Prince Edward Island
CANADA
Education and Early Years

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

Grade 5

Acknowledgments

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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base in its creation. From examining the curriculum proposed throughout Canada to securing the latest research in the teaching of mathematics, the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work and study today and in the future. Essential graduation learnings are cross-curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will be able to demonstrate knowledge, skills and attitudes in the following essential graduation learnings:

- Respond with critical awareness to various forms of the arts and be able to express themselves through the arts.
- Assess social, cultural, economic and environmental interdependence in a local and global context.
- Use the listening, viewing, speaking and writing modes of language(s), and mathematical and scientific concepts and symbols to think, learn and communicate effectively.
- Continue to learn and to pursue an active, healthy lifestyle.
- Use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts.
- Use a variety of technologies, demonstrate an understanding of technological applications and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate high expectations for students in mathematics education to all educational partners. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to:

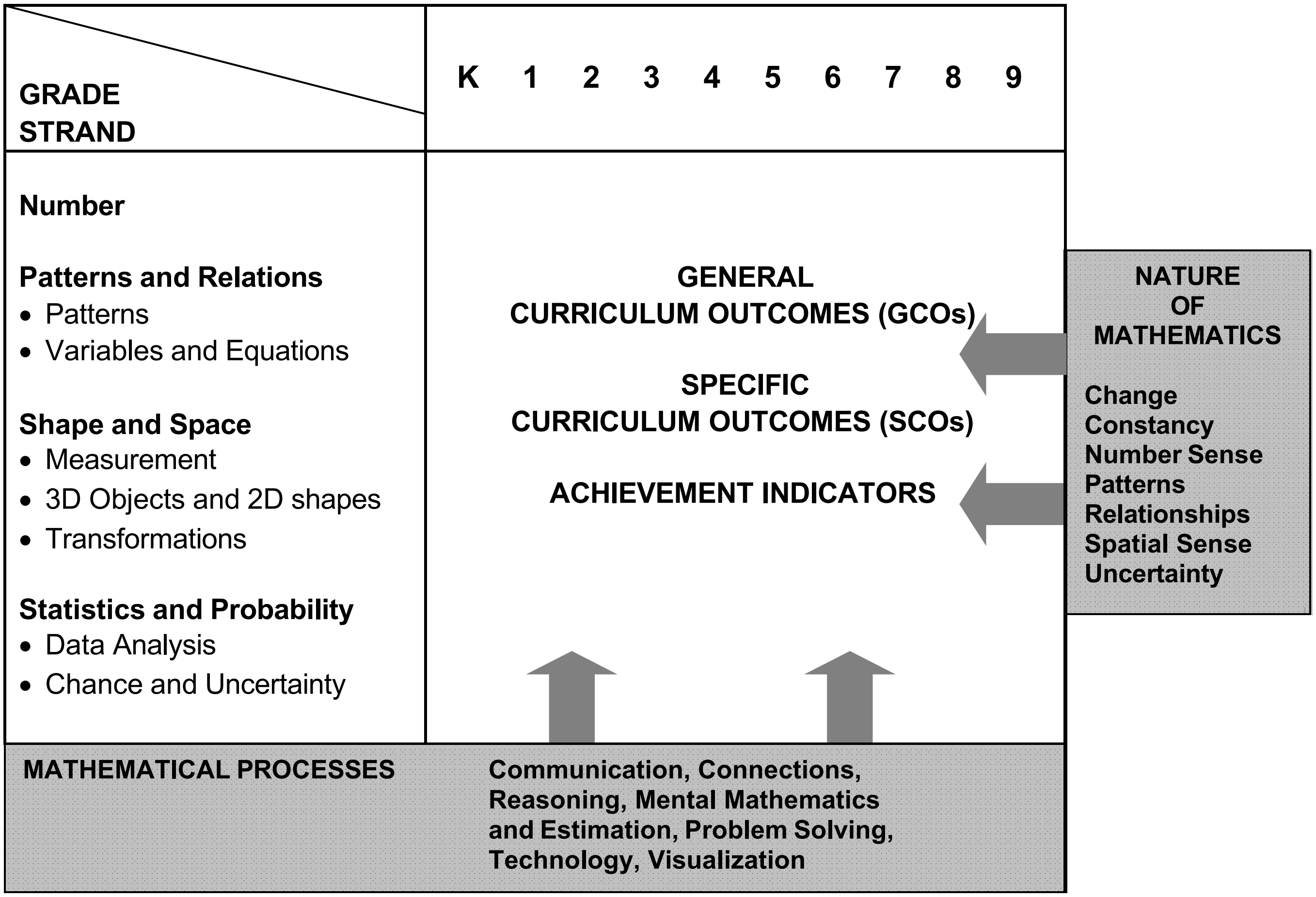
- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning; and
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks; and
- exhibit curiosity.

Conceptual Framework for K – 9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes:



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely: **Number**, **Patterns and Relations**, **Shape and Space**, and **Statistics and Probability**. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to:

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]** and
- develop visualization skills to assist in processing information, making connections and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

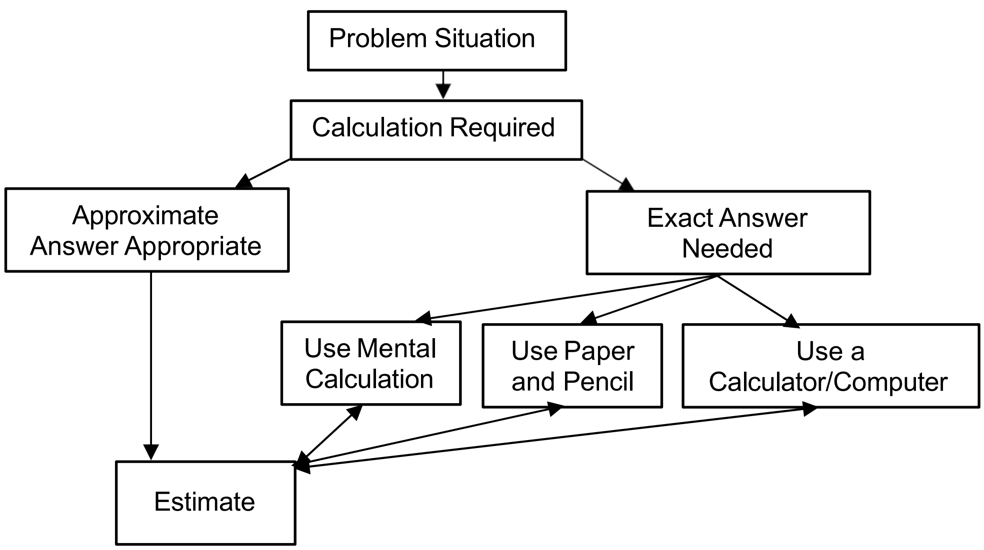
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001). Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision making process as described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you. . . ?" or "How could you. . . ?" the problem-solving approach is being modeled. Students develop their own problem-solving strategies by being open to listening, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modeled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- use estimation
- guess and check
- look for a pattern
- make an organized list or table
- use a model
- work backwards
- use a formula
- use a graph, diagram or flow chart
- solve a simpler problem
- use algebra

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations; and
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K–3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3D objects and 2D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, to determine when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **patterns**, **relationships**, **spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution;
- the sum of the interior angles of any triangle is 180^0; and
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3D and 2D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3D objects and 2D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3D or 2D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations;
- the volume of a rectangular solid can be calculated from given dimensions; and
- doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking and critical thinking and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Connections across the Curriculum

There are many possibilities for connecting Grade 5 mathematical learning with the learning occurring in other subject areas. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should reduce some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a parent will have a clearer understanding of the mathematics curriculum and the progress of his or her child in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every classroom comprises students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters but should be designed to help all students, whether strong, weak or average, to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson, from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean that not only should enrolments of students of both genders and various cultural backgrounds in public school mathematics courses reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English proficiency and cultural differences must not be a barrier to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and coordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education (p.60)." The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers that will facilitate "communicating to learn mathematics and learning to communicate mathematically (NCTM, p.60)."

To this end:

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counselors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated, with appropriate language support, to both students and parents; and
- to verify that barriers have been removed, educators should monitor enrollment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development – such as poverty alleviation, human rights, health, environmental protection and climate change – into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental and economic perspective and explores how those factors are inter-related and inter-dependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database

Resources for Rethinking, found at http://r4r.ca/en. It provides teachers with access to materials that integrate ecological, social and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, whether teaching has been effective or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated and how results are communicated send clear messages to students and others.

➢ Assessment

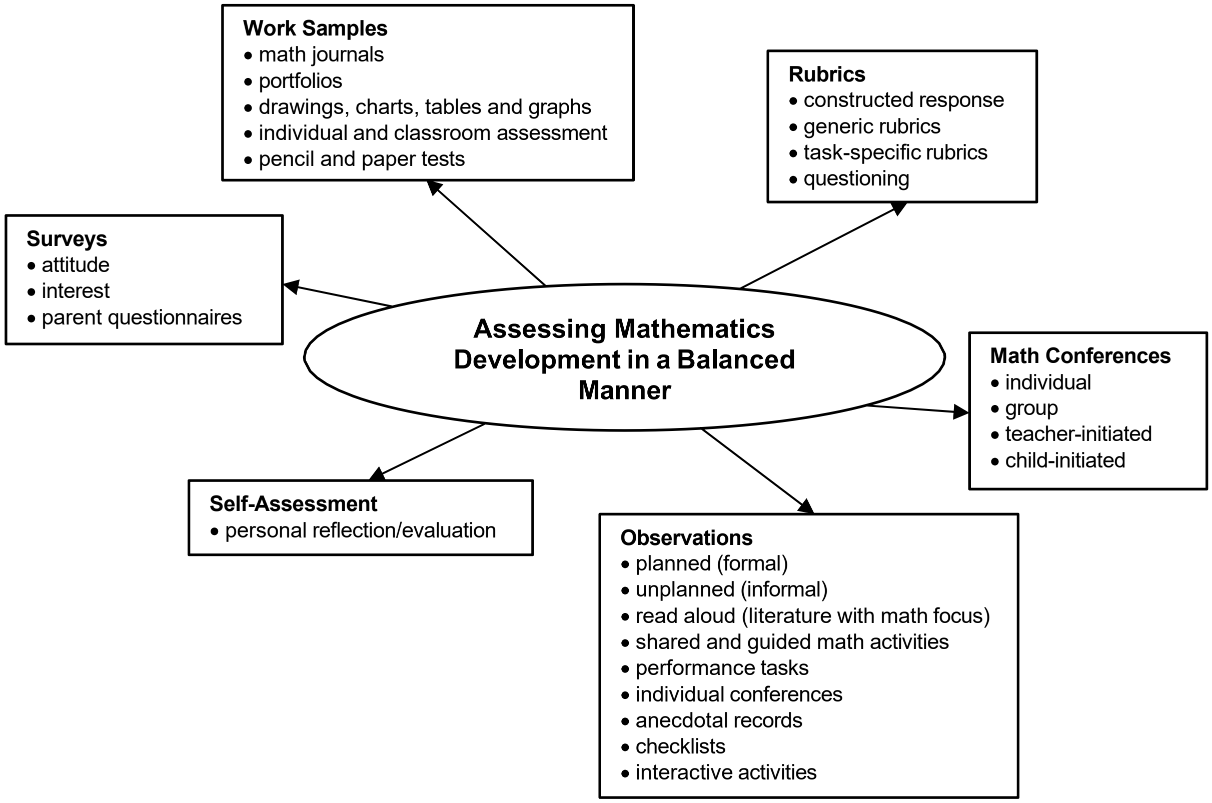
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as:

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources including:

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment

This balanced approach for assessing mathematics development is illustrated in the diagram below.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used:

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used:

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used:

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;
- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon and summarizing assessment information and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires:

- student learning;
- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information; and
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes and phone calls.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.

- Assessment reports should be clear, accurate and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that:

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes; and
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are **Number, Patterns and Relations, Shape and Space**, and **Statistics and Probability**. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense.
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems.
	3D Objects and 2D Shapes: Describe the characteristics of 3D objects and 2D shapes, and analyze the relationships among them.
	Transformations: Describe and analyze position and motion of objects and shapes.
Statistics and Probability (SP)	**Data Analysis**: Collect, display, and analyze data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

The first two pages for each outcome contain the following information:

- the corresponding **strand** and **General Curriculum Outcome;**
- the **Specific Curriculum Outcome**(s) and the mathematical **processes** which link this content to instructional methodology;
- the **scope and sequence** of concept development related to this outcome(s) from grades 4- 6;
- a list of **achievement indicators**; and

  Students who have achieved a particular outcome should be able to demonstrate their understanding in the manner specified by the achievement indicators. It is important to remember, however, that these indicators are not intended to be an exhaustive list for each outcome. Teachers may choose to use additional indicators as evidence that the desired learning has been achieved.
- an **elaboration** of the outcome.

The last two pages for each outcome contain lists of **instructional strategies** and **strategies for assessment**.

The primary use of this section of the guide is as an ***assessment for learning*** (formative assessment) tool to assist teachers in planning instruction to improve learning. However, teachers may also find the ideas and suggestions useful in gathering ***assessment of learning*** (summative assessment) data to provide information on student achievement.

