractions are used.**

[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Present **three models** for fractions: 1) part of a region, 2) part of a set, 3) part of a length or measures.
- Ensure students develop an understanding that a fraction is not meaningful without knowing what the "whole" is.
- Develop conceptual understanding of **fractions** by allowing students to use physical materials to model and compare fractions:
  in which the denominators are the same; e.g., five-eighths is greater than three-eighths
  in which the numerators of the fractions are the same; e.g., three-quarters is greater than three-fifths
  in which the numerator of a fraction is compared to the denominator in deciding its relation to a given benchmark; e.g., three-eighths is less than one-half because three is less than half of eight (Van de Walle and Lovin, vol. 1, 2006, p. 265)
- Remember to use a horizontal line when writing fractions, instead of a slash: e.g., $\frac{2}{3}$

Suggested Activities

- Have the students order a set of fractions. Use sticky notes and put a fraction on 4-8 students' foreheads. The students need to place themselves in order without speaking. Invite students to determine what fraction of the letters in their names are vowels.
- Have students explore fraction relationships among pattern blocks and other materials.
- Show examples and non-examples of specified fractional parts. Have students identify the wholes that are correctly divided into requested fractional parts and those that are not. For each response, have students explain their reasoning. The activity should be done with a variety of models, including length and set models.
- Tell the student that you have 8 coins. Half of them are pennies. More than 1/8 of them are quarters. The others are nickels. Have the student use coins to represent the situation. How much money might you have? Students are then asked to create coin problems using proper fraction notation.
- Provide students with different sizes and shapes of paper and have them estimate and then tear-off different fractional parts, such as one-fifth. Have them explain their thinking. Students can compare their "fifths" as the size of these will vary depending on the size of the whole.

SCO: **4.N8 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

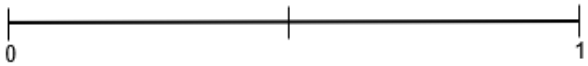
- **name and record fractions for the parts of a whole or a set**
- **compare and order fractions**
- **model and explain that for different wholes, two identical fractions may not represent the same quantity**
- **provide examples of where fractions are used.**

[C, CN, PS, R, V]

Assessment Strategies

- Have students place the following fractions on the number line below and verify their positions using models.

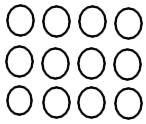
$\frac{5}{6}, \frac{1}{4}, \frac{7}{8}, \frac{5}{8}, \frac{1}{10}$



- Present the following problem to students: Kim ate $\frac{1}{4}$ of her pizza and David ate $\frac{3}{4}$ of his pizza. Kim said that she ate more pizza than David. Explain how Kim could be right by using diagrams and words.
- Place the following pairs of fractions before the student, one at a time. Tell the student to circle the larger fraction and explain orally how he or she knows that the fraction is larger. Then, have them select a manipulative and model the fractions to verify their choice.

$\frac{1}{5}$ $\frac{3}{5}$ $\frac{3}{8}$ $\frac{3}{5}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{4}{8}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{9}{10}$

- Ask the student to tell why, whenever you see a representation of, $\frac{1}{3}$ there is always a $\frac{2}{3}$ associated with it.
- Ask the student to colour $\frac{1}{4}$ of the circles.



SCO: **4.N9 Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**
[C, CN, R, V]
4.N10 Relate decimals to fractions (to hundredths).
[CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
	4.N9 Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically. **4.N10** Relate decimals to fractions (to hundredths).	**5.N8** Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically. **5.N9** Relate decimals to fractions and fractions to decimals (to thousandths). **5.N10** Compare and order decimals (to thousandths), by using: • benchmarks • place value • equivalent decimals.

Achievement Indicators

Students who have achieved these outcomes should be able to:

4.N9

A. write the decimal for a given concrete or pictorial representation of part of a set, part of a region or part of a unit of measure;
B. represent a given decimal using concrete materials or a pictorial representation;
C. explain the meaning of each digit in a given decimal with all digits the same;
D. represent a given decimal using money values (dimes and pennies);
E. record a given money value using decimals;
F. provide examples of everyday contexts in which tenths and hundredths are used; and
G. model, using manipulatives or pictures, which a given tenth can be expressed as hundredths, e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies.

4.N10

A. read decimals as fractions, e.g., 0.5 is zero and five tenths;
B. express orally and in written form a given decimal in fractional form;
C. express orally and in written form a given fraction with a denominator of 10 or 100 as a decimal;
D. express a given pictorial or concrete representation as a fraction or decimal, e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or $\frac{15}{100}$; and
E. express orally and in written form the decimal equivalent for a given fraction, e.g., $\frac{50}{100}$ can be expressed as 0.5.

SCO: **4.N9 Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**
[C, CN, R, V]
4.N10 Relate decimals to fractions (to hundredths).
[CN, R, V]

Elaboration

Number sense with fractions and decimals requires that the students develop a conceptual understanding of fractions and decimals as numbers. To work effectively with fractions and decimals, the students should demonstrate the ability to:

- Represent numbers using words, models, diagrams and symbols and make connections among various representations.
- Give other names for numbers and justify the procedures used to generate the equivalent forms.
- Describe the relative magnitude of numbers by comparing them to common benchmarks, given simple estimates, ordering a set of number, and finding a number between two numbers.

Conceptual understanding of decimals requires that the students connect decimals to **whole numbers** and to fractions. Decimals are shown as an extension of the whole number system by introducing a new place value, the **tenth**'s place, to the right of the one's place. The tenth's place follows the pattern of the base ten number system by iterating one-tenth ten times to make one whole or a unit (Wheatley and Abshire 2002, p. 152). Similarly, the hundredth's place to the right of the tenth's place iterates one-**hundredth** ten times to make one-tenth.

The connection between decimals and fractions is developed conceptually when the students read decimals as fractions and represent them using the same visuals. For example, 0.8 is read as eight-tenths and can be represented using fraction strips or decimal strips (Wheatley and Abshire, 2002). Students should use a variety of materials to model and interpret decimal tenths and hundredths.

4.N9: This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 5, Lesson 9, pp. 197-199
- Unit 5, Lesson 10, pp. 200-202
- Unit 5, Lesson 11, pp. 203, 204
- Unit 5, Lesson 14, p. 212

4.N10: This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 5, Lesson 9, pp. 197-199
- Unit 5, Lesson 10, pp. 200-202
- Unit 5, Lesson 11, pp. 203, 204
- Unit 5, Lesson 14, p. 212

SCO: **4.N9 Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**
[C, CN, R, V]
4.N10 Relate decimals to fractions (to hundredths).
[CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Foster understanding of decimals by ensuring that they be read correctly. For example, 3.4 should be read as 3 and 4 tenths, not 3 point 4, or 3 decimal 4. It is also important that students understand the relationship between fractions and decimals. Example, 12.56 reads as 12 and 56 hundredths.
- Saying decimal numbers correctly will assist students in gaining an understanding of how decimals relate to fractions. By saying 12 and 56 hundredths, 56 is the numerator and 100 is the denominator. Plus, saying the number correctly reinforces that the digits to the right of the decimal are part of the whole number.
- Help students extend the place-value system to decimals by focusing on the basic pattern of ten. Remind students that 10 ones make 1 ten, 10 tens make 1 hundred, etc. Then, extend this pattern to help students understand that it takes 10 equal parts (tenths) to make 1 whole and 100 equal parts (hundredths) to make 1 whole. Explain the place value of the digits to the right of the one are tenths and hundredths.
- Investigate the relationship between 0.01, 0.1 and 1.0 by making analogies and using real-life objects which are sized proportionally.