The **Mental Math Guide,** which outlines the **Fact Learning, Mental Computation and Estimation** strategies for this grade level, can be found at learn.edu.pe.ca. Included is an **Overview of the Thinking Strategies in Mental Math** for grades one to six complete with a description of each strategy as well as a scope and sequence table of the strategies for the elementary grades.

NUMBER

SPECIFIC CURRICULUM OUTCOMES

5.N1 – Represent and describe whole numbers to 1 000 000.

5.N2 – Use estimation strategies including front-end rounding, compensation, and compatible numbers in problem-solving contexts.

5.N3 – Apply mental mathematics strategies and number properties, such as skip counting from a known fact, using doubling or halving, using patterns in the 9s facts, and using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts.

5.N4 – Apply mental mathematics strategies for multiplication, such as annexing then adding zero, halving and doubling, and using the distributive property.

5.N5 – Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.

5.N6 – Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.

5.N7 – Demonstrate an understanding of fractions by using concrete and pictorial representations to create sets of equivalent fractions and compare fractions with like and unlike denominators.

5.N8 – Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.

5.N9 – Relate decimals to fractions (to thousandths).

5.N10 – Compare and order decimals (to thousandths) by using benchmarks, place value, and equivalent decimals.

5.N11 – Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).

SCO: **5.N1 Represent and describe whole numbers to 1 000 000.**
[C, CN, V, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.N1 Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.	**5.N1** Represent and describe whole numbers to 1 000 000.	**6.N1** Demonstrate an understanding of place value for numbers • *greater than one million* • *less than one thousandth.*

Achievement Indicators

Students who have achieved this outcome should be able to:

A. write a given numeral using proper spacing without commas, e.g., 934 567;
B. describe the pattern of adjacent place positions moving from right to left;
C. describe the meaning of each digit in a given numeral;
D. provide examples of large numbers used in print or electronic media;
E. express a given numeral in expanded notation, e.g., 45 321 = (4 × 10 000) + (5 × 1000) + (3 × 100) + (2 × 10) + (1 × 1) or 40 000 + 5000 + 300 + 20 + 1;
F. write the numeral represented by a given expanded notation; and
G. read a given numeral without using the word “and,” e.g., 574 321 is five hundred seventy-four thousand three hundred twenty-one, NOT five hundred AND seventy-four thousand three hundred AND twenty-one. Note: The word “and” is reserved for reading decimal numbers.

SCO: **5.N1 Represent and describe whole numbers to 1 000 000.**
[C, CN, V, T]

Elaboration

Students will continue to use whole numbers as they perform computations or measurements and as they read and interpret data. To have a better understanding of large numbers, such as a million, students need opportunities to investigate problems involving these numbers.

Students should have many opportunities to:
- read numbers several ways. For example, 879 346 is read eight hundred seventy-nine thousand, three hundred forty-six but might also be renamed as 87 ten thousands, 9 thousands, 346 ones (other examples may include:8 hundred thousands 79 thousands, 34 tens and 6 ones or 879 thousands, 3 hundreds 30 tens and 16 ones);
- record numbers. For example, ask students to write eight hundred thousand sixty; a number which is eighty less than one million; as well as write numbers in expanded notation (741 253 = 700 000 + 40 000 + 1000 + 200 + 50 + 3), and standard form.
- establish personal referents to develop a sense of larger numbers.

Through these experiences, students will develop flexibility in identifying and representing numbers up to 1 000 000. It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real life contexts that are personally meaningful. Students should establish personal referents to think about large numbers. Benchmarks that students may find helpful are multiples of 100, 1000, 10 000 and 100 000, as well as 250 000, 500 000, and 750 000 (quarter, half, and three quarters of a million).

Include situations in which students use a variety of models, such as:
- base ten blocks (e.g., recognize that 1000 large cubes would represent 1 000 000.)
- money (e.g., How many $100 bills are there in $9347?)
- place value charts.

Millions			Thousands			Ones		
		O	H	T	O	H	T	O

The focus of instruction should be on ensuring students develop a strong sense of number. The development of this outcome should be ongoing throughout the year.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:
- Unit 2, Launch, p. 35
- Unit 2, Lesson 1, pp. 36-38
- Unit 2, Lesson 2, pp. 40-42
- Unit 2, Lesson 3, pp. 43-47

SCO: **5.N1 Represent and describe whole numbers to 1 000 000.**
[C, CN, V, T]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use large numbers from students' experiences, such as populations and professional sport salaries.
- Use visual models based on the cubic centimetre and cubic metre.
- Share children's books to explore number concepts, such as *How Much Is A Million?* by David Schwartz.
- Provide students with frequent opportunities to read, write, and say numbers in standard and expanded form. Note: insist that students use proper spacing (not commas) when writing large numbers and reserve the use of "and" for reading decimal numbers.
- Discuss how large numbers can represent either a large amount or a small amount depending on the context used.
- Use various manipulatives (number cubes, spinners, number cards, etc.) to generate six-digit numbers. Students can then be asked to explore these numbers in many different ways.

Suggested Activities

- Have students locate large numbers in newspapers or magazines. Ask them to read, write, and represent the numbers in different ways.
- Collect, as a class, some type of object with the objective of reaching a specific quantity. For example, 100 000 buttons, pieces of junk mail or pop can tabs. If collecting is not possible, students could start a project where they draw a specific number of dots each week until the objective is reached.
- Identify how many $100 bills it would take to make $ 1 000 000.
- Identify how long a line of 1 million unit cubes would be.
- Ask students questions about the reasonableness of numbers, such as "Have you lived 1 million hours yet?" "Are there 1 million people in any Prince Edward Island city?" Have students explain their thinking.
- Create 2 page spreads for a class book about 1 million. Each spread could begin: "If you had a million _________, it would be__________." Alternatively the sentences could start, "I wish I had a million _________ , but I would not want a million _________ ."
- Ask students to create six-digit numbers by rolling a number cube six times and order the numbers.
- Have students explore the way numbers have been expressed in examples of whole numbers found in various types of media and personal conversations, and discuss why variations in saying and writing numbers might occur.
- Ask students to compare 10 000 steps to 10 000 metres. If you walked 10 000 steps per day, in how many days would you have walked 1 million steps?
- Ask students to list three non-consecutive numbers between 284 531 and 285 391.
- Have students place counters on a place value chart to represent a number stated orally. The digital form can be written once the chart is filled in, and the number can be read back.

SCO: **5.N1 Represent and describe whole numbers to 1 000 000.**
[C, CN, V, T]

Assessment Strategies

- Ask students to record a series of numbers that have been read to them. Ensure students include correct spacing without commas. Have students express those same numbers in expanded notation.
- Ask, “How does a million compare to 1000, 10 000, 100 000?”
- Have students record a number that is 100 000 more than a given number (or variations of this such as 20 000 less, etc.).
- Ask the students to use newspapers or catalogues to find items that would total $1 million.
- Ask the student how she/he knows that 1 000 000 is the same as 1000 thousands.
- Tell students you bought a car with 50 hundred dollar bills, 20 thousand dollar bills, 100 ten dollar bills and 46 loonies. Ask students to determine the cost of the car.
- Have students explain at least three things they know about a number with 7 digits.
- Ask a student to describe when 1 000 000 of something might be a big amount? A small amount?

SCO: **5.N2 Use estimation strategies including:**
- **front-end rounding**
- **compensation**
- **compatible numbers**

in problem-solving contexts.
[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.N3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4- digit numerals) by: using personal strategies for adding and subtracting, estimating sums and differences solving problems involving addition and subtraction. **4.N6** Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: using personal strategies for multiplication with and without concrete materials, using arrays to represent multiplication, connecting concrete representations to symbolic representations, and estimating products.	**5.N2** Use estimation strategies including: • front-end rounding • compensation • compatible numbers in problem- solving contexts.	

Achievement Indicators

Students who have achieved this outcome should be able to:

A. provide a context for when estimation is used to:
   make predictions,
   check reasonableness of an answer, and
   determine approximate answers;
B. describe contexts in which overestimating is important;
C. determine the approximate solution to a given problem not requiring an exact answer;
D. estimate a sum or product using compatible numbers;
E. estimate the solution to a given problem using compensation and explain the reason for compensation;
F. select and use an estimation strategy for a given problem; and
G. apply front-end rounding to estimate:
   sums, e.g., 253 + 615 is more than 200 + 600 = 800
   differences, e.g., 974 – 250 is close to 900 – 200 = 700
   products, e.g., the product of 23 × 24 is greater than 20 × 20 (400) and less than 25 × 25 (625)
   quotients, e.g., the quotient of 831 ÷ 4 is greater than 800 ÷ 4 (200).

SCO: **5.N2 Use estimation strategies including:**
- **front-end rounding**
- **compensation**
- **compatible numbers**

in problem-solving contexts.
[C, CN, ME, PS, R, V]

Elaboration

Students need to recognize that estimation is a useful skill in their lives. To be efficient when estimating sums and differences mentally, students must be able to access a strategy quickly and they need a variety from which to choose. Students should be aware that in real-life estimation contexts **overestimating** is often important.

The context and the numbers and operations involved affect the estimation strategy chosen.
- **Rounding** – There are a number of things to consider when rounding to estimate for a multiplication calculation. If one of the factors is a single digit, consider the other factor carefully. For example, when estimating 8 x 693, rounding 693 to 700 and multiplying by 8 is a much closer estimate than multiplying 10 by 700. Explore rounding one factor up and the other one down, even if it does not follow the "rule". For example, when estimating 77 by 35, compare 80 x 30 and 80 x 40 to the actual answer of 2695.
- **Compensation** – In this case, compensation refers to increasing one value and decreasing the other. For example, 35 + 57 might be estimated as 30 + 60 (rather than 40 + 60) as this is a more accurate estimation.
- **Compatible numbers** or “nice numbers” – Clustering compatible (or near compatible) numbers is useful for addition. For example, to solve 134 + 55 + 68 + 46, the 46 and 55 together make about 100; the 134 and 68 make about another 200 for a total of 300. Look for compatible numbers when rounding for a division estimate. For 4719 ÷ 6, think "4800 ÷ 6". For 3308 ÷ 78, think "3200 ÷ 80”.

Students and teachers should note that multiplication and division estimations are typically further from the actual value because of the nature of the operations involved.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:
- Unit 2, Lesson 4, pp. 48-52
- Unit 2, Lesson 5, pp. 53-56
- Unit 2, Lesson 6, pp. 57-59
- Unit 2, Lesson 7, pp. 60-63
- Unit 2, Lesson 8, pp. 64, 65
- Unit 2, Unit Problem, pp. 68, 69
- Unit 3, Lesson 4, pp. 84-87
- Unit 3, Lesson 7, pp. 97-99

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to the Grade 5 Mathematics page at learn.edu.pe.ca for the Mental Math Guide.)

SCO: **5.N2 Use estimation strategies including:**
- **front-end rounding**
- **compensation**
- **compatible numbers**

in problem-solving contexts.
[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Support students in exploring personal strategies for estimation, but then guide students toward more efficient and accurate strategies as needed.
- Have students share personal strategies. Begin with the least efficient strategies and then share progressively more complex as this encourages participation from all and does not discourage others.
- Accept a range of estimates, but focus on “good” estimate.
- Provide real-world contexts for estimations, as most situations require estimations and not precise answers.
- Practice strategy selection and explain the choice for estimation.

Suggested Activities

- Tell the student that ☐834 ÷ 6 is about 300. Ask the student to decide what digit should go in the box.
- Ask the students to find two numbers with a difference of about 150 and a sum of about 500 or two numbers with a difference of about 80 and a sum of about 200.
- Ask the student to estimate what one might subtract in each case below so that the answer is close to, but not exactly, fifty: 384 - ____, 219 - ____, 68 - _____
- Have students describe a real world situation when overestimating is appropriate.
- Ask students if their age in days would be closer to 400, 4000, or 40 000 days and to explain their thinking.

SCO: **5.N2 Use estimation strategies including:**
- **front-end rounding**
- **compensation**
- **compatible numbers**

in problem-solving contexts.
[C, CN, ME, PS, R, V]

Assessment Strategies

- Ask: Which pair of factors would you choose to estimate 37 x 94? Explain why.
  30 x 90   40 x 100   35 x 95   40 x 95   40 x 90
- Have students estimate each sum and explain their strategies: 1976 + 3456   69 423 + 21 097
- Have students estimate each difference and explain their strategies: 99 764 – 17 368   5703 – 755
- Have students add 6785 + 1834. Explain how they know their answer is reasonable.
- Have students solve problems that require an estimate; for example*: "Jeff has 138 cans of soup. He wants to collect 500 cans for the food bank.* ***About*** *how many more does he need to collect?"*
- Ask students for an estimate if a number between 300 and 400 is divided by a number between 60 and 70.
- Say to students*, "A bus holds 58 students. How would you estimate how many buses are needed to transport 3000 students?"*
- Tell students that you have multiplied a 3-digit number by a 1-digit number and the answer is about 1000. Ask them to write three possible pairs of factors.

SCO: **5.N3 Apply mental mathematics strategies and number properties, such as:**
- **skip counting from a known fact**
- **using doubling or halving**
- **using patterns in the 9s facts**
- **using repeated doubling or halving**

to determine answers for basic multiplication facts to 81 and related division facts.
[C, CN, ME, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.N4 Explain the properties of 0 and 1 for multiplication and the property of 1 for division. **4.N5** Describe and apply mental mathematics strategies, such as: • *skip counting from a* • *known fact* • *using doubling or halving* • *using doubling or halving and adding or subtracting one more group* • *using patterns in the 9s facts* • *using repeated doubling* to determine basic multiplication facts to 9 × 9 and related division facts.	**5.N3** Apply mental mathematics strategies and number properties, such as: • *skip counting from a known fact* • *using doubling or halving* • *using patterns in the 9s facts* • *using repeated doubling or halving* to determine answers for basic multiplication facts to 81 and related division facts.	**6.N2** Demonstrate an understanding of factors and multiples by: • *determining multiples and factors of numbers less than 100 identifying prime and composite numbers* • *solving problems involving multiples*.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. describe the mental mathematics strategy used to determine a given basic fact, such as:
   - skip count up by one or two groups from a known fact, e.g., if 5 × 7 = 35, then 6 × 7 is equal to 35 + 7 and 7 × 7 is equal to 35 + 7 + 7
   - skip count down by one or two groups from a known fact, e.g., if 8 × 8 = 64, then 7 × 8 is equal to 64 – 8 and 6 × 8 is equal to 64 – 8 – 8
   - doubling, e.g., for 8 × 3 think 4 × 3 = 12, and 8 × 3 = 12 + 12
   - patterns when multiplying by 9, e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63
   - repeated doubling, e.g., if 2 × 6 is equal to 12, then 4 × 6 is equal to 24 and 8 × 6 is equal to 48
   - repeated halving, e.g., for 60 ÷ 4, think 60 ÷ 2 = 30 and 30 ÷ 2 = 15;

B. explain why multiplying by zero produces a product of zero;

C. explain why division by zero is not possible or undefined, e.g., 8 ÷ 0; and

D. recall multiplication facts to 81 and related division facts.

SCO: **5.N3 Apply mental mathematics strategies and number properties, such as:**
- **skip counting from a known fact**
- **using doubling or halving**
- **using patterns in the 9s facts**
- **using repeated doubling or halving**

to determine answers for basic multiplication facts to 81 and related division facts.
[C, CN, ME, R, V]

Elaboration

This is an extension of the grade 4 outcomes, 4.N4 and 4.N5. The goal for grade 5 is **automaticity**, which means that students are able to recall multiplication facts with little or no effort. The facts have been committed to memory as a result of extensive use of strategies.

Students need to understand and use the relationship between multiplication and division. Students should recognize that multiplication can be used to solve division situations. Contextual problems are key to emphasizing this connection.

Along with understanding why multiplication by 0 produces a product of 0, students must be able to explain why **division by 0 is undefined** or **not possible**. It is not possible to make a set of zero from a given group, nor is it possible to make zero sets from a given group. When demonstrated as repeated subtraction, removing groups of zero will never change your dividend. Rather than telling students these properties, pose problems involving 0.

In grade 4, students will have become proficient at **doubling** (4 x 3 = (2 x 3) x 2). This idea is extended in grade 5 to include **repeated doubling**. For example, to solve 8 x 6, students can think 2 x 6 = 12; 4 x 6 = 24, so 8 x 6 = 48. The same principle applies to **halving** and **repeated halving**. For example, for 36 ÷ 4, think 36 ÷ 2 = 18; so 18 ÷ 2 = 9.

Skip counting up or down from a known fact reinforces the meanings of multiplication and division as students must be thinking about the addition or subtraction of "groups". For example, for 8 x 7, think 7 x 7 = 49 and then add another group of 7; 49 + 7 = 56.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:
- Unit 3, Lesson 1, pp. 72-75
- Unit 3, Lesson 2, pp. 76-79

Mental Math strategies will strengthen student understanding of this specific curriculum outcome.(Refer to the Grade 5 Mathematics page at learn.edu.pe.ca for the Mental Math Guide.)

SCO: **5.N3 Apply mental mathematics strategies and number properties, such as:**
- **skip counting from a known fact**
- **using doubling or halving**
- **using patterns in the 9s facts**
- **using repeated doubling or halving**

to determine answers for basic multiplication facts to 81 and related division facts.
[C, CN, ME, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Introduce and practise strategies. When students are proficient at more than one strategy, have them explain why one strategy may be better than another in a given situation.
- Have students start with what they know.
- Allow students to use counters or base ten blocks as they continue to develop strategies.
- Have students skip count aloud.
- Play games to practise strategies that lead to fact recall.
- Use a problem solving context to practice facts.
- Ensure students understand why strategies work. Fact strategies should not become “rules without reasons” (Van de Walle & Lovin, vol. 2, 2006; p. 90).
- Avoid using drill until students have mastered a strategy. Unless students have mastered a strategy, drills are not effective.

Suggested Activities

- Use counters to model 6 x 6 in an array. Add another row or column to demonstrate a related fact.
- Ask students, “If you didn’t know the answer to 6 x 8, how could you figure it out?” (see Van de Walle 3-5)
- Ask students, “Jennifer reads a chapter of a novel each day. How many chapters will she have read in 8 weeks. Describe your strategy.
- Have students agree or disagree with this statement, “There are more than 2 ways to figure out any multiplication fact”. Have students use a fact of their choice.
- Provide students with food labels showing 0 g of fat per serving. Ask them how many grams of fat are in 4 servings.
- Ask students if they agree or disagree with this statement, “If you know your multiplication facts, you already know your division facts.” Have students provide a rationale.
- Provide small groups with a square piece of paper. Have them fold the paper in half and record how many sections they have. Have them fold it again and record how many sections. Have them continue until they see a pattern of doubling. Relate this to halving.
- Have students fill in all the facts they know in a multiplication table. Ask them to work with a partner to identify what strategies they could use to fill in the rest of the table.
- Use sets of “loop cards” (I have ___, who has ____) where the answer for one card answers the question on another to form a loop of questions and answers. For example, one card could read, “I have 24. Who has 3×4 ?”

SCO: **5.N3 Apply mental mathematics strategies and number properties, such as:**
- **skip counting from a known fact**
- **using doubling or halving**
- **using patterns in the 9s facts**
- **using repeated doubling or halving**

to determine answers for basic multiplication facts to 81 and related division facts.
[C, CN, ME, R, V]

Assessment Strategies

- Ask students to list three multiplication facts they can use to help calculate 5 x 8, and explain how they can use each fact.
- Ask, “If you buy muffins in boxes of 6, how many muffins are in 7 boxes? How would the number of muffins change if you bought 9 boxes? If you needed 36 muffins for a party, how many boxes would you buy?”
- Ask students how they could use multiplication to find the perimeter of a square.
- Tell the students that you have eight boxes, each of which holds six markers, and one other box that has only five markers in it. Ask the students to describe at least two ways one could find the total number of markers, and to explain which way they would prefer and why.

SCO: **5.N4 Apply mental mathematics strategies for multiplication, such as:**
- **annexing then adding zero**
- **halving and doubling**
- **using the distributive property.**

[C, ME, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.N4 Explain the properties of 0 and 1 for multiplication and the property of 1 for division. **4.N5** Describe and apply mental mathematics strategies, such as: • skip counting from a known fact • using doubling or halving • using doubling or halving • adding or subtracting one more group • using patterns in the 9s facts using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts.	**5.N4** Apply mental mathematics strategies for multiplication, such as: • annexing then adding zero • halving and doubling • using the distributive property.	

Achievement Indicators

Students who have achieved this outcome should be able to:

A. determine the products when one factor is a multiple of 10, 100 or 1000 by annexing zero or tacking on zeros, e.g., for 3 × 200 think 3 × 2 hundreds which equals six hundreds (600);

B. apply halving and doubling when determining a given product, e.g., 32 × 5 is the same as 16 × 10; and

C. apply the distributive property to determine a given product involving multiplying factors that are close to multiples of 10, e.g., 98 × 7 = (100 × 7) – (2 × 7)

SCO: **5.N4 Apply mental mathematics strategies for multiplication, such as:**
- **annexing then adding zero**
- **halving and doubling**
- **using the distributive property.**

[C, ME, R]

Elaboration

A **mental computation** is one that produces the actual answer, not an estimate. In grade 5, students are extending the strategies learned in grade four to multiply mentally. It is important to recognize that these strategies develop and improve over the years with regular practice. This means that mental mathematics must be a consistent part of instruction in computation from primary through the elementary and middle grades. Mental strategies must be taught both explicitly as well as being embedded in problem solving situations. Sharing of computational strategies within the context of problem-solving situations is essential.

Students should perform and discuss the following types of mental multiplication on a regular basis:

- **Annexing then adding zero** for multiplication by 10, 100 and 1000 and multiplication of single-digit multiples of powers of ten (e.g., for 30×400, students should think "Tens times hundreds is thousands. How many thousands? 3×4 or 12 thousands.")
- **Halving and doubling:** For example, to solve 4×16, students can change it to 2×32 or 8×8.
- **Distributive property:** The ability to break numbers apart is important in multiplication. For example, to multiply 5×43, think 5×40 (200) and 5×3 (15) and then add the results. This principle also applies to multiplication questions in which one of the factors ends in a nine (or eight or seven). For such questions, one could use a **compensating strategy** - multiply by the next multiple of ten and compensate by subtracting to find the actual product. For example, when multiplying 39 by 7 mentally, one could think, "7 times 40 is 280, but there were only 39 sevens so I need to subtract 7 from 280 which gives an answer of 273."

Whenever presented with problems that require computations, students should be encouraged to first check to see if it can be done mentally. Students should select an efficient strategy that makes sense to them.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 3, Lesson 3, pp. 80-83
- Unit 3, Lesson 5, pp. 88-91

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to the Grade 5 Mathematics page at learn.edu.pe.ca for the Mental Math Guide.)

SCO: **5.N4 Apply mental mathematics strategies for multiplication, such as:**
- **annexing then adding zero**
- **halving and doubling**
- **using the distributive property.**

[C, ME, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with many experiences to construct a personal strategy and then be guided to use the most efficient strategy available. Mental strategies encourage students to think about the whole number and not just the digits.
- Provide students with frequent opportunities to share their mental strategies.
- Provide problem-based situations that support the use of mental strategies.
- Use materials and pictorial representations to demonstrate mental strategies.
- Introduce a strategy with the use of materials, practice the strategy, and continue to introduce and practice new strategies. When students have two or more strategies, it is important to encourage them to choose the most efficient strategy for the student.
- Encourage students to visualize the process for the strategy they are using.
- Place students in pairs to practise strategies as well as strategy selection.
- Avoid timed tests until students have developed and practised specific mental strategies in other contexts.
- Ask students to keep track of when they use their mental math strategies outside of the classroom and to write about these experiences.
- Ask the students to keep a list of mental math strategies that they use regularly.

Suggested Activities

- Use two recipe cards and have students write a series of mental math questions. Students take the cards home to have a “race” with a parent/guardian. The student can then “teach” the strategy being practiced at home.
- Ask students to explain how they could calculate 23 x 8 if the “eight” key on the calculator was broken.
- Mix cards with number sentences that can be solved using two or more strategies into a single package. Prepare simple pictures or labels for the strategies in the package. Have students sort the problems and then solve them using the appropriate strategy.
- Ask the student to use square tiles to show that if the length of a rectangle is halved and the width is doubled, the area remains the same.
- Ask the student to provide an explanation and examples for how to multiply a 1-digit number by 99 mentally.

SCO: **5.N4 Apply mental mathematics strategies for multiplication, such as:**
- **annexing then adding zero**
- **halving and doubling**
- **using the distributive property.**

[C, ME, R]

Assessment Strategies

- Tell the student that when asked to multiply 36 × 11, Kelly said, "I think 360 + 36 = 396." Ask the student to explain Kelly's thinking.
- Ask: Why is it easy to calculate the questions below mentally?
  48 × 20 50 × 86
- Ask the student why Lynn multiplied 11 × 30 to find 22 × 15.
- Provide students with a problem situations to solve, such as:
  1. Fourteen students raised $20 each in pledges for “Save the Wetlands Walk”. How much money was raised? How much money would be raised if the pledges were increased to $50 each?
  2. A hotel has 7 floors with 39 windows on each floor. How many windows are in the hotel? Explain how you know.
- Explain how you know that 48 × 50 is the same as 24 × 100.

SCO: **5.N5 Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.**
[C, CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Grade Four	Grade Five	Grade Six
4.N6 Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: using personal strategies for multiplication with and without concrete materials; using arrays to represent multiplication; connecting concrete representations to symbolic representations; estimating products.	**5.N5** Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.	**6.N6** Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).