Suggested Activities

- Use a variety of materials to model number with decimals to the hundredths. Ensure that some models show equivalent fractions/decimals. Shading in 2 tenths of a 100 grid represents the equivalent fraction/decimal of 20 hundredths. Provide students with ample opportunities to write the decimal and the fraction of what the model represents.
- Show students how to make a calculator “count” by ones by pressing +, 1, =, =, … Now have students press +, 0.1, =, =, … when the display reaches 0.9, stop and discuss what this means and what the display will look like with the next press. Many students will predict 0.10 (thinking that 10 comes after 9). When the tenth press produces a display of 1 (Note: calculators never display trailing zeroes), the discussion should revolve around regrouping 10 tenths for a whole. How many presses to get from one whole number to the next? Repeat counting by 0.01 (Van de Walle and Lovin, 2006)
- Ask students to show 2 tenths if a large cube represents one whole; if a flat represents one whole; if a rod represents one whole. Extend this to explore hundredths.

SCO: **4.N9 Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**
[C, CN, R, V]
4.N10 Relate decimals to fractions (to hundredths).
[CN, R, V]

Assessment Strategies

- Have students use a hundred grid to show a capital “T” that takes up more than 0.20 or of the grid and one that takes less than 0.20 of the grid. Express the shaded and un-shaded areas as fractions.
- Ask students where they would find decimal numbers in their daily lives.
- Ask the student to use a model of choice to explain why 0.40 and 0.4 are equivalent.
- Ask the student to give the number that is 0.01 more than, or less than, 3.24.
- Explain to the student that someone forgot to put the decimal in the number 1427. Ask where it would be if the number is less than 100.
- Ask students to read decimal numbers orally. Example: 2.5, 26.9, $127.60, 44.09, 0.02
- Have students write the numbers that you say to them. Example: 3.2, 87.06, 0.14, $5.40
- Plot common fraction and decimal equivalents on a number line. For example: one half, one fourth, one tenth, seventy-five hundredths.
- Have students count forward and backward from any number. For example, count on in tenths from 4.7 or count backwards in hundredths from 4.05.

SCO: **4.N11 Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**
- **using compatible numbers**
- **estimating sums and differences**
- **using mental math strategies to solve problems.**

[C, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.N8 Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by: • using personal strategies for adding and subtracting with and without the support manipulatives • creating and solving problems that involve addition and subtraction concretely, pictorially and symbolically	**4.N11** Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by: • using compatible numbers • estimating sums and differences • using mental math strategies to solve problems.	**5.N11** Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).

Achievement Indicators

Students who have achieved this outcome should be able to:

A. predict sums and differences of decimals using estimation strategies;
B. solve problems, including money and measurement, which involve addition and subtraction of decimals, limited to hundredths;
C. ask students to determine which problems do not require an exact solution;
D. determine the approximate solution of a given problem using compatible numbers;
E. determine an exact solution using mental computation strategies; and
F. count back change for a given purchase.

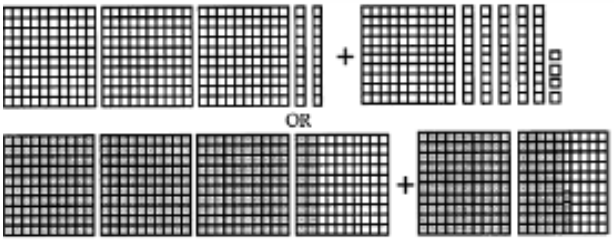
SCO: **4.N11 Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**
- **using compatible numbers**
- **estimating sums and differences**
- **using mental math strategies to solve problems.**

[C, ME, PS, R, V]

Elaboration

It is essential that students recognize that all of the properties and techniques established for the addition and subtraction of whole numbers apply to decimals. Students should recognize that adding or subtracting **tenths** (e.g., 3 tenths and 4 tenths are 7 tenths) is similar to adding or subtracting quantities of other items (e.g., 3 apples and 4 apples are 7 apples). The same is true with **hundredths**. Rather than simply telling students to line up decimals vertically, or suggesting that they "add zeroes," they should be directed to think about what each **digit** represents and what parts go together. For example: 1.62 + 0.3, a student might think, 1 whole, 9 (6 + 3) tenths and 2 hundredths, or 1.92.

Base-ten blocks and hundredths grids continue to be useful models. If a flat represents one whole unit, then 3.2 + 1.54 would be modeled as:



Students need to recognize that **estimation** is a useful skill in their lives. To be efficient when estimating **sums** and **differences** mentally, students must be able to access a strategy quickly and they need a variety from which to choose. Situations must be provided regularly to ensure that students have sufficient practice with mental math strategies and that they use their skills as required. When a problem requires an exact answer, students should first determine if they are able to calculate it mentally; this should be an automatic response.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:
- Unit 5, Lesson 12, pp. 205-208
- Unit 5, Lesson 13, pp. 209-211
- Unit 5, Lesson 14, pp. 212-215
- Unit 5, Unit Problem, pp. 218, 219

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to the Grade 4 Mathematics page at learn.edu.pe.ca for the Mental Math Guide.)

SCO: **4.N11 Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**
- **using compatible numbers**
- **estimating sums and differences**
- **using mental math strategies to solve problems.**

[C, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Encourage students to estimate prior to calculating answers.
- Use a variety of appropriate models, such as base ten blocks and number lines to assist students in their initial consideration of estimation.
- Use ***estimation strategies*** including: Compatible numbers: e.g., 0.72 + 0.23 are close to 0.75 and 0.25 which are compatible numbers so the sum of the decimal numbers must be close to 1. Front-end addition: e.g., 32.3 + 24.5 + 14.1; a student might think "30 + 20 + 10 is 60 and the ones and tenths clustered together make about another 10 for a total of 70." Front-end subtraction: e.g., 4.76 – 3.48; a student might think "4 ones – 3 ones is 1 and 7 tenths – 4 tenths is 3 tenths for a difference of approximately 1 and 3 tenths." Rounding: e.g., 4.39 + 5.2 is approximately 4+ 5 for an estimate of 9.
- Use a think-aloud strategy to model a variety of mental computation strategies.

Suggested Activities

- Give students word problems that require the addition and/or subtraction of whole numbers and decimals. Particularly appropriate contexts are money and measurement (e.g., 3.45 m + 721.6 m; 12.4 kg - 7.25 kg).
- Ask students to determine how best to calculate various problems without a calculator. If they decide to use mental strategies, have them compute and share their strategies.
- Ask students which questions from a group of computations that they could solve mentally. Explain their thinking and identify the strategy they used.
- Ask the student to generate addition or subtraction number sentences using only decimal numbers that would result in an answer which is close to 50. Share their work.
- Ask the students to use a calculator, the digits 7, 5, 1, and 2, the symbols +, =, and the decimal point to produce 7.8 on the display.
- Have the student model a subtraction problem using base-ten, number line, pictorial representations or other suitable materials. Have them record their procedure using paper-pencil.

SCO: **4.N11 Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**
- **using compatible numbers**
- **estimating sums and differences**
- **using mental math strategies to solve problems.**

[C, ME, PS, R, V]

Assessment Strategies

- Ask the student to count back the change for $5.00, if the bill totaled $3.59
- Ask the student to make up a problem with multi-digit numbers for which the calculation could be done mentally. Have them solve it and explain their thinking.
- Ask: How can you know that 265 + 535 < 1000 without actually completing the addition? (Observe if they applied the compatible number strategy).
- Show the student $44.98 + $3.98 + $10.99. Ask him/her to predict the sum and then actually calculate the sum mentally.
- Ask the students to find the difference for 2.3 – 1.8 or other similar computations and explain how they got their answer.
- Tell the students that to solve 9.7 – 8.6, Syesha thought 86 + 11 are 97. Explain her thinking.