ACHIEVEMENT INDICATORS

Students who have achieved this outcome should be able to:

A. illustrate partial products in expanded notation for both factors, e.g., for 36 × 42, determine the partial products for (30 + 6) × (40 + 2);

B. represent both 2-digit factors in expanded notation to illustrate the distributive property, e.g., to determine the partial products of 36 × 42
(30 + 6) × (40 + 2) = (30 × 40) + (30 × 2) + (6 × 40) + (6 × 2) = 1200 + 60 + 240 + 12 = 1512;

C. model the steps for multiplying 2-digit factors using an array and base ten blocks, and record the process symbolically;

D. describe a solution procedure for determining the product of two given 2-digit factors using a pictorial representation, such as an area model; and

E. solve a given multiplication problem in context using personal strategies and record the process.

SCO: **5.N5 Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.**
[C, CN, PS, V]

ELABORATION

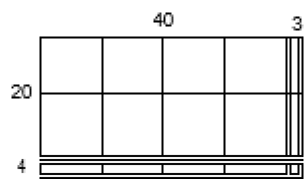
Strategies for multiplication can be more complex than those for addition and subtraction. Students need to be flexible in the way they think about the factors, and should be thinking about numbers, not just digits. Students should have many opportunities to practise and share their ideas.

Model multiplying two 2-digit numbers concretely:

- Model the product as the **area** of a rectangle with the dimensions of the two numbers. This can be done using base-ten blocks and grid paper. Students should relate the model to an **algorithm**. The symbolic steps should be recorded and related to each physical manipulation.
- When the students understand the **area model**, they may choose to use a grid-paper drawing as an explanation, but it is important to record the process. A standard algorithm might be presented, but it is important that an explanation with models be provided, not just procedural rules.

The **distributive property** of multiplication allows students to record **partial products**. For example:

$$43 \times 24 = (40 + 3) \times (20 + 4)$$
$$\left.\begin{array}{r} 40 \times 20 \\ 40 \times 4 \\ 3 \times 20 \\ 3 \times 4 \end{array}\right\} \text{add the products}$$



The **commutative property** of multiplication means the order in which you multiply does not matter. The above example closely resembles what students may already think of as **front end multiplication**.

- As always, students should be given the choice of using a standard algorithm. If, however, students are using inefficient algorithms, they should be guided to select more appropriate ones.
- As for all computational questions, students should estimate before calculating. Immediate recall of basic multiplication facts is a necessary prerequisite not only for paper-and-pencil algorithmic procedures, but also for estimation and mental computation.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 3, Lesson 6, pp. 92-95
- Unit 3, Lesson 10, pp. 109-111
- Unit 3, Lesson 11, pp. 112, 113

SCO: **5.N5 Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.**
[C, CN, PS, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Model multiplication concretely (base ten blocks, grid paper).
- Develop the symbolic representation from the model.
- Introduce the traditional algorithm only after students have conceptual understanding.
- Use place value language. (e.g., 24 × 62 is **twenty** × 62 + **four** × 62)
- Have students estimate the product first.
- Encourage frequent use of mental math strategies.
- Guide students to use efficient strategies.
- Use the language of multiplication, such as factor, product, distributive and commutative property.

Suggested Activities

- Provide students with a large rectangle (e.g., 24 cm × 13 cm). Have students fill the rectangle with base ten materials to find the area. Have them write the related multiplication equation.
- Use known facts and combinations of facts that students know to solve more complex computations. For example, provide students with 31 × 24 and use 31 × 10, 31 × 4, 30 × 24, 1 × 24 to solve.
- Ask students to explore the pattern in these products: 15 × 15, 25 × 25, 35 × 35, etc. Have them describe the pattern and tell how the pattern could be used to predict 85 × 85 or 135 × 135. They might then test their predictions using a calculator. Alternatively, students might explore the pattern in these products: 19 × 21, 29 × 31, 39 × 41, and use it to make a prediction for 79 × 81 and 109 × 111.
- Find the product of 25 × 25. How can the product of 25 × 25 be used to help find the products of 25 × 24, 25 × 50 and 25 × 75?
- Discuss multiplication strategies. Have students share which strategies they prefer for particular situations and why.
- Ask students to explore the following: 24 + 35 is the same as 25 + 34. Is 24 × 35 the same as 25 × 34? Have students provide an explanation.

SCO: **5.N5 Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.**
[C, CN, PS, V]

Assessment Strategies

- Have the student use a model to show how to find the total money collected for photos if 43 students each bring in $23.
- Ask the students to explain why the product of two different 2-digit numbers is always greater than 100.
- Have students draw an array to represent 32 × 16.
- Present problems such as the following to students:
    - *"Hardcover books are being sold at a book sale for $26 each. If 48 hardcover books were sold, how much did they cost?"*
    - *"If a cheetah can run 29 m per second, how far can it run in 1 minute?"*
- Show students the following:

$$\begin{array}{r} 31 \\ \times\ \underline{25} \\ 124 \\ \underline{62} \\ 186 \end{array}$$

Ask students to identify the error and explain how to correct it.

SCO: **5.N6 Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.**
[C, CN, PS]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.N7 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: • *using personal strategies for dividing with and without concrete materials* • *estimating quotients* • *relating division to multiplication*.	**5.N6** Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.	**6.N6** Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).

Achievement Indicators

Students who have achieved this outcome should be able to:

A. model the division process as equal sharing using base ten blocks and record it symbolically;
B. explain that the interpretation of a remainder depends on the context:
- ignore the remainder, e.g., making teams of 4 from 22 people
- round up the quotient, e.g., the number of five passenger cars required to transport 13 people
- express remainders as fractions, e.g., five apples shared by two people
- express remainders as decimals, e.g., measurement and money; and

C. solve a given division problem in context using personal strategies and record the process.

SCO: **5.N6 Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.**
[C, CN, PS]

Elaboration

Division problems can involve either **sharing** or finding **how many groups**.

1 5 1 3) 4 5 3 - 3 15 -15 0 3	“4 hundreds shared among 3, each gets 1 with 1 hundred left. Trade 1 hundred for 10 tens; now 15 tens to share, each gets 5 tens, etc”	3) 4 5 3 - 3 0 0 | 100 1 5 3 -1 5 0 | 50 3 | 1	“Make 100 sets of 3, using 300; 153 left. Make 50 sets of 3 using 150; 3 left…etc” The number of sets at each stage tends to be a multiple of 10 or 100 to facilitate computation.

Division should be connected to **estimation** and multiplication to check the reasonableness of the answer. It is necessary that division be modelled as “**equal sharing**” using base ten materials. Recording the process symbolically will support student’s understanding of division. The traditional long-division algorithm, whether modelled with base-ten blocks or not, is best described using “sharing words” as seen above.

Students also need to know that the answer for a division sentence is the **quotient** and the number to be divided is the **dividend**.

The context of **remainders** must be discussed with students. They must understand why the number of units leftover after the sharing must be less than the **divisor**. Models help to clarify this idea. Students need many opportunities to explore the interpretation of the remainder in problem solving situations to decide if it should be **ignored**, **rounded up**, expressed as a **fraction** or a **decimal**. A common mistake of students is to write a remainder as a decimal when the divisor is other than 10 (e.g., a remainder of 7 is written as “.7”). This should be addressed through a discussion of remainders and the meaning of tenths.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions of personal interest. These opportunities provide students with a chance to practice their computational skills and clarify their mathematical thinking.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 3, Lesson 8, pp. 100-103
- Unit 3, Lesson 9, pp. 104-107
- Unit 3, Lesson 10, pp. 109-111
- Unit 5, Lesson 9, pp. 194-196

SCO: **5.N6 Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.**
[C, CN, PS]

Instructional Strategies

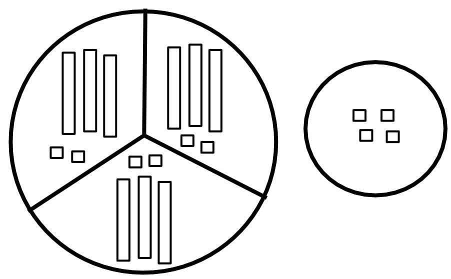
Consider the following strategies when planning lessons:

- Provide students the opportunity to solve division problems using base ten materials.
- Present division questions in a problem solving situation.
- Provide regular practice and discussion of estimation strategies to support division.
- Have students create and share problems involving division.

Suggested Activities

- Ask students to write a word problem involving division by a 2-digit number for each of the following:
  a) a situation in which the remainder would be ignored.
  b) a situation in which the remainder would be rounded up.
  c) a situation in which the remainder would be part of the answer.
- Ask the student to tell what division is being modelled below and to provide a word problem that would apply to the model.

$100 \div 32 = 3 \text{ R}4$



SCO: **5.N6 Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.**
[C, CN, PS]

Assessment Strategies

- Ask the student to use materials to model how to divide 489 by 7.
- Ask: At the T-Shirt Shop, you can buy t-shirts in packages of 8. One package costs $130. At “Big Deals”, a t-shirt costs $18. Does “Big Deals” have the better price? How do you know?
- Ask: Jenna solved a problem by dividing 288 by 4. She said the answer was 72. What could the problem have been?

SCO: **5.N7 Demonstrate an understanding of fractions by using concrete and pictorial representations to:**
- **create sets of equivalent fractions**
- **compare fractions with like and unlike denominators.**

[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.N8 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: • *name and record fractions for the parts of a whole or a set* • *compare and order fractions* • *model and explain that for different wholes, two identical fractions may not represent the same quantity* • *provide examples of where fractions are used.*	**5.N7** Demonstrate an understanding of fractions by using concrete and pictorial representations to: • *create sets of equivalent fractions* • *compare fractions with like and unlike denominators.*	**6.N3** Relate improper fractions to mixed numbers.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. create a set of equivalent fractions and explain why there are many equivalent fractions for any given fraction using concrete materials;
B. model and explain that equivalent fractions represent the same quantity;
C. determine if two given fractions are equivalent using concrete materials or pictorial representations;
D. formulate and verify a rule for developing a set of equivalent fractions;
E. identify equivalent fractions for a given fraction;
F. compare two given fractions with unlike denominators by creating equivalent fractions; and
G. position a given set of fractions with like and unlike denominators on a number line and explain strategies used to determine the order.

SCO: **5.N7 Demonstrate an understanding of fractions by using concrete and pictorial representations to:**
- **create sets of equivalent fractions**
- **compare fractions with like and unlike denominators.**

[C, CN, PS, R, V]

Elaboration

Developing number sense with fractions takes time and is best supported with a conceptual approach and the use of materials. Using a variety of manipulatives helps students understand properties of fractions and realize that the relationship between the two numbers in a fraction is the focus. A fraction does not say anything about the size of the whole.

Students should continue to use conceptual methods to compare fractions. These methods include: i) comparing each to a **benchmark**; ii) comparing the two **numerators** when the fractions have the same denominator; and iii) comparing the two **denominators** when the fractions have the same numerator. A common error made by students at this level is to think, for example, that $\frac{4}{7}$ is greater than $\frac{4}{6}$ because of their experience comparing whole numbers.

Considerable time needs to be spent on activities and discussion to develop number sense of fractions. Provide students with a variety of experiences using different models (number lines, pattern blocks, counters, etc.) and different representations of the whole with the same model. Students should recognize that a fraction can **name part of a set** as well as **part of a whole** and the size of these can change. Students also need to understand that fractions can only be compared if they are parts of the same whole. Half of a cake cannot be compared to half a brownie. When comparing one half and one quarter, the whole is the “unit” (1).

It is important that students are able to **visualize equivalent fractions** as the naming of the same **region** or **set** partitioned in different ways. At this stage, a rule about multiplying numerators and denominators to form equivalents should not be offered. Such a rule could be confirmed if students observe it; however, the explanation should be connected to manipulatives.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 5, Lesson 1, pp. 166-169
- Unit 5, Lesson 2, pp. 170-173
- Unit 5, Lesson 3, pp. 174, 175

SCO: **5.N7 Demonstrate an understanding of fractions by using concrete and pictorial representations to:**
- **create sets of equivalent fractions**
- **compare fractions with like and unlike denominators.**

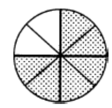
[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide the students with a variety of activities that include the three interpretations of fractions: 1) part of a whole (e.g., part a chocolate bar); 2) part of a set (e.g., part of 30 marbles); and 3) part of a linear measurement (e.g., part of a 4 m baseboard).
- Provide many opportunities for students to model fractions both concretely and pictorially, using a variety of models such as, pattern blocks, grid paper, fraction pieces, fraction towers, Cuisenaire® rods, counters, egg cartons, number lines, etc.
- Point out to the student that to rename $\frac{6}{8}$ as $\frac{3}{4}$, you can "clump" the 8 sections of the whole into 2s.

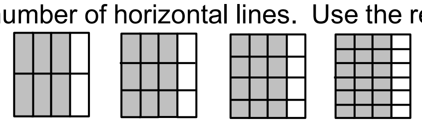
  There are then four groups of 2 sections; three of the four groups are shaded.



- Use number lines and other models to compare fractions.

Suggested Activities

- Have the student make a diagram and identify the "clump size" that should be used to show that $\frac{10}{15} = \frac{2}{3}$. Ask how one might predict the "clump size" without drawing the diagram.
- Fold a piece of paper into fourths. Colour $\frac{1}{4}$. Fold the paper again. What equivalent fraction is represented? Fold the paper again. What equivalent fraction is shown? Discuss the pattern.
- Have students prepare a poster showing all the equivalent fractions they can find using a set of no more than 30 pattern blocks.
- Give students a sheet with 4 squares. Have them shade $\frac{3}{4}$ on each square vertically. Have them subdivide each square with a different number of horizontal lines. Use the resulting pictures to find possible equivalent fractions for $\frac{3}{4}$.



- Ask the student to use his/her fingers and hands to show that $\frac{1}{2}$ and $\frac{5}{10}$ are equivalent fractions.

  Alternatively, the student might be asked to choose a manipulative of choice to show this or some other equivalence.

SCO: **5.N7 Demonstrate an understanding of fractions by using concrete and pictorial representations to:**
- **create sets of equivalent fractions**
- **compare fractions with like and unlike denominators.**

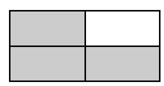
[C, CN, PS, R, V]

Assessment Strategies

- Have students create a diagram to show why $\frac{4}{8} = \frac{1}{2}$ are equivalent.
- Give students an equation expressing equivalence between two fractions but with one of the terms missing. Find the missing term and explain your solution.

  $$\frac{4}{10} = \frac{\square}{5}$$

- Have students place the following fractions on a number line: $\frac{1}{2}$, $\frac{9}{10}$, $\frac{4}{5}$, $\frac{1}{5}$.
- Have students write two equivalent fractions for the following diagram.



SCO: **5.N8 Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.**
[C, CN, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.N9 Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.	**5.N8** Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.	**6.N1** Demonstrate an understanding of place value for numbers: • greater than one million less than one thousandth.

Achievement Indicators

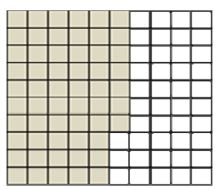
Students who have achieved this outcome should be able to:

A. write the decimal for a given concrete or pictorial representation of part of a set, part of a region or part of a unit of measure;
B. represent a given decimal using concrete materials or a pictorial representation;
C. represent an equivalent tenth, hundredth or thousandth for a given decimal using a grid;
D. express a given tenth as an equivalent hundredth and thousandth;
E. express a given hundredth as an equivalent thousandth; and
F. describe the value of each digit in a given decimal.

SCO: **5.N8 Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.**
[C, CN, R, V]

Elaboration

Students need to understand that anything to the right of the decimal represents a quantity less than one. Students should continue to use physical materials to represent or model decimals. In this way, they can better see the relationship between **hundredths** and **thousandths**.



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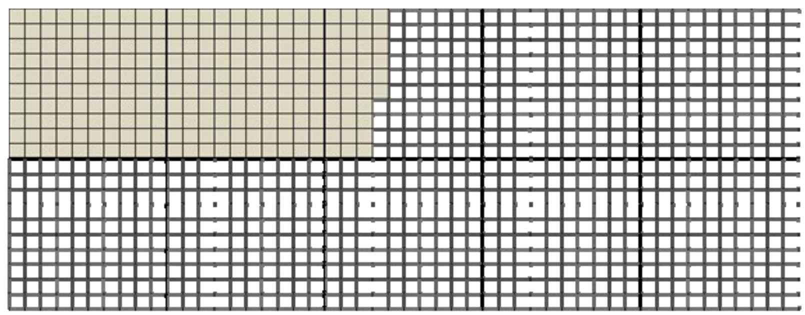
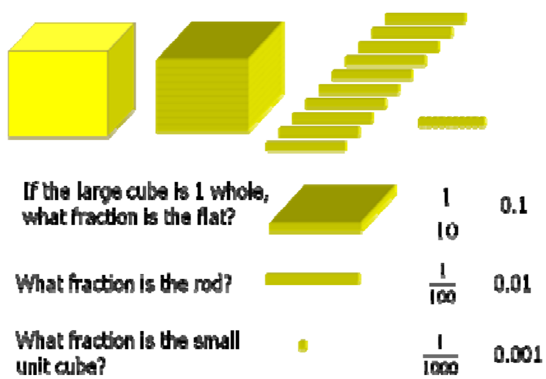


0.238

Alternatively, base ten blocks might be used to illustrate the relationship.



Students can also represent thousandths using **a meter stick or tape measure** since 1 mm = 0.001 m. For example, 0.423 m can be represented as 423 mm or 42.3 cm ("a little more than 42 cm").

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 5, Lesson 4, pp. 176-179
- Unit 5, Lesson 6, pp. 183-186
- Unit 5, Lesson 8, pp. 191-193

SCO: **5.N8 Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.**
[C, CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Writing decimals using place value language and expanded notation can help explain equivalence of decimals.

  0.4 = 4 tenths
  0.40 = 4 tenths + 0 hundredths
  0.400 = 4 tenths + 0 hundredths + 0 thousandths

  Since adding zeros have no effect, 0.4 must equal 0.40 and 0.400

- Use the same sized tenth, hundredths and thousandths grid squares to draw equivalent decimals.
- Help students extend the place value system to decimals by focusing on the basic pattern of ten. While building on their understanding of tenths and hundredths from grade 4, students need to know that it takes 1000 equal parts (thousandths) to make one whole.
- Vary the representation of the whole. Use a cube, flat, or rod. Students will have a fixed notion of what these models represent and it is important to reinforce the idea that a decimal relates a part to a whole the same way that fractions do.
- Provide opportunities for students to read decimals in context. Saying decimals correctly will help students make the connection between decimals and fractions (5.N9) 3.147 should be read as "three and one hundred forty-seven thousandths" not "three point one four seven".

Suggested Activities

- Present a riddle to the class such as, "I have 25 hundredths and 4 tenths. What am I?" Have students use a model of their choice to represent the solution to the riddle.
- Make sets of cards showing decimals in different forms including expanded form, pictorial representations and equivalent decimals. Students can play matching games or decimal snap.
- Provide opportunities for students to find and share how large numbers are represented in newspapers and magazines. For example, a CEO's salary may be written as 4.5 million dollars.
- Place five different displays of combinations of base-ten blocks. Ask the students to visit the centre and record the five decimals displayed.
- Provide students with two hundredths disks, each of a different color. Cut each disk along one radius so they can be fit together. Students can use these to model given decimals, or to write decimals from a given model.
- Give students three number cubes. Have them make the greatest and least possible decimals using the numbers rolled as the digits. Have students read the decimal numbers aloud.
- Use the calculator to "count". Enter + 0.1 =, =… when the display shows 0.9 have students predict what number will be next. Extend this to use 0.01 and 0.001 to demonstrate the relative magnitude of hundredths and thousandths.
- Give students a drawing of an irregular shape and have them shade in approximately 0.247 of it.

SCO: **5.N8 Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.**
[C, CN, R, V]

Assessment Strategies

- Ask the student to express 0.135 in at least three different ways.
- Tell the student that gasoline is priced at 83.9¢ per litre. Ask: What part of a dollar is this?
- Ask the student to explain why newspapers might record a number as 2.5 million rather than as 2 500 000. Ask him/her to discuss whether or not this is a good idea.
- Ask the students to write 10 different decimal numbers that have tenths, hundredths and/or thousandths. Have them make base-ten block pictures that would represent their numbers.
- Ask students use hundredths and thousandths grids or base ten blocks to model equivalent decimals.
- Ask the student to write the numerals for "two hundred fifty-six thousandths" and "two hundred and fifty-six thousandths". Ask the student to explain why watching and listening for "and" is important when interpreting numbers.

SCO: **5.N9 Relate decimals to fractions (to thousandths).**
[CN, R, V]
5.N10 Compare and order decimals (to thousandths) by using:
- **Benchmarks**
- **place value**
- **equivalent decimals.**

[CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.N10 Relate decimals to fractions (to hundredths).	**5.N9** Relate decimals to fractions and fractions to decimals (to thousandths). **5.N10** Compare and order decimals (to thousandths), by using: • *benchmarks* • *place value* • *equivalent decimals.*	**6.N4** Demonstrate an understanding of percent, (limited to whole numbers) concretely, pictorially and symbolically.

Achievement Indicators

Students who have achieved these outcomes should be able to:

5.N9

A. write a given decimal in fractional form;
B. write a given fraction with a denominator of 10, 100 or 1000 as a decimal; and
C. express a given pictorial or concrete representation as a fraction or decimal, e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $\frac{250}{1000}$.

5.N10

A. order a given set of decimals by placing them on a number line that contains benchmarks, 0.0, 0.5, 1.0;
B. order a given set of decimals including only tenths using place value;
C. order a given set of decimals including only hundredths using place value;
D. order a given set of decimals including only thousandths using place value;
E. explain what is the same and what is different about 0.2, 0.20 and 0.200; and
F. order a given set of decimals including tenths, hundredths and thousandths using equivalent decimals.

SCO: **5.N9 Relate decimals to fractions (to thousandths).**
[CN, R, V]
5.N10 Compare and order decimals (to thousandths) by using:
- **Benchmarks**
- **place value**
- **equivalent decimals.**

[CN, R, V]

Elaboration

Decimals are simply another way of writing fractions. Students should continue to build their conceptual understanding of the relationship of decimals to fractions as they explore numbers to the **thousandths**. One thousandth (0.001) can be written as $\frac{1}{1000}$. Students should be encouraged to read decimals as fractions. e.g., 0.246 is read as 246 thousandths and can be written as $\frac{246}{1000}$. Measurement contexts provide valuable learning experiences for decimal numbers because any measurement can be written in an equivalent unit that requires decimals e.g., one metre is $\frac{1}{1000}$ of a kilometre, (1 m = 0.001 km).

To develop decimal and fractional number sense, it is essential to discuss the **magnitude** of the number, such as 493 thousandths is about one half, and 1.761 is about $1\frac{3}{4}$. By using number lines with benchmarks such as $\frac{1}{4}$ (0.25), $\frac{1}{2}$ (0.5), $\frac{3}{4}$ (0.75) we can help to create a visual for students.

Students should be able to determine which of two decimal numbers is greater by comparing the whole number parts first and then the amounts to the right of the decimals. It is important that students understand decimal numbers do not need the same number of digits after the decimal to be compared. For example, one can quickly conclude that 0.8 > 0.423, without converting 0.8 to 0.800, because the former is much more than half and the latter is less than half. Other common misconceptions include students thinking 0.101 is greater than 0.11 because 101 is larger than 11; others thinking it is less just because it has thousandths while the other number has only hundredths. These same students would also say 0.101 is less than 0.1 because it has thousandths while 0.1 has only tenths. Such misconceptions are best dealt with by having students make base ten block representations of numbers that are being compared. **Decimal equivalence** becomes apparent when the connection is made between decimals and fractions e.g., 0.3 = $\frac{3}{10}$ or $\frac{30}{100}$ or $\frac{300}{1000}$.

5.N9: This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:
- Unit 5, Lesson 4, pp. 176-179
- Unit 5, Lesson 5, pp. 180-182
- Unit 5, Lesson 6, pp. 183-186

5.N10: This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:
- Unit 5, Lesson 5, pp. 180-182
- Unit 5, Lesson 7, pp. 187-190

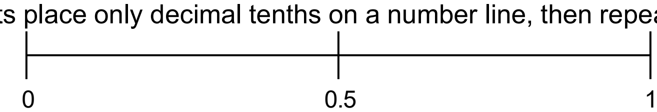
SCO: **5.N9 Relate decimals to fractions (to thousandths).**
[CN, R, V]
5.N10 Compare and order decimals (to thousandths) by using:
- **Benchmarks**
- **place value**
- **equivalent decimals.**

[CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students place only decimal tenths on a number line, then repeat for decimal hundredths and thousandths.



- Have students use thousandths grids to model the equivalency of tenths, hundredths and thousandths (e.g., 0.3, 0.30, 0.300) and explain what is the same and what is different.
- Have students order a given set of decimals including tenths, hundredths and thousandths using equivalent decimals. For example, to order 0.402, 0.39 and 0.4, students should be encouraged to think of them as thousandths (0.402, 0.390, 0.400).
- Have students begin to explore the relationship between fraction and decimal benchmarks. For example, 0.5 is another name for $\frac{1}{2}$; 0.25 is another name for $\frac{1}{4}$; 0.75 is another name for $\frac{3}{4}$.
- Represent decimals in a variety of ways. For example: 0.452 is $\frac{452}{1000}$ and can be expressed as

  0.4 + 0.05 + 0.002 or $\frac{4}{10} + \frac{5}{100} + \frac{2}{1000}$
- Provide a variety of models, stressing the magnitude of the number, e.g., 0.452 could be modeled using a number line (about one half), base 10 blocks, thousandths grids, and place value chart.

Suggested Activities

- Have students express given numbers as fractions and decimals e.g., sixty-four hundredths ($\frac{64}{100}$, 0.64)
- Ask students to investigate where in the media fractions and decimals are used, and to write a report on their findings.
- Give students a "number of the day" and have them express this number in as many ways as they can. For example: 0.752 could be shown as: $\frac{752}{1000}$ or $\frac{7}{10} + \frac{5}{100} + \frac{2}{1000}$; about $\frac{3}{4}$; plotted on a number

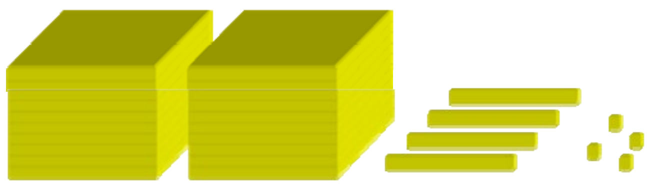
  line; modelled with base ten materials on a place value chart; shown on a thousandths grid; or described in a variety of ways ("It's 0.248 less than one whole", etc.).
- Give each student a different irregular shape and ask him/her to tear off about 0.256 of that shape. Have students explain how they estimated 0.256, and why pieces may not be the same size or shape.