PATTERNS AND RELATIONS

SPECIFIC CURRICULUM OUTCOMES

Patterns

4.PR1 – Identify and describe patterns found in tables and charts, including a multiplication chart.

4.PR2 – Reproduce a pattern shown in a table or chart using concrete materials.

4.PR3 – Represent and describe patterns and relationships using charts and tables to solve problems.

Variables and Equations

4.PR4 – Express a given problem as an equation in which a symbol is used to represent an unknown number.

4.PR5 – Solve one-step equations involving a symbol to represent an unknown number.

SCO: **4.PR1 Identify and describe patterns found in tables and charts, including a multiplication chart.**
[C, CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.PR1 Demonstrate an understanding of decreasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).	**4.PR1** Identify and describe patterns found in tables and charts, including a multiplication chart.	**5.PR1** Determine the pattern rule to make predictions about subsequent elements.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. identify and describe a variety of patterns in a multiplication chart;
B. determine the missing element(s) in a given table or chart;
C. identify error(s) in a given table or chart; and
D. describe the pattern found in a given table or chart

SCO: **4.PR1 Identify and describe patterns found in tables and charts, including a multiplication chart.**
[C, CN, PS, V]

Elaboration

Mathematics is often referred to as the science of patterns, as they are found in every mathematical concept and in everyday contexts. Patterns are found in physical and geometric situations as well as in numbers. The same pattern can be found in many different forms (Van de Walle and Lovin 2006, p. 290).

Students should be encouraged to identify and explain patterns that can be found in a variety of tables and charts, including addition and multiplication tables. These patterns can then be used to help students determine an unknown sum or difference. Students should be encouraged to find and explain patterns that occur in tables. It is important that students understand they can use these patterns to determine unknown products or quotients. Students should be familiar with tables that list either all the multiplication facts or some portion of them. For example, the three times table might be shown as:

×	0	1	2	3	4	5	6	7	8	9
3	0	3	6	9	?	15	18	21	24	27

Students should also explore the many patterns in the hundred chart. The hundred chart is a useful model to provide opportunities for students to find and describe a variety of patterns as well as identifying missing elements and errors. Students should use vocabulary, such as **vertical**, **horizontal**, **diagonal**, **row**, and **column** to help describe patterns.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 1, Lesson 1, pp. 6-9
- Unit 3, Lesson 5, pp. 98, 99

SCO: **4.PR1 Identify and describe patterns found in tables and charts, including a multiplication chart.**
[C, CN, PS, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Explore patterns found on a **multiplication grid**, such as:
  numbers in each row and column increase by the same amount
  square numbers are found on the left-right diagonal
  numbers on the left-right diagonal increase by 1, 3, 5, 7,...
  row 4 is double row 2, row 6 is double row 3
  the grid is symmetrical (i.e., numbers are the same both above and under the left-right diagonal)
  when you add the corresponding products of rows 2 and 3, you get the product in row 5; for example, 2 × 4 (8) plus 3 × 4 (12) is the same as 5 × 4 (20)
  when you "cross multiply" any 4 numbers that form a square on the grid, the product is always the same; for example, 2 × 6 = 3 × 4 - also, when you "cross add" these numbers and subtract the sums, you get 1
- Explore patterns found on an **addition chart,** such as:
  even numbers are located on the main diagonal (upper left to lower right), so the sum of a number with itself is always even
  numbers increase by ones across a row, since one more is added for each step to the right
  there are three 2s, four 3s, five 4s, etc.
  the diagonals of any four numbers that form a square will have the same sum
- Explore the many patterns in the **hundred chart**. For example:
  Select four numbers that form a square. Add the two numbers on the diagonal, such as, 59 + 68 and 58 + 69. The sums are equal.

Suggested Activities

- Ask students to look for the even and the odd numbers on a chart and see if they can find the pattern.
- Provide the student with a multiplication grid, addition grid, hundred chart. Ask him/her to describe some of the patterns he/she observes.
- Have students extend several hundreds charts so they can see from 1 to 100, 101 to 200, up to 999. On these charts, use coloured counters to cover numbers forming a pattern and encourage the students to explore the place value representation of the covered numbers; for example, the pattern 13, 23, 33, 43, .., depicted as a vertical column of counters, represents increasing the number by 10 each time.
- Have students explore different versions of hundred charts by changing the order of the numbers, for example, the numbers can spiral or be placed in a different shaped chart, such as a triangle.
- Ask the students to show how one could use the multiplication grid to practice skip counting.

SCO: **4.PR1 Identify and describe patterns found in tables and charts, including a multiplication chart.**
[C, CN, PS, V]

Assessment Strategies

- Ask the students to explain why some column/rows on a multiplication grid have both even and odd numbers.
- Provide a chart or grid with missing numbers and ask students to fill in the missing numbers.
- Provide the students with a multiplication grid. Ask him/her to describe some of the patterns he/she observes.
- Provide a chart/grid/table that has not been used in the class as a model and ask students to identify and explain the patterns that can be found on the chart/grid/table.
- Provide a chart/grid containing errors and ask students to identify and correct them.

SCO: **4.PR2 Reproduce a pattern shown in a table or chart using concrete materials.**
[C, CN, V]
4.PR3 Represent and describe patterns and relationships using charts and tables to solve problems.
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.PR1 Demonstrate an understanding of decreasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).	**4.PR2** Reproduce a pattern shown in a table or chart using concrete materials. **4.PR3** Represent and describe patterns and relationships using charts and tables to solve problems.	**5.PR1** Determine the pattern rule to make predictions about subsequent elements.

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

4.PR2

A. create a concrete representation of a given pattern displayed in a table or chart; and
B. explain why the same relationship exists between the pattern in a table and its concrete representation.

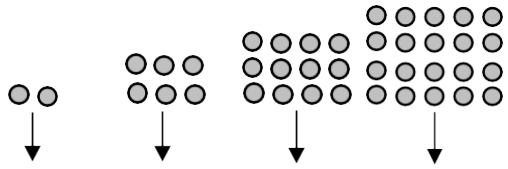
4.PR3

A. extend patterns found in a table or chart to solve a given problem;
B. translate the information provided in a given problem into a table or chart; and
C. identify and extend the patterns in a table or chart to solve a given problem.

SCO: **4.PR2 Reproduce a pattern shown in a table or chart using concrete materials.**
[C, CN, V]
4.PR3 Represent and describe patterns and relationships using charts and tables to solve problems.
[C, CN, PS, R, V]

Elaboration

Once a table or chart is developed, students have two representations of a pattern: the one created with the drawing or materials and the numeric version that is in the table/ grid. When looking for relationships, some students focus on the table and others will focus on the physical pattern. It is important for students to see that whatever relationships they discover, they exist in both forms. When a relationship is found in a table, challenge students to see how that pattern plays out in a physical version (Van de Walle and Lovin 2006, p. 295).



Step	1	2	3	4	5	6	?	…	10
No. of dots	2	6	12	20	30	?	?	…	?

Growing patterns also have a numeric component, the number of objects in each step. A table or "T-chart" can be constructed. Once a table is used for the growing pattern, the materials may become unnecessary. This also leads to the next step which would be to predict what will happen at a particular step (Van de Walle and Lovin 2006, p. 293-294).