SCO: **5.N9 Relate decimals to fractions (to thousandths).**
[CN, R, V]
5.N10 Compare and order decimals (to thousandths) by using:
- **Benchmarks**
- **place value**
- **equivalent decimals.**

[CN, R, V]

Assessment Strategies

- Have students compare and order decimals tenths, hundredths and thousandths and express them as fractions.
- Have students model decimal thousandths using base ten materials.



2.044

- Have students place decimals and fractions on a number line, such as: $\frac{3}{4}$, 0.31, $\frac{6}{10}$, $\frac{102}{1000}$.

- Tell students that they have properly placed 796 pieces of the 1000-piece jigsaw puzzle. Ask what part (fractional and decimal) of the puzzle has been completed? What part of the puzzle has yet to be finished? ($\frac{204}{1000}$, 0.204)

SCO: **5.N11 Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).** [C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.N11 Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by: • *using compatible numbers* • *estimating sums and differences* • *using mental math strategies* to solve problems.	**5.N11** Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).	**6.N6** Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).

Achievement Indicators

Students who have achieved this outcome should be able to:

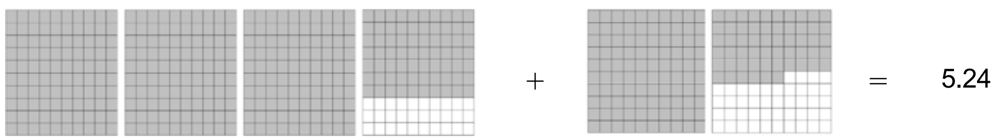
A. place the decimal point in a sum or difference using front-end estimation, e.g., for 6.3 + 0.25 + 306.158, think 6 + 306, so the sum is greater than 312;
B. correct errors of decimal point placements in sums and differences without using paper and pencil;
C. explain why keeping track of place value positions is important when adding and subtracting decimals;
D. predict sums and differences of decimals using estimation strategies; and
E. solve a given problem that involves addition and subtraction of decimals, limited to thousandths.

SCO: **5.N11 Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).**
[C, CN, PS, R, V]

Elaboration

It is essential that students recognize that all of the properties established and strategies developed for the addition and subtraction of whole numbers apply to decimals. For example, adding or subtracting **tenths** (e.g., 3 tenths and 4 tenths are 7 tenths) is similar to adding or subtracting quantities of other items (e.g., 3 apples and 4 apples are 7 apples). This could be extended to addition with tenths that total more than one whole (e.g., 7 tenths and 4 tenths are 11 tenths or 1 and 1 tenth). The same is true with **hundredths** and **thousandths**. Rather than simply telling students to line up decimals vertically, or suggesting that they “add zeroes,” they should be directed to think about what each **digit** represents and what parts go together. For example, to add 1.625 and 0.34, a student might think using front end addition, 1 whole, 9 (6 + 3) tenths and 6 (2 + 4) hundredths, and 5 thousandths or 1.965.

Base-ten blocks and hundredths grids continue to be useful models. If a flat represents one whole unit, then 3.7 + 1.54 would be modeled as:



Students need to recognize that **estimation** is a useful skill in their lives. To be efficient when estimating **sums** and **differences** mentally, students must be able to access a strategy quickly and they need a variety from which to choose. An example may be using front estimation e.g., for 9.65 + 8.106, think of 9 + 8, so the sum is greater than 17. Situations must be provided regularly to ensure that students have sufficient practice with mental math strategies and that they use their skills as required. When a problem requires an exact answer, students should first determine if they are able to calculate it mentally; this should be an automatic response.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 5, Lesson 10, pp. 197-199
- Unit 5, Lesson 11, pp. 200-203
- Unit 5, Lesson 12, pp. 205-209
- Unit 5, Lesson 13, pp. 211-215

SCO: **5.N11 Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).**
[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide opportunity for students to model and solve addition and subtraction questions involving tenths, hundredths and thousandths concretely, pictorially, and symbolically (e.g., thousandths and hundredths grids, base ten blocks, and number lines.)
- Present addition and subtraction questions both horizontally and vertically to encourage alternative computational strategies. For example, for 1.234 + 1.990, students might calculate:
  1.234 + 2 = 3.234 followed by 3.234 - 0.01 = 3.224.
- Have students investigate the relationship between adding decimals numbers and whole numbers. For example, 356 + 232 = 588; this looks similar to 0.356 + 0.232 = 0.588.
- Provide problem solving situations that require students to add or subtract decimals using a variety of strategies.
- Have students estimate first when asked to solve problems involving adding and or subtracting decimals.

Suggested Activities

- Provide base ten blocks or thousandths grids. Have the student choose addition or subtraction questions involving decimal numbers to represent with the models.
- Model 4.23 and 1.359 with base ten blocks or thousandths grids. Ask the student to use the materials to explain how to find the difference between the two numbers.
- Provide students with the batting averages of some baseball players. Have them calculate the spread between the player with the highest average and the one with the lowest. Have students create problems using the averages on the list.
- Request that the students provide examples of questions in which two decimal numbers are added and the answers are whole numbers.
- Tell students that you have added three numbers, each less than 1, and the result is 2.4. Ask if all the decimal numbers could be less than one half and to explain why or why not. Once students realize the numbers cannot all be less than one-half, ask them how many could be less.
- Have students find situations in which decimals are added and subtracted beyond the classroom and present their findings to the class.

SCO: **5.N11 Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).**
[C, CN, PS, R, V]

Assessment Strategies

- Ask the students to fill in the boxes so that the answer for each question is 0.4. The only stipulation is that the digit 0 cannot be used to the right of the decimal points.

$$\begin{array}{r} \square.\square\square \\ +\ \square.\square\square \\ \hline \end{array} \qquad \begin{array}{r} \square.\square\square \\ -\ \square.\square\square \\ \hline \end{array}$$

- Present the following situation in which Jane made an error when she subtracted. Ask the students what could be said to Jane to help her understand why the answer is incorrect: 5.23 - 1.453 = 3.783.
- Provide students with addition and subtraction questions in which the decimal is missing from the sum or difference. Have students place the decimal in the correct position.
- Use an example to explain why it is important to keep track of place value positions when adding and subtracting decimal numbers. (This can be written in a journal.)
- Have students solve problems such as (ensure that students provide an estimate):
  - John needs 2 kg of hamburger for a recipe. He has a 0.750 kg package. How much more does he need to buy?
  - Sasha bought two books at the book fair. One was $6.95 and the other was $7.38. How much change will she get from a $20 bill

PATTERNS AND RELATIONS

SPECIFIC CURRICULUM OUTCOMES

Patterns

5.PR1 – Determine the pattern rule to make predictions about subsequent elements.

Variables and Equations

5.PR2 – Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

SCO: **5.PR1 Determine the pattern rule to make predictions about subsequent elements.**
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.PR1 Identify and describe patterns found in tables and charts, including a multiplication chart. **4.PR2** Reproduce a pattern shown in a table or chart using concrete materials. **4.PR3** Represent and describe patterns and relationships using charts and tables to solve problems.	**5.PR1** Determine the pattern rule to make predictions about subsequent elements.	**6.PR1** Demonstrate an understanding of the relationship within tables of values to solve problems. **6.PR2** Represent and describe patterns and relationships using graphs and tables.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. extend a given pattern with and without concrete materials, and explain how each element differs from the preceding one;

B. describe, orally or in writing, a given pattern using mathematical language, such as one more, one less, five more;

C. write a mathematical expression to represent a given pattern, such as $r + 1$, $r – 1$, $r + 5$;

D. describe the relationship in a given table or chart using a mathematical expression;

E. determine and explain why a given number is or is not the next element in a pattern;

F. predict subsequent elements in a given pattern;

G. solve a given problem by using a pattern rule to determine subsequent elements; and

H. represent a given pattern visually to verify predictions.

SCO: **5.PR1 Determine the pattern rule to make predictions about subsequent elements.**
[C, CN, PS, R, V]

Elaboration

Patterns are key to understanding mathematical concepts. The ability to create, recognize and extend patterns is essential for making generalizations, seeing relationships, and understanding the order and logic of mathematics (Burns, 2007; p.144). These skills provide the foundation for **algebraic reasoning** and inquiry.

Patterns represent identified regularities based on rules describing the patterns' **elements**. Unless a pattern rule is provided there is no single way to extend a pattern (e.g., 1, 3, 5, 7 might be an odd number sequence, or a repeating sequence 1, 3, 5, 7, 1, 3, 5, 7…).

Patterns can be used to represent a situation and to solve problems. They can be **extended** with and without concrete materials and can be described using mathematical language. When discussing a pattern, students should be encouraged to determine how each step in the pattern is different from the preceding step.

			xxx
		xxx	xxx
	xxx	xxx	xxx
xxx	xxx	xxx	xxx

Step	1	2	3	4	5	6	?	…	20
Number of X's	3	6	9	12	?	?	?	…	?

Tables and charts provide an opportunity to display patterns and see relationships. For most students, these tables and charts make it easier to see the patterns from one step to the next. When a chart has been constructed, the differences from one step to the next can be written by it. Students will probably first observe the pattern from one step to the next; however, using the chart to find the twentieth or hundredth step is not reasonable. If a rule or relationship can be discovered, any table entry can be determined without building or calculating all of the intermediate entries. Some students might recognize that the rule can be described as a **mathematical expression**. For example, in the above pattern, the rule could be described as $3n$. The 20^{th} step would be 20×3 (60).

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 1, Launch, pp. 4, 5
- Unit 1, Lesson 1, pp. 6-8
- Unit 1, Lesson 2, pp. 9-12
- Unit 1, Lesson 3, pp. 13-16
- Unit 1, Lesson 4, pp. 18, 19

SCO: **5.PR1 Determine the pattern rule to make predictions about subsequent elements.**
[C, CN, PS, R, V]

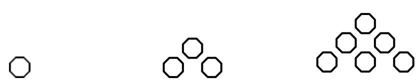
Instructional Strategies

Consider the following strategies when planning lessons:

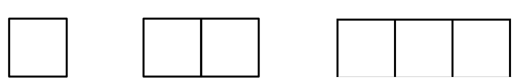
- Have students practice extending patterns with materials and drawings and then translate the pattern elements to a table or T-chart. Ask them to describe what is happening as the pattern grows, and how the new step is related to the previous one.
- Have students describe, using mathematical language (e.g., one more, seven less) and symbolically (e.g., $r + 1$, $p - 7$), a pattern represented concretely, pictorially, or from a chart.
- Have students verify whether or not a particular number belongs to a given pattern.
- Have students solve problems and make decisions based upon the mathematical analysis of a pattern and other contributing factors.

Suggested Activities

- Show students the first three or four steps of a pattern. Provide them with appropriate materials and/or grid paper and have them extend the patterns recording each step, and explain why their extension follows the pattern. Have them determine the pattern rule.



- Have students examine number sequences to determine subsequent terms and explain their extensions. Ask students to determine the pattern rule.
  1, 4, 7, 10, 13, …   20, 19, 16, 11, …   0, 2, 6, 14, 30, …   1, 2, 5, 11, 23, …
- Ask students to work in pairs to explore the many patterns on a multiplication chart (e.g., square numbers on the diagonal, sums of rows and columns, adjacent square patterns, doubling between columns, such as the 2's, 4's, and 8's).
- Provide students with a growing pattern and have them extend it. They should make a table showing how many items are needed to make each step of the pattern. Have them predict the number of items in the tenth or twentieth step of the pattern. For example, four people can sit at one table, six people can sit at two tables pushed together, eight people can sit at three tables. How many can sit at ten tables? Twenty? How many tables are needed for 24 people?



Number of tables	1	2	3	4	…
Number of seats	4	6	8	?	…

This pattern could be displayed on a T-chart:

Tables	Seats

SCO: **5.PR1 Determine the pattern rule to make predictions about subsequent elements.**
[C, CN, PS, R, V]

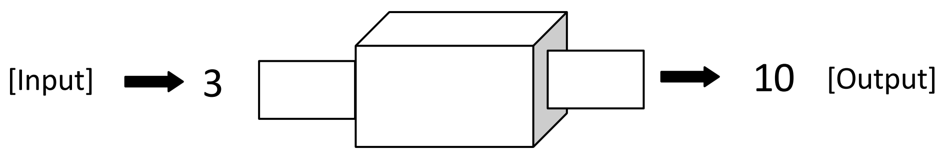
Assessment Strategies

- Have students fill in missing elements from number sequences and identify pattern rules.
  e.g., 1, 4, _____, 16, ___, 36

  2, 5, 11, 23, _____, _____

  2.4, 2.7, _____ , _____ , 3.6

- Give the students an input/output machine and ask them to provide different possible rules.



What's My Rule?

Some possibilities: Add 7; 3*n* + 1; 4*n* - 2

- Have students extend patterns concretely or pictorially, then complete tables and identify pattern rules.
- Have students solve real-world problems that require identifying a pattern rule to determine subsequent elements. For example, to bake cookies for a school bake sale, the quantities of ingredients in the recipe must be determined for multiple batches of cookies. If $\frac{2}{3}$ cups of sugar, and $1\frac{1}{2}$ cups flour is needed for one batch, how much is needed for 4 batches? 7 batches?

SCO: **5.PR2 Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.**
[C, CN, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.PR4 Express a given problem as an equation in which a symbol is used to represent an unknown number. **4.PR5** Solve one-step equations involving a symbol to represent an unknown number.	**5.PR2** Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.	**6.PR3** Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. explain the purpose of the letter variable in a given addition, subtraction, multiplication or division equation with one unknown; e.g., $36 \div n = 6$;

B. express a given pictorial or concrete representation of an equation in symbolic form;

C. express a given problem as an equation where the unknown is represented by a letter variable;

D. create a problem for a given equation with one unknown;

E. solve a given single-variable equation with the unknown in any of the terms; e.g., $n + 2 = 5$, $4 + a = 7$, $6 = r - 2$, $10 = 2c$; and

F. identify the unknown in a problem; represent the problem with an equation; and solve the problem concretely, pictorially or symbolically.

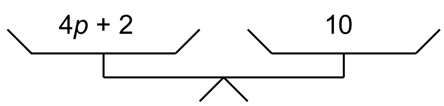
SCO: **5.PR2 Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.**
[C, CN, PS, R]

Elaboration

Exploring patterns leads to algebraic thinking. Algebra is a system that allows us to represent and explain mathematical relationships. Students are thinking algebraically when they solve open number sentences like $5 + \square = 13$, first using boxes or open frames, then using letters, $5 + n = 13$. When letters or open frames are used in mathematics, they are called **variables**. Students usually progress from the use of open frame to letters. It is useful for students to think of variables as numbers that can be operated on and manipulated like other numbers.

An **equation** is a mathematical sentence with an equal sign. An **expression** does not include an equal sign and is used most frequently to describe a pattern rule. A **coefficient** is a quantity (usually a numerical constant), which is multiplied by another quantity following it in an expression or an equation; e.g., in the algebraic equation $4p + 2 = 10$, the coefficient is 4.

In order to solve an equation, we need to find the value of the variable to make the equation true. Using the balance concept on a regular basis will help students develop a visual image for solving equations.



Students have been exploring the concept of equality since grade two. It is important for students to recognize that the equal sign indicates that both sides of the equation are balanced and does not simply mean “the answer is”.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:
- Unit 1, Lesson 5, pp. 20-22
- Unit 1, Lesson 6, pp. 23-25
- Unit 1, Lesson 7, pp. 26-28

SCO: **5.PR2 Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.**
[C, CN, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Build on the students' knowledge from the previous grades to write addition, subtraction, multiplication and division equations. Connect the concrete (use models such as counters and balance scales) with pictorial and symbolic representations consistently as the students develop and demonstrate understanding of equations.
- Use everyday contexts for problems to which the students can relate so that they can translate the meaning of the problem into an appropriate equation using a letter to represent the unknown number.
- Have the students create problems for a variety of number sentences using the four operations.
- Explain that if the same variable, or unknown, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown; e.g., for $n + n = 20$ can be written as $2n = 20$.
- Have students complete tables such as:

n	$3n$
3	9
8	
	30
12	

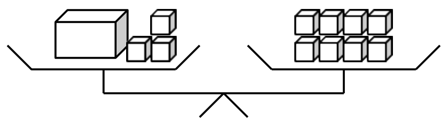
Suggested Activities

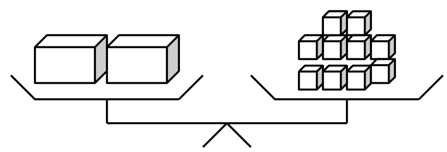
- Have students play “Solve for my variable”
  I subtract 6 from n and have 13 left. What is n?
  Four more than p is 37. What is p?
  Possible extension: Two more than $3x$ is 23. What is x? or One less than $4k$ is 27. What is k?
- Provide simple story problems and ask students to write equations. Include stories for all four operations. For example:
  I had birthday money and spent \$6.25. I now have \$8.75. (n - \$6.25 = \$8.75 or \$6.25 + \$8.75 = n)
  There are 3 full boxes of pencils. There are 36 pencils in all. ($3a = 36$)
- Provide one-step single-variable equations and have students create story problems.
- Have students write equations for balances such as the ones below, using letters for the variables.

SCO: **5.PR2 Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.**
[C, CN, PS, R]

Assessment Strategies

- Have students solve single-variable, one-step equations such as $18 + n = 31$; $9 = 43 – p$; $8x = 56$; $m \div 6 = 7$
- Write word problems that could be represented by each of the equations in the previous task.
- Have students complete tables using whole number coefficients.
- Have students write equations to describe the balance representations, such as the following:





SHAPE AND SPACE

SPECIFIC CURRICULUM OUTCOMES

Measurement

5.SS1 – Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.

5.SS2 – Demonstrate an understanding of measuring length (mm) by selecting and justifying referents for the unit mm and modelling and describing the relationship between mm and cm units, and between mm and m units.

5.SS3 – Demonstrate an understanding of volume by selecting and justifying referents for cm3 or m3 units, estimating volume by using referents for cm3 or m3, measuring and recording volume (cm3 or m3) and constructing rectangular prisms for a given volume.

3D Objects and 2D Shapes

5.SS4 – Identify and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombuses according to their attributes.

SCO: **5.SS1 Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.**
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.SS3 Demonstrate an understanding of area of regular and irregular 2D shapes by • *recognizing that area is measured in square units* • *selecting and justifying referents for the units* cm^2 *or* m^2 • *estimating area by using referents for* cm^2 *or* m^2 • *determining and recording area* • *constructing different rectangles for a given area (*cm^2 *or* m^2*) in order to demonstrate that many different rectangles may have the same area.*	**5.SS1** Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and draw conclusions.	**6.SS3** Develop and apply a formula for determining the: • *perimeter of polygons* • *area of rectangles* • *volume of right rectangular prisms.*

Achievement Indicators

Students who have achieved this outcome should be able to:

A. construct or draw two or more rectangles for a given perimeter in a problem-solving context;
B. construct or draw two or more rectangles for a given area in a problem-solving context;
C. illustrate that for any given perimeter, the square or shape closest to a square will result in the greatest area;
D. illustrate that for any given perimeter, the rectangle with the smallest possible width will result in the least area; and
E. provide a real-life context for when it is important to consider the relationship between area and perimeter.

SCO: **5.SS1 Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.**
[C, CN, PS, R, V]

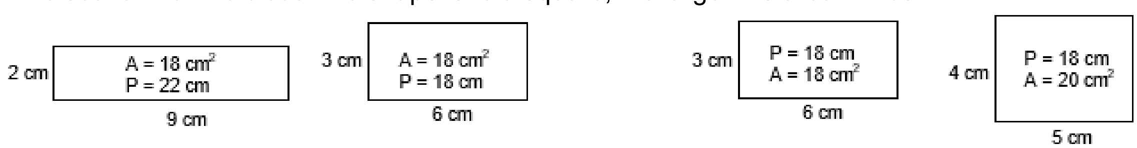
Elaboration

Area is the measure of the space inside a region or how much it takes to cover a region; **perimeter** is the distance around a region. Students in grade five often do not make the distinction between area and perimeter and may calculate the area instead of perimeter or vice versa. Therefore, it is important that they have many opportunities to construct rectangles of different areas and perimeters concretely and pictorially.

Area and perimeter involve measuring length. Formulas may be part of what comes out of the activities but is not an essential learning at this point. When the students are able to measure efficiently and effectively using standard units, their learning experiences can be directed to situations that encourage them to construct measurement formulas. When determining the area of a rectangle, students may realize as they count squares that it would be quicker to find the number of squares in one row and multiply this by the number of rows. When finding perimeters of rectangles, students may discover more efficient methods instead of adding all four sides to find the answer (e.g., add the length and width and double).

It is important that students learn about area and perimeter together. Through explorations, students will:

- discover that it is possible for a rectangle of a certain area to have different perimeters
- discover that is possible for rectangles with the same perimeter to have different areas.
- discover that the closer the shape is to a square, the larger the area will be.



This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 4, Lesson 2, pp. 126, 127
- Unit 4, Lesson 3, pp. 128-130
- Unit 4, Lesson 4, pp. 132-134

SCO: **5.SS1 Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.**
[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students use geoboards to construct rectangles with specified perimeters and discuss the areas.
- Assign specific areas (e.g., 12 square units) and have students use colour tiles to create rectangles and find the possible perimeters.
- Ask the students to use dot paper to compare the areas of rectangles with the following dimensions: 2 cm × 3 cm, 4 cm × 3 cm, 6 cm × 3 cm. Ask what they observe and have them give another set of dimensions that follows the same pattern and draw conclusions.

Suggested Activities

- Ask the student to explain why the perimeter of rectangles with whole number side lengths is always even. Have them use words, drawings, and/or numbers in their explanation.
- Have students relate perimeters to areas. For example, give pairs of students 24 colour tiles and ask them to find different rectangles, each with the area of 24 square units, but with different perimeters. Ask them to find a way to keep track of their rectangles and perimeters. What rectangle has the largest perimeter? The smallest? Have students draw conclusions.
- Ask the students to draw three different rectangles with the same perimeter.
- Construct rectangles on grid paper with a given perimeter, and then compare the side lengths and the areas. Have students discuss their findings and conclusions with regard to side length and area.
- Use a 2 × 3 rectangle, have students predict what would happen to the area and perimeter if the side lengths were doubled, halved? Have students draw conclusions.

SCO: **5.SS1 Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.**
[C, CN, PS, R, V]

Assessment Strategies

- Have students compare and contrast a given pair of rectangles with the same perimeter.
- Ask students to choose the dimensions of the rectangle with the largest area and the smallest area from a set of rectangles with the same perimeter.
- Have students construct (concretely or pictorially) and record the dimensions of two or more rectangles with a specified perimeter. Have students select and justify dimensions that would be most appropriate in a particular situation.
- Have students construct (concretely or pictorially) and record the dimensions of as many rectangles as possible with a specified area and select, with justification, the rectangle that would be most appropriate in a particular situation.
- Ask students to identify situations relevant to self, family, or community where the solution to problems would require the consideration of both area and perimeter, and solve the problems.

SCO: **5.SS2 Demonstrate an understanding of measuring length (mm) by:**
- **selecting and justifying referents for the unit mm**
- **modeling and describing the relationship between mm and cm units, and between mm and m units.**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.SS3 Demonstrate an understanding of area of regular and irregular 2D shapes by • *recognizing that area is measured in square units* • *selecting and justifying referents for the units cm^2 or m^2* • *estimating area by using referents for cm^2 or m^2* • *determining and recording area* • *constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.*	**5.SS2** Demonstrate an understanding of measuring length (mm) by: • *selecting and justifying referents for the unit mm* • *modeling and describing the relationship between mm and cm units, and between mm and m units.*	

Achievement Indicators

Students who have achieved this outcome should be able to:

A. provide a referent for one millimetre and explain the choice;

B. provide a referent for one centimetre and explain the choice;

C. provide a referent for one metre and explain the choice;

D. show that 10 millimetres is equivalent to 1 centimetre using concrete materials, e.g., ruler;

E. show that 1000 millimetres is equivalent to 1 metre using concrete materials, e.g., meter stick.; and

F. provide examples of when millimetres are used as the unit of measure.

SCO: **5.SS2 Demonstrate an understanding of measuring length (mm) by:**
- **selecting and justifying referents for the unit mm**
- **modelling and describing the relationship between mm and cm units, and between mm and m units.**

[C, CN, ME, PS, R, V]

Elaboration

Measurement is fundamentally about making comparisons. At this point in their learning, students are able to compare two objects directly by accurately using standard units of length such as **millimetres**, **centimetres**, and **metres**.

Students will choose a **personal referent** for one millimetre and explain their choice. They should continue to use their referents for one centimetre and one metre developed in grade three. For example, one millimetre is about the thickness of a dime, one centimetre is about the width of your baby finger, and one metre is about the height of the doorknob.

Students need to learn how to choose the appropriate unit or combination of units for the task at hand. This choice depends on the magnitude of the length to be measured and the level of precision required by the task (Small, 2008; p. 379). For example, millimetres can be used to measure small objects or to measure larger objects with more precision. Students should recognize that 1 metre is 100 centimetres or 1000 millimetres and 1 centimetre is 10 millimetres and change from the smaller unit to the larger unit. Flexibility with using the different measurements is in the developmental stage and needs to be supported with a variety of materials. Students need to be able to rename measurements such as a pencil that is 11 cm long could also be described as 110 mm or 0.11 m, but also able to identify which unit is the most appropriate.