4.PR2: This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:
- Unit 1, Lesson 3, pp. 14-17

4.PR3: This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:
- Unit 1, Lesson 3, pp. 10-13
- Unit 1, Unit Problem, pp. 30, 31
- Unit 3, Lesson 6, pp. 102, 103
- Unit 8, Lesson 4, pp. 288, 289
- Unit 8, Lesson 6, pp. 293, 294

SCO: **4.PR2 Reproduce a pattern shown in a table or chart using concrete materials.**
[C, CN, V]
4.PR3 Represent and describe patterns and relationships using charts and tables to solve problems.
[C, CN, PS, R, V]

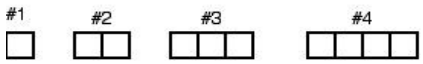
Instructional Strategies

Consider the following strategies when planning lessons:

- Have students not only practice extending patterns with materials and drawings but translate patterns from one medium to another. For example, red and blue pattern blocks become letters or triangles and squares translate to colored tiles. Have students explain how these patterns are mathematically alike.
- Engage students in constructing growing patterns with different materials (toothpicks, multi-link cubes, etc.) They may draw them on grid paper as well. Ask them to describe what is happening as the pattern grows? How is the new step related to the previous one?

Suggested Activities

- Present students with a geometric design series and have them extend the pattern and develop a "T- chart" to go with it. Ask students what the 10th step would be, 12th, 20th, etc. For example:



Squares are the basic building blocks for this series.

Design #	Number of squares
1	1
2	2
3	3
4	4

- Provide a table or "T-chart" involving one arithmetic operation in the pattern, such as the one below. Describe what the data could be about and complete the table.

1	2	3	4	5	6	7	8	9
3	6	9	12	**?**	**?**	**?**	**?**	**?**

SCO: **4.PR2 Reproduce a pattern shown in a table or chart using concrete materials.**
[C, CN, V]
4.PR3 Represent and describe patterns and relationships using charts and tables to solve problems.
[C, CN, PS, R, V]

Assessment Strategies

- Provide a table or chart and have students create a concrete representation of the given pattern displayed in the table/chart.
- Provide several examples of tables and their concrete representations. Ask students to find the pairs.
- Ask students to fill in the missing parts of a table or graph. Drawings or materials may be used to complete these.

SCO: **4.PR4 Express a given problem as an equation in which a symbol is used to represent an unknown number.**
[CN, PS, R]
4.PR5 Solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.PR2 Solve one-step addition and subtraction equations involving a symbol to represent an unknown number.	**4.PR4** Express a given problem as an equation in which a symbol is used to represent an unknown number. **4.PR5** Solve one-step equations involving a symbol to represent an unknown number.	**5.PR2** Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

4.PR4

A. explain the purpose of the symbol, such as a triangle or circle, in a given addition, subtraction, multiplication or division equation with one unknown, e.g. 36 ÷ ☐ = 6;
B. express a given pictorial or concrete representation of an equation in symbolic form;
C. identify the unknown in a story problem, represent the problem with an equation and solve the problem concretely, pictorially or symbolically; and
D. create a problem in context for a given equation with one unknown.

4.PR5

A. solve a given one-step equation using manipulatives;
B. solve a given one-step equation using guess and test;
C. describe, orally, the meaning of a given one-step equation with one unknown;
D. solve a given equation when the unknown is on the left or right side of the equation;
E. represent and solve a given addition or subtraction problem involving a “part-part-whole” or comparison context using a symbol to represent the unknown; and
F. represent and solve a given multiplication or division problem involving equal grouping or partitioning (equal sharing) using symbols to represent the unknown.

SCO: **4.PR4 Express a given problem as an equation in which a symbol is used to represent an unknown number.**
[CN, PS, R]
4.PR5 Solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

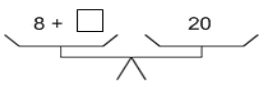
Elaboration

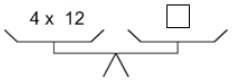
The various representations of patterns, including unknowns, provide valuable tools in making generalizations of mathematical relationships.

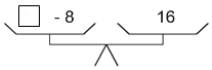
Equality is used to express **relationships**. The symbols used on either side of the equal sign represent a quantity. The equal sign is "a symbol of equivalence and balance" (NCTM 2000, p. 39).

Students should be comfortable using various symbols to represent the unknown, for example, a square, circle, or triangle.

Display a number of samples of balance scales, such as those below. Have students write an equation for each balance scale and then solve it. For example, for the first one: 8 + ☐ = 20, so ☐ = 12.







4.PR4: This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:
- Unit 1, Lesson 4, pp. 18-21
- Unit 1, Lesson 5, pp. 22-24
- Unit 1, Lesson 6, pp. 26, 27

4.PR5: This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:
- Unit 1, Lesson 4, pp. 18-21
- Unit 1, Lesson 5, pp. 22-24
- Unit 1, Unit Problem, pp. 30, 31

SCO: **4.PR4 Express a given problem as an equation in which a symbol is used to represent an unknown number.**
[CN, PS, R]
4.PR5 Solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Build on the students' knowledge from the previous grade in using equations to write addition, subtraction, multiplication and division equations. Connect the concrete, pictorial and symbolic representations consistently as the students develop and demonstrate understanding of equations.
- Use everyday contexts for problems to which the students can relate so that they can translate the meaning of the problem into an appropriate equation using a symbol to represent the unknown number.
- Review the relationship between addition and subtraction number sentences as well as the relationship between multiplication and division number sentences.
- Have the students create problems for a variety of number sentences using the 4 operations.
- Encourage the students to write equations in various ways to represent the meaning of a given problem. $14 + \Delta = 37$ or $\Delta + 14 = 37$; $5 \times \square = 30$ or $\square \times 5 = 30$. Note that the order (commutative) property does not apply to subtraction and division.
- Explain that if the same variable, or unknown, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown; e.g. for $\square + \square = 20$; the unique solution is to place 10 in each of the squares. If, however, two different symbols are used, there may be a number of possible solutions, e.g., $\square + \Delta = 16$, some solutions include 0 + 16, 7 + 9, 12 + 4.

Suggested Activities

- Ask students to create problems to represent the following equations:

$15 + \square = 24$ $\quad$ $\bigcirc + 15 = 24$ $\quad$ $24 = 15 + \hexagon$ $\quad$ $24 = \triangle + 15$

$24 - \pentagon = 15$ $\quad$ $24 - 15 = \diamond$ $\quad$ $15 = 24 - \square$ $\quad$ $\angle = 24 - 15$

- Show students a balance scale and ask them to work with a partner to find an equation that is represented for each of the examples below;

SCO: **4.PR4 Express a given problem as an equation in which a symbol is used to represent an unknown number.**
[CN, PS, R]
4.PR5 Solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

Assessment Strategies

- Tell what the box represents in the following equation: 15 – ☐ = 8.
- You have 24 marbles and your friend gives you some more marbles. Now you have 32 marbles in all. How many marbles did your friend give you?
  a. Write an equation to show what is happening in this problem.
  b. Solve the problem. Explain your thinking.
- Solve the following equation and use a diagram to explain the process.
  34 + 5 = ☐ + 12
- Solve the following equation and explain your thinking.
  Δ – 13 = 20
- Lori said that the box in the equation 6 + 8 = ☐ + 4 could represent more than one number. Is Lori correct? Why or why not?
- Ask the student to explain how to find the missing number in $4 \times \Delta = 100$.

SHAPE AND SPACE

SPECIFIC CURRICULUM OUTCOMES

Measurement

4.SS1 – Read and record time using digital and analog clocks, including 24-hour clocks.

4.SS2 – Read and record calendar dates in a variety of formats.

4.SS3 – Demonstrate an understanding of area of regular and irregular 2D shapes by recognizing that area is measured in square units, selecting and justifying referents for the units cm^2 or m^2, estimating area by using referents for cm^2 or m^2, determining and recording area (cm^2 or m^2), constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.

SCO: **4.SS1 Read and record time using digital and analog clocks, including 24-hour clocks.** [C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.SS1 Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). **3.SS2** Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.	**4.SS1** Read and record time using digital and analog clocks, including 24-hour clocks.	

Achievement Indicators

Students who have achieved this outcome should be able to:

A. state the number of hours in a day;
B. express the time orally and numerically from a 12-hour analog clock;
C. express the time orally and numerically from a 24-hour analog clock;
D. express the time orally and numerically from a 12-hour digital clock;
E. describe time orally and numerically from a 24-hour digital clock;
F. describe time orally as “minutes to” or “minutes after” the hour; and
G. explain the meaning of AM and PM, and provide an example of an activity that occurs during the AM and another that occurs during the PM.

SCO: **4.SS1 Read and record time using digital and analog clocks, including 24-hour clocks.** [C, CN, V]

Elaboration

Although students have not had any explicit teaching related to reading and recording time using clocks by grade four, they have had opportunities in previous grades to explore the passage of time and have an understanding that there are 60 minutes in an hour. As well, students will have had many opportunities to use time through their own experiences with the real world.

By the end of grade four, students should be able to read and record time on 12-hour and 24-hour **analog** and **digital** clocks. Students should read times on clocks to provide information about relevant situations, such as - comparing start and finish times to determine how much time has passed - focusing on times when special events are going to happen.

Students may want to investigate the meaning for some terminology such as, A.M. and P.M. (A.M. is the abbreviation for *ante meridiem* meaning being before noon. P.M. is the abbreviation for *post meridiem* meaning being after noon.)

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 4, Lesson 2, pp. 128-131
- Unit 4, Lesson 3, pp. 132-135
- Unit 4, Lesson 4, pp. 136-138
- Unit 4, Lesson 5, pp. 139-141
- Unit 4, Lesson 6, pp. 142-145

SCO: **4.SS1 Read and record time using digital and analog clocks, including 24-hour clocks.** [C, CN, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students explore that the minute hand and hour hand on an analog clock are different lengths, and that the minute hand is at 6 for the :30 and at 12 or 24 for the :00. Students will be aware that the hour hand moves during the course of the hour, and that, at the :30, it is halfway between two numbers.
- Use an analog clock to introduce the terms “half past”, “quarter after”, and “quarter to”.
- Have students read time to the nearest five minutes. It is important that students are comfortable with skip counting by 5. This provides the opportunity for students to relate the numbers on a clock to the five times table.
- Use a clock that shows not only the numbers from 1 to 12, but also the minute amounts from 5 to 55 beside the numbers from 1 to 11. Furthermore, students will be aware that there are 5 minutes between the numbers on the clock. The long hand on the 3 represents 15 minutes, so two one-minute spaces past the 3 is 17 minutes, etc.

Suggested Activities

- Present students with an analog clock showing just the hour hand. Ask them to predict what the time might be. For example, if the hour hand is somewhere between the 4 and the 5, the time could be anything from *five **past** four* to *five **to** five* depending on the exact placement of the hour hand. Students could also be asked to name an event/activity that often happens at about that time of day.
- Introduce the terms of analog, a.m. and p.m. and discuss the difference between the two and brainstorm activities for each.
- Ask the student to show, on an analog clock, the time (to the nearest half hour) at which they arrive at school, have lunch, go to bed, etc.
- Discuss when a 24-hour clock would be more appropriate to use than a 12-hour clock.
- Have students track events throughout a specific day by means of a time line divided into 15-minute segments. Students should record the time of the activity or event and note it at the appropriate spot on a time line.
- Have the student make a list of the times when the minute hand and the hour hand just about line up as well as other patterns, such as all of the times that include a 4 in a 24-hour period.
- Have students work in pairs to set up a schedule in which every student will get 10 minutes on the computer, starting at 8:30 a.m. Ask them if all students can have time on the computer before noon and, if not, how long it will take to finish after lunch. At what time will the last one finish? (Remind them to leave time for recess.)

SCO: **4.SS1 Read and record time using digital and analog clocks, including 24-hour clocks.** [C, CN, V]

Assessment Strategies

- Ask students what time might it be if the minute hand and hour hand are opposite one another.
- Ask the student to move the hands of an analog clock to match the time shown on a digital clock.
- Ask students to express the time orally and numerically that has been created on a 12-hour analog clock, 24-hour analog clock, and 12-hour digital clock.
- Ask students to name an activity they would typically do in the p.m.? a.m.?
- Ask students how many hours are in a day and a half.

SCO: **4.SS2 Read and record calendar dates in a variety of formats.**
[C, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.SS1 Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). **3.SS2** Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.	**4.SS2** Read and record calendar dates in a variety of formats.	

Achievement Indicators

Students who have achieved this outcome should be able to:

A. write dates in a variety of formats, e.g., *yyyy/mm/dd*, *dd/mm/yyyy*, March 21, 2006, *dd/mm/yy;*
B. relate dates written in the format *yyyy/mm/dd* to dates on a calendar; and
C. identify possible interpretations of a given date, e.g., 06/03/04.

SCO: **4.SS2 Read and record calendar dates in a variety of formats.**
[C, V]

Elaboration

By grade four, students should already know the days of the week, the months of the year, and the four seasons. As well, students will have already developed a sense of the arrangement of our year in relation to the months and seasons; for example, January is the first month of a new year and is early in our winter season.

Using **calendars** throughout the school year strengthens the students' sense of time. Each month brings a new calendar to explore. Students would be familiar with calendars through their home and school experiences by grade 4. In previous grades, teachers may have explored calendars during explorations of units of time, such as days, weeks, months, and years. Calendars may also have been used to assist in developing number sense and for exploring patterns.

Students need to become aware of the variety of ways **dates** can be recorded. In grade four, students are expected to read, record, and interpret calendar dates in a variety of ways.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 4, Lesson 1, pp. 124-127

SCO: **4.SS2 Read and record calendar dates in a variety of formats.**
[C, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Send students on a scavenger hunt and have them bring in different dates from magazines, posters, items printed from the Internet, cheques and newspapers. Share, discuss and display the variety of formats as a class.
- Have students predict how many days and/or weeks there are in a year. Verify using calendars.
- Have them explore what calendar dates can be confused with other dates when they are interpreted using various formats.
- Investigate a special holiday which has a date that fluctuates, such as Labour Day. Have students record the date(s) of this holiday over the past five years. Share their findings.

Suggested Activities

- Ask students to write about their favourite format for recording a calendar date and justify their choice.
- Have students interpret a particular date such as 06/04/03. Discuss that there is no standard or consistent format and why some dates may be misinterpreted unless you know the format.
- Provide students with a list of dates recorded in metric notation and have them order them from past to present.

SCO: **4.SS2 Read and record calendar dates in a variety of formats.**
[C, V]

Assessment Strategies

- Show the student a calendar for the year. Ask him/her to point out the day's date. Have them record it using the format month/day/year.
- Ask the student to identify two calendar dates which cannot be confused with other dates when they are interpreted regardless of the format.
- Have the student write their birth date using 3 different formats.
- Have the student identify their favourite day of year and write the date in metric notation. (year, month, day)

SCO: **4.SS3 Demonstrate an understanding of area of regular and irregular 2D shapes by:**

- **recognizing that area is measured in square units**
- **selecting and justifying referents for the units cm^2 or m^2**
- **estimating area by using referents for cm_2 or m^2**
- **determining and recording area (cm^2 or m^2)**
- **constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.SS5 Demonstrate an understanding of perimeter of regular and irregular shapes by: estimating perimeter, using referents for cm or m; measuring and recording perimeter (cm, m); constructing different shapes for a given perimeter (cm, m); to demonstrate that many shapes are possible for a perimeter.	**4.SS3** Demonstrate an understanding of area of regular and irregular 2D shapes by: • recognizing that area is measured in square units • selecting and justifying referents for the units cm^2 or m^2 • estimating area by using referents for cm^2 or m^2 • determining and recording area • constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.	**5.SS1** Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and make generalizations. **5.SS2** Demonstrate an understanding of measuring length (mm) by: • selecting and justifying referents for the unit mm • modeling and describing the relationship between mm and cm units, and between mm and m units.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. describe area as the measure of surface recorded in square units;
B. identify and explain why the square is the most efficient unit for measuring area;
C. provide a referent for a square centimetre and explain the choice;
D. provide a referent for a square metre and explain the choice;
E. determine which standard square unit is represented by a given referent;
F. estimate the area of a given 2D shape using personal referents;
G. determine the area of a regular 2D shape and explain the strategy;
H. determine the area of an irregular 2D shape and explain the strategy;
I. construct a rectangle for a given area; and
J. demonstrate that many rectangles are possible for a given area by drawing at least two different rectangles for the same given area.

SCO: **4.SS3 Demonstrate an understanding of area of regular and irregular 2D shapes by:**

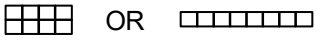
- **recognizing that area is measured in square units**
- **selecting and justifying referents for the units cm^2 or m^2**
- **estimating area by using referents for cm_2 or m^2**
- **determining and recording area (cm^2 or m^2)**
- **constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.**

[C, CN, ME, PS, R, V]

Elaboration

Students should understand that the area of a shape can be expressed as the number of units required to cover a certain surface. Van de Walle and Lovin define area as "a measure of the space inside a region or how much it takes to cover a region" (2006, p. 234). The **square unit** is the most efficient unit to use for measuring area. It is helpful for students to use a **referent** for the single unit of measure and iterate this unit mentally to obtain the estimate; e.g., use the size of the fingernail on your smallest finger as a referent for 1 cm^2.

Once students have developed the meaning of measurement, it is time to move on to connect multiplication in an **array** format to determine the area of rectangles (Van de Walle and Lovin 2006, p.263). Students should relate the area of a rectangle to the product of the numbers describing its length and width. Conversely, any factor of the number representing the area of a rectangle can be one dimension of a rectangle with that area. For example,
consider rectangles with an area of 8 square units.

 OR

It is important for students to explore not only the areas of rectangles, but areas of other shapes as well. Through these investigations students should recognize that objects of different shapes can have the same area. Encourage students to find shapes using partial squares.

Opportunities should be provided for students to estimate and calculate the area of various surfaces. Laying an acetate centimetre grid over objects is helpful when determining surface area. Students might investigate the area of shapes drawn on centimetre dot paper. Strategies for doing this include adding squares and half squares within the figure; placing a rectangle around the shape, determining its area, and subtracting the area of the "extra" pieces.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 4, Lesson 7, pp. 146-149
- Unit 4, Lesson 8, pp. 150-152
- Unit 4, Lesson 9, pp. 153-155
- Unit 4, Lesson 10, pp. 156-158
- Unit 4, Lesson 11, pp. 159-161
- Unit 4, Lesson 12, pp. 162, 163
- Unit 4, Lesson 13, pp. 164, 165
- Unit 4, Unit Problem, pp. 168, 169

SCO: **4.SS3 Demonstrate an understanding of area of regular and irregular 2D shapes by:**

- **recognizing that area is measured in square units**
- **selecting and justifying referents for the units cm^2 or m^2**
- **estimating area by using referents for cm^2 or m^2**
- **determining and recording area (cm^2 or m^2)**
- **constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.**

[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use referents for area and estimating area. Review that referents are familiar objects which students can use to which students can refer for estimating (e.g., the width of the pinky finger is about 1 cm). Ask the students to suggest a suitable referent for 1 cm^2 and explain why they think it would work. Have them use this referent to estimate the area of a book cover. Ask them to check their estimate by finding the area of the book cover. Discuss possible referents for 1 m^2. Have the students use their referents and estimate the area of a large tabletop or a section of the classroom floor and check their estimates.
- Have students use colour tiles or grid paper to investigate the numbers from 1 to 30 to see how many different rectangles can be made for each. Students should record their results and look for patterns.
- Use a transparency of a centimetre grid to confirm the estimate of an area of an irregular shape.

Suggested Activities

- Have students explore how a diagonal of rectangle(s) divides the shape in half.

- Provide the students with rectangular papers that each measure 10 cm by 13 cm. Have them estimate how many copies of each shape of pattern block it would take to cover the rectangle. Then have the students measure the area using each of the shapes in turn.

____ ____ ____ 20 squares ___

- Make the design below on an overhead geoboard and ask a student to explain to the class how to find the area. Have the students alter the shape on their geoboards to increase the area by 1 cm^2.

- Provide the students with tiles and centimetre grid paper. Give them the following instructions: For each of the areas from 1 cm^2 to 20 cm^2, find all the possible rectangular arrays using whole numbers. For example, the possible arrays for an area of 6 cm^2 would be as follows:

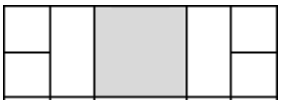
SCO: **4.SS3 Demonstrate an understanding of area of regular and irregular 2D shapes by:**
- **recognizing that area is measured in square units**
- **selecting and justifying referents for the units cm^2 or m^2**
- **estimating area by using referents for cm_2 or m^2**
- **determining and recording area (cm^2 or m^2)**
- **constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.**

[C, CN, ME, PS, R, V]

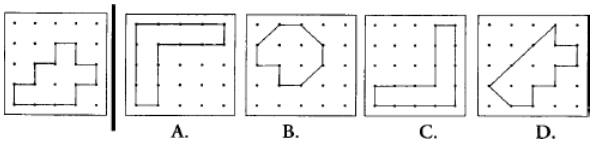
Assessment Strategies

- Ask students to predict how many different arrays can be made to represent 36 cm^2. Draw the arrays to check your prediction.
- Have students estimate the area for each of the following pairs of congruent shapes. Decide if the shaded part has the same area in each pair of shapes. Explain your thinking.

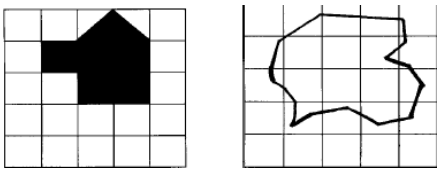
- Ask the student to estimate the area of a rectangle and explain what referent he or she used.
- Explain why area is measured in square units.
- Find the area of the shaded part.
  The area of the entire design below is 12 m^2.
  Explain your thinking.



- Ask students to circle the letters of the shapes that have the same area as the one on the left.



- Ask why it is easier to find the area of the shape on the left than the one on the right.
  Give an estimate for the area of the shape on the right.



STATISTICS AND PROBABILITY

SPECIFIC CURRICULUM OUTCOMES

Data Analysis

4.SP1 – Demonstrate an understanding of many-to-one correspondence.

4.SP2 – Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.

SCO: **4.SP1 Demonstrate an understanding of many-to-one correspondence.**
[C, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
3.SP1 Collect first-hand data and organize it using: tally marks; line plots; charts; lists to answer questions.	**4.SP1** Demonstrate an understanding of many-to-one correspondence.	**5.SP2** Construct and interpret double bar graphs to draw conclusions.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. compare graphs in which different intervals or correspondences are used and explain why the interval or correspondence was used;
B. compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences and explain how they are the same and different;
C. explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence; and
D. find examples of graphs in which many-to-one correspondence is used in print and electronic media, such as newspapers, magazines and the Internet, and describe the correspondence used.

SCO: **SP1 Demonstrate an understanding of many-to-one correspondence.**
[C, R, T, V]

Elaboration

Prior to grade four students have had opportunities to collect and display data in **pictographs** and **bar graphs**. As they investigate a wider range of topics, they may discover that the data they collect is too large to display in a graph using a one-to-one correspondence (i.e. having each symbol or number on the bar graph represent one piece of data). Students need to be introduced to the concept of using a **many-to-one correspondence** or **scale** when they are creating graphs to display large amounts of data. In grade four, students should begin to make decisions about what symbol to use and what that symbol should represent. These decisions are based on the data being used.

Students need to be given many opportunities to explore what scale is most appropriate for their set of data. For example, if they want to display a graph to show their marble collection and they have 36 blue, 28 red, and 42 clear, students may decide to draw symbols to each represent 5 marbles or create a scale in a bar graph that increases by 2. If the numbers were all less than 20, it is usually more appropriate to use a one-to-one correspondence. However, if the numbers were much larger, students may find it better to use 10 as the scale. Students should discuss their data displays and be able to explain why they chose their scale. It is important for students to ensure that the scale in their data displays is consistent. For example, if they are creating a bar graph that has a scale of two, all of the numbers need to increase by 2 (2, 4, 6, 8, 10, 12 … and not 2, 4, 6, 7, 8, 9, 10, 12…). Depending on the data and the scale that is selected, it may become necessary to create partial symbols and bars that fall between numbers.

As students compare their own graphs and those from other sources, they should examine how the graphs are similar and different. Students should discuss why they think the interval or correspondence was chosen and what other scales may have also been used.

Deciding on what scale to use requires students to apply their knowledge of multiplication and therefore, it is very helpful for students to have a good knowledge of these facts.

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 7, Lesson 1, pp. 254-257
- Unit 7, Lesson 2, pp. 258-261
- Unit 7, Lesson 3, pp. 262-265
- Unit 7, Unit Problem, pp. 274, 275

SCO: **SP1 Demonstrate an understanding of many-to-one correspondence.**
[C, R, T, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Allow students to decide on the scale for their data displays but ensure that they can justify their choice.
- Have students work with a variety of sets of data, so they will have experiences creating different scales.
- Use data display software or websites for students to quickly compare graphs that have different scales.

Suggested Activities

- Have the student redraw a pictograph so that each symbol represents 4, instead of 2. Ask the student which graph he/she prefers and to give reasons for the choice. Ask if there is another way to display the data which might be clearer.
- Provide data for a bar graph: such as Favourite Sports (hockey 36, baseball 22, basketball 30, volleyball 16). Have the student select a scale.
- Ask the students to determine the scale for a bar graph to display the number of students traveling on each different school bus in the morning. Each step along a bar is to represent more than one student.
- Have students explore other applications of many-to-one correspondence, such as the use of scale in mapping.

SCO: **SP1 Demonstrate an understanding of many-to-one correspondence.**
[C, R, T, V]

Assessment Strategies

- Ask why a symbol in a pictograph usually represents more than 1.
- Provide students with two graphs: one that displays one-to-one correspondence and the other displays many-to-one correspondence. Explain the similarities and differences.
- Provide students with a set of data and have them create a scale for it. Ask the student to support their choice.
- Ask the student for an example of when it would be appropriate to use a one-to-one correspondence using a real-life context.
- Ask the student for an example of when it would be more appropriate to use a many-to-one correspondence using a real-life context.

SCO: **4.SP2 Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions. [C, PS, R, V]**

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

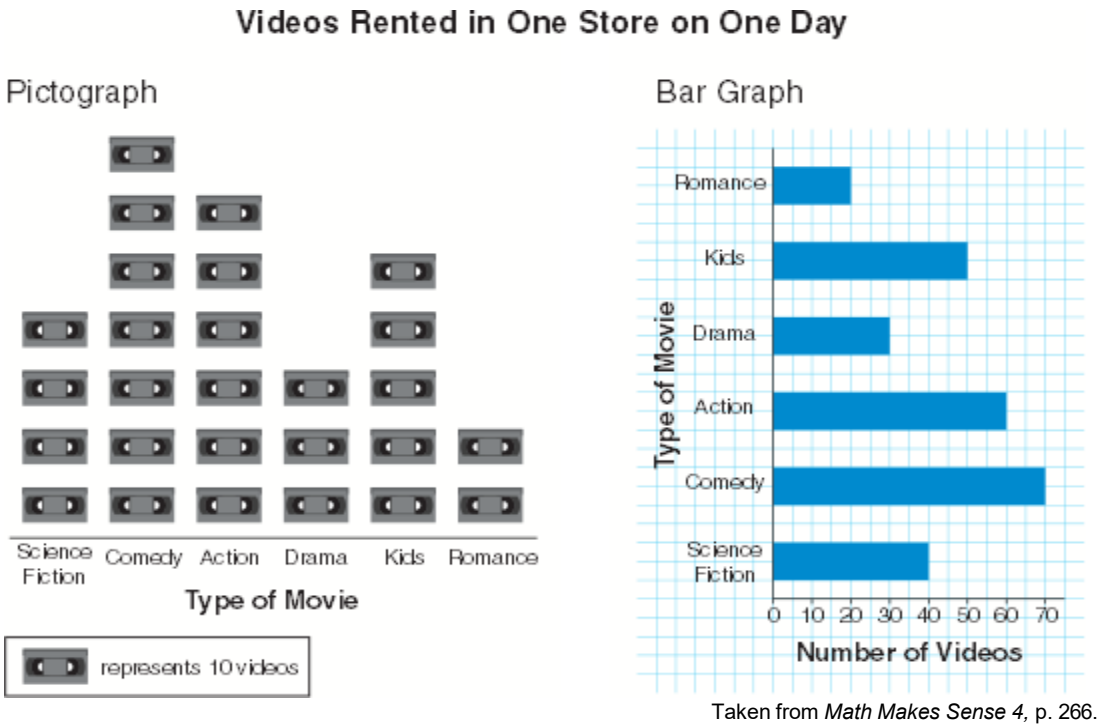
Scope and Sequence

Grade Three	Grade Four	Grade Five
3.SP1 Collect first-hand data and organize it using: tally marks; line plots; charts; lists to answer questions.	**4.SP2** Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.	**5.SP2** Construct and interpret double bar graphs to draw conclusions.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. identify an interval and correspondence for displaying a given set of data in a graph and justify the choice;
B. create and label (with categories, title and legend) a pictograph to display a given set of data using many-to-one correspondence, and justify the choice of correspondence used;
C. create and label (with axes and title) a bar graph to display a given set of data using many-to-one correspondence, and justify the choice of interval used; and
D. answer a given question using a given graph in which data is displayed using many-to-one correspondence.



Taken from *Math Makes Sense 4,* p. 266.

SCO: **SP2 Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions. [C, PS, R, V]**

Elaboration

Statistical literacy is a life skill that effective citizens use to read, question, and interpret data in our world. In elementary school, students are expected to develop an understanding of that graphs tell us about information and that different types of representations tell different things about the data. Whether data is presented in graphical form or in numeric form, students learn to pose and answer questions about the information. The value of having students actually construct their own graphs is not so much that they learn the techniques but that they are personally invested in the data and that they learn how a graph conveys information (Van de Walle and Lovin 2006, p. 329).

Bar graphs use the lengths or heights of bars to represent quantities; whereas, **pictographs** use pictures or symbols to display data. Students extend their understanding of constructing graphs and interpreting data from previous grades by exploring **vertical** and **horizontal** displays that require a **many-to-one correspondence**. When creating **pictographs** and **bar graphs**, it is important for their displays to include a **title**, **labelled categories** (including units if needed),**labelled axes**, an **appropriate scale, correctly plotted data** and a **legend** or **key** (when applicable). When creating graphs, it is helpful for students to use *grid paper* and to become comfortable using a *ruler* to aid in drawing a straight line.

Once students have constructed a graph, it is important for students to have an opportunity to make *observations* and *interpret* the data. They should also have experiences discussing other graphs that they can find, such as in newspapers and magazines, and on television and the Internet. Questioning should be ongoing whenever students use graphs to encourage students to *interpret* the data presented and to draw *inferences*. It is important to ask questions that go beyond simplistic reading of a graph. Both literal questions and inferential questions should be posed. For example:

- What can you tell about …..by looking at this graph?
- Why do you think the creator chose this scale / legend?
- How many more/less than …..?
- Based on the information presented in the graph, what other conclusions can you make?
- Why do you think…..?

This specific curriculum outcome is addressed in *Math Makes Sense 4* in the following units:

- Unit 7, Lesson 1, pp. 254-257
- Unit 7, Lesson 2, pp. 258-261
- Unit 7, Lesson 3, pp. 262-265
- Unit 7, Lesson 4, pp. 266-269
- Unit 7, Unit Problem, p. 274, 275

SCO: **SP2 Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.**
[C, PS, R, V]

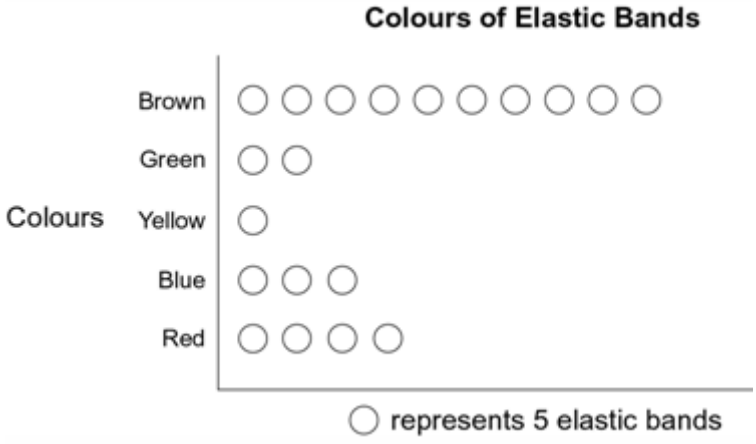
Instructional Strategies

Consider the following strategies when planning lessons:

- Provide experiences to ensure that when creating bar graphs and pictographs, student have an understanding of the importance of a common base line and many-to-one matching of objects in the various categories.
- Use pictographs based on a many-to-one correspondence (i.e., a picture represents a group of items).
- Have students interpret and create various bar graphs and pictographs that run horizontally and vertically.
- Create graphs primarily in the context of other investigations, rather than as an isolated activity to achieve the curriculum outcome.

Suggested Activities

- Suggest that students create a graph that shows the most popular authors, movies, types of food, etc. of class members. Have some students create a bar graph that shows the results of data in scale of 2 and other groups can use a scale of 3, 4, and 5. Have students explain which graph displays the most appropriate use of the data.
- Show a graph like the one below. Discuss that each circle represents 5 elastic bands. Ask questions, such as: “What is the favourite colour of elastic bands? How many more blue bands than yellow bands? How many elastic bands were counted? If you wanted to sell elastic bands, which colours would you choose to sell and why?” Order the elastic bands from most plentiful to least plentiful.



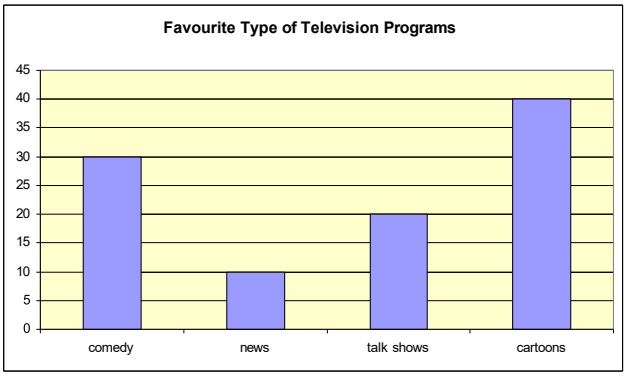
Taken from *Math Makes Sense 4*, Unit 7 Test Sample Solutions

- Have students discuss what kinds of information they can get from reading pre-made bar graphs and pictographs that display the use of many-to-one correspondence.

SCO: **SP2 Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.**
[C, PS, R, V]

Assessment Strategies

- Was this graph created correctly? If not, what is missing? What questions might be answered by interpreting this graph?



- Create and correctly label a pictograph and bar graph using the table below about Favourite Movies using many-to-one correspondence and justify the choice of correspondence used.

Adventure	9
Comedy	8
Drama	5
Science Fiction	10

REFERENCES

American Association for the Advancement of Science [AAAS-Benchmarks]. *Benchmark for Science Literacy*. New York, NY: Oxford University Press, 1993.

Banks, James A. and Cherry A. McGee Banks. *Multicultural Education: Issues and Perspectives*. Boston: Allyn and Bacon, 1993.

Black, Paul and Dylan Wiliam. "Inside the Black Box: Raising Standards Through Classroom Assessment." *Phi Delta Kappan*, 20, October 1998, pp.139-148.

British Columbia Ministry of Education. *The Primary Program: A Framework for Teaching*, 2000.

Council of Atlantic Ministers of Education and Training. *Atlantic Canada Mathematics Assessment Resource Entry-3*, 2008.

Davies, Anne. *Making Classroom Assessment Work*. British Columbia: Classroom Connections International, Inc., 2000.

Hope, Jack A.. *et. al*. *Mental Math in the Primary Grades*. Dale Seymour Publications, 1988.

National Council of Teachers of Mathematics. *Mathematics Assessment: A Practical Handbook*. Reston, VA: NCTM, 2001.

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

New Brunswick Department of Education. *Mathematics: GRADE 4 Curriculum* (draft). January 2008.

Rubenstein, Rheta N. *Mental Mathematics Beyond the Middle School: Why? What? How?* September 2001, Vol. 94, Issue 6, p. 442.

Shaw, Jean M. and Mary Jo Puckett Cliatt. "Developing Measurement Sense." In P.R. Trafton (ed.), *New Directions for Elementary School Mathematics* (pp. 149–155). Reston, VA: NCTM, 1989.

Steen, Lynn Arthur (ed.). *On the Shoulders of Giants – New Approaches to Numeracy.* Washington, DC: National Research Council, 1990.

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 5-8.* Boston: Pearson Education, Inc. 2006.

Western and Northern Canadian Protocol. *Common Curriculum Framework for K-9 Mathematics*, 2006.