Using rulers, meter sticks, Cuisenaire® rods and base ten materials will provide students with benchmarks when estimating lengths. Students should be encouraged to estimate measurements before actually verifying them using a measuring device.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 4, Lesson 1, pp. 122-125
- Unit 4, Lesson 8, pp. 191-193

SCO: **5.SS2 Demonstrate an understanding of measuring length (mm) by:**
- **selecting and justifying referents for the unit mm**
- **modeling and describing the relationship between mm and cm units, and between mm and m units.**

[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Help the students develop mental images of various measurement standards. To provide estimation practice, involve students in activities such as, “Show me (with hands or arms): 75 centimetres; 20 millimetres; 0.5 metres”.
- Have students use the relationships between standard metric units to rename measurements when comparing them.
- Share a short paragraph describing the measurements of a variety of classroom items. Ask the students to insert the appropriate unit for each. For example, the table was 1524 ___ long. On it was a pencil that was 0.17 ___ long.
- Encourage students to think of their ruler, as well as a meter stick or base ten blocks, when estimating length. Most rulers are 30 cm (or 300 mm) long and serve as good benchmarks. For example, 62 cm can be thought of as the length of about 2 rulers.

Suggested Activities

- Ask the students to show, with fingers or arms, the following lengths: 550 mm, 60 cm, 0.25 m. Have them describe the length using another unit of measure.
- Ask the student to rewrite 2.3 m using other metric units.
- Ask: If you change metres to centimetres, will the numerical value become greater or less? Why?
- Have students measure objects that do not measure exact centimetres, thus stressing the importance of millimetres when striving for precision of measurement.
- Hold a “Measurement Scavenger Hunt” in the classroom. Students should estimate the length of objects first, and then measure for accuracy.

SCO: **5.SS2 Demonstrate an understanding of measuring length (mm) by:**
- **selecting and justifying referents for the unit mm**
- **modelling and describing the relationship between mm and cm units, and between mm and m units.**

[C, CN, ME, PS, R, V]

Assessment Strategies

- Have students choose and use referents for 1 mm, 1 cm, 1 m to determine approximate linear measurements in situations relevant to self, family, or community and explain the choice.
- Have students generalize measurement relationships between mm, cm, and m from explorations using concrete materials (e.g., 10 mm = 1 cm, 0.01m = 1 cm).
- Have students provide examples of situations that are relevant to their life, family, or community in which linear measurements would be made and identify the standard unit (mm, cm, or m) that would be used for that measurement and justify the choice (e.g., heights of people, crafts).
- Ask students to draw, construct, or physically act out a representation of a given linear measurement.
- Have students pose and solve problems that involve hands-on linear measurements using either referents or standard units.

SCO: **5.SS3 Demonstrate an understanding of volume by:**
- **selecting and justifying referents for cm^3 or m^3 units**
- **estimating volume by using referents for cm^3 or m^3**
- **measuring and recording volume (cm^3 or m^3)**
- **constructing rectangular prisms for a given volume.**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Four	Grade Five	Grade Six
	5.SS3 Demonstrate an understanding of volume by: • *selecting and justifying referents for cm^3 or m^3 units* • *estimating volume by using referents for cm^3 or m^3* • *measuring and recording volume (cm^3 or m^3)* • *constructing rectangular prisms for a given volume.*	**6.SS3** Develop and apply a formula for determining the: • *perimeter of polygons* • *area of rectangles* • *volume of right rectangular prisms.*

Achievement Indicators

Students who have achieved this outcome should be able to:

A. identify the cube as the most efficient unit for measuring volume and explain why;
B. provide a referent for a cubic centimetre and explain the choice;
C. provide a referent for a cubic metre and explain the choice;
D. determine which standard cubic unit is represented by a given referent;
E. estimate the volume of a given 3D object using personal referents;
F. determine the volume of a given 3D object using manipulatives and explain the strategy;
G. construct a rectangular prism for a given volume; and
H. explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same given volume.

SCO: **5.SS3 Demonstrate an understanding of volume by:**
- **selecting and justifying referents for cm^3 or m^3 units**
- **estimating volume by using referents for cm^3 or m^3**
- **measuring and recording volume (cm^3 or m^3)**
- **constructing rectangular prisms for a given volume.**

[C, CN, ME, PS, R, V]

Elaboration

Volume and **capacity** are both terms used for measuring the size of three-dimensional regions. Although these two concepts are related, we will concentrate on volume for this specific outcome. Volume typically refers to the amount of space that an object takes up. Volume is measured with cubic centimetres and cubic metres. Capacity is the amount a container can hold (only hollow objects have capacity). Capacity units introduced in grade 5 are millilitres (mL) and litres (L). Volume can also be used to refer to the capacity of a container.

Students should develop personal referents the units. The use of personal referents helps students establish the relationships between the units. Students should realize that the small cube in the base-ten set is 1 cm on a side and would hold 1 mL and the large base-ten cube is 1000 cubic centimetres and would hold 1L Being able to estimate the volume of various containers and then to measure in the appropriate unit is important as students begin to construct rectangular prisms of various sizes.

Students should be given opportunities to explore the size of a million using visualization. By building a cubic metre with meter sticks, they will to have a good mental image of $1m^3$.

Students should have a sense of which volume or capacity unit is more appropriate to use in various circumstances.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:
- Unit 4, Lesson 5, pp. 135-137
- Unit 4, Lesson 6, pp. 138-141
- Unit 4, Lesson 7, pp. 142-144
- Unit 4, Lesson 8, pp. 145-147
- Unit 4, Lesson 9, pp. 148-150
- Unit 4, Lesson 10, pp. 151-154
- Unit 4, Lesson 11, pp. 155-157

SCO: **5.SS3 Demonstrate an understanding of volume by:**
- **selecting and justifying referents for cm^3 or m^3 units**
- **estimating volume by using referents for cm^3 or m^3**
- **measuring and recording volume (cm^3 or m^3)**
- **constructing rectangular prisms for a given volume.**

[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Identify the cube as the most efficient unit for measuring volume and explain why.
- Provide a referent for a cubic centimetre and explain the choice.
- Provide a referent for a cubic metre and explain the choice.
- Determine which standard cubic unit is represented by a given referent.
- Estimate the volume of a given 3D object using personal referents.
- Determine the volume of a given 3D object using manipulatives and explain the strategy.
- Construct a rectangular prism for a given volume.
- Explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same given volume.

Suggested Activities

- Measure the volume of small rectangular prisms by counting the number of centimetre cubes it takes to build a duplicate of it.
- Use base ten blocks or linking cubes to build several different structures each with a set volume. Discuss the different dimensions of the rectangular prisms.
- Provide students with a pair of small boxes, exactly one block and a ruler. Have students decide which box has the greater volume. Students use words, drawings and numbers to explain their conclusions. Sample box dimensions: ($6 \times 3 \times 4$) ($5 \times 4 \times 4$) ($6 \times 6 \times 2$).
- Have students build a cubic metre using metre sticks or other materials. Keep a model to use as a referent for m^3.
- Provide students with a pair of containers and ask them to predict which has the largest capacity (which holds more). Have them verify their predictions.
- Use centimetre grid paper to make nets for two prisms that have the same volume, but different shapes.
- Have students research the volumes of moving trucks. Ask what is a reasonable estimate for the volume of all the furniture in a school or in a house?

SCO: **5.SS3 Demonstrate an understanding of volume by:**
- **selecting and justifying referents for cm^3 or m^3 units**
- **estimating volume by using referents for cm^3 or m^3**
- **measuring and recording volume (cm^3 or m^3)**
- **constructing rectangular prisms for a given volume.**

[C, CN, ME, PS, R, V]

Assessment Strategies

- Ask students to calculate the volume of each size of base-ten blocks.

- Ask the student to estimate the volume of the classroom in cubic metres and give an explanation as to how the estimate was determined.
- Tell the student that you need a box with a volume of 400 cubic centimetres to hold a gift you have purchased. Ask: What might that gift be?
- Give students the volume of a rectangular prism and have them construct it.
- Ask students to describe the strategy they would use to estimate the volume of certain common rectangular prisms such as lunchboxes, pasta boxes, tissue boxes, etc.

SCO: **5.SS4 Identify and sort quadrilaterals, including:**
- **rectangles; squares**
- **trapezoids**
- **parallelograms**
- **rhombuses**

according to their attributes.
[C, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Four	Grade Five	Grade Six
	5.SS4 Identify and sort quadrilaterals, including: • rectangles and squares • trapezoids • parallelograms • rhombuses according to their attributes.	**6.SS2** Demonstrate that the sum of interior angles is: • 180° in a triangle • 360° in a quadrilateral. **6.SS5** Describe and compare the sides and angles of regular and irregular polygons.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. identify and describe the characteristics of a pre-sorted set of quadrilaterals;

B. sort a given set of quadrilaterals and explain the sorting rule;

C. sort a given set of quadrilaterals according to the lengths of the sides; and

D. sort a given set of quadrilaterals according to whether or not opposite sides are parallel.

SCO: **5.SS4 Identify and sort quadrilaterals, including:**
- **rectangles; squares**
- **trapezoids**
- **parallelograms**
- **rhombuses**

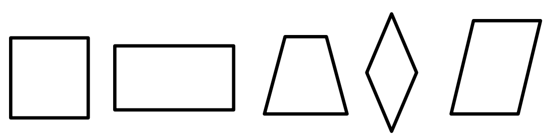
according to their attributes.
[C, R, V]

Elaboration

Quadrilaterals are four-sided polygons. Although rectangles are the most common quadrilateral that you see in everyday life, students will soon discover that there are many classes of quadrilaterals (Small, 2008; p. 295). Students will be exploring the attributes of various quadrilaterals such as **rectangles**, **squares**, **trapezoids**, **parallelograms**, **rhombuses**. They will compare the similarities and differences and sort them according to their attributes.

There is a gradual progression from identifying and describing two- and three-dimensional objects in students' own words to identifying and describing them in the formal language of geometry. It is important that students become familiar with the vocabulary associated with describing the **attributes** of 2D shapes such as **parallel**, **perpendicular**, **vertical, horizontal,** and **symmetry**

Common attributes will be side lengths, pairs of opposite sides parallel, and lines of symmetry.



This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:
- Unit 6, Lesson 1, pp. 222-225
- Unit 6, Lesson 2, pp. 226-229
- Unit 6, Lesson 3, pp. 230-233
- Unit 6, Lesson 4, pp. 234-239
- Unit 6, Lesson 5, pp. 240, 241

SCO: **5.SS4 Identify and sort quadrilaterals, including:**
- **rectangles; squares**
- **trapezoids**
- **parallelograms**
- **rhombuses**

according to their attributes.
[C, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use models, drawings and real life examples of quadrilaterals, to identify and describe the characteristics of each and classify them. Have students explain their classification system.
- Encourage students to use the formal language of geometry as they describe the attributes of classroom or real-life 2D shapes and 3D objects (e.g., the opposite walls in the classroom are parallel).

Suggested Activities

- Have students go on a “Quadrilateral Scavenger Hunt”. Have them sort their quadrilaterals with similar attributes and explain their rules for sorting.
- Have students prepare property lists with headings: sides, parallel, perpendicular, symmetries. Using a collection of quadrilaterals made on recipe cards, have students describe the shapes using language such as: at least 2 lines of symmetry.
- Provide students with a list of attributes and have them construct a quadrilateral that has the set of attributes. Have students share and compare with the class.
- Prepare a “Guess What Quadrilateral I Am?” game with clues about their attributes.

SCO: **5.SS4 Identify and sort quadrilaterals, including:**
- **rectangles; squares**
- **trapezoids**
- **parallelograms**
- **rhombuses**

according to their attributes.
[C, R, V]

Assessment Strategies

- Provide students with several different quadrilaterals to sort and have them justify their classification scheme.
- Have students draw quadrilaterals that satisfy a given set of attributes.

STATISTICS AND PROBABILITY

SPECIFIC CURRICULUM OUTCOMES

Data Analysis

5.SP1 – Differentiate between first-hand and second-hand data.

5.SP2 – Construct and interpret double bar graphs to draw conclusions.

SCO: **5.SP1 Differentiate between first-hand and second-hand data.**
[C, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Four	Grade Five	Grade Six
	5.SP1 Differentiate between first-hand and second-hand data.	

Achievement Indicators

A. explain the difference between first-hand and second-hand data;
B. formulate a question that can best be answered using first-hand data and explain why;
C. formulate a question that can best be answered using second-hand data and explain why; and
D. find examples of second-hand data in print and electronic media, such as newspapers, magazines and the Internet.

SCO: **5.SP1 Differentiate between first-hand and second-hand data.**
[C, R, T, V]

Elaboration

Students are familiar with collecting and organizing data from previous grades. They will learn about **first-hand data**, that they collect themselves, and **second-hand data** that other people have collected. The focus will be on comparing the collection methods and communicating results.

First-hand data: Collecting first-hand data can be done using a variety of methods such as interviews, surveys, experiments and observations. Students will then analyze the data and use **reasoning** to draw conclusions.

Students will need to determine what data they want to collect, and then design a survey with an appropriate question that will provide the information they want. They also have to determine who to survey, and whether their choice will influence their results, e.g., surveying people at the hockey rink about their favourite sport.

Second-hand data: Some data is difficult to obtain first-hand, but can be found in print and electronic media. Students will need to create appropriate questions that can be answered using second-hand data, then use that data to communicate different conclusions.

This outcome provides an opportunity for students to work with large numbers in context (relating to outcome N1), for example, comparing populations.

This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 7, Lesson 1, pp. 258-260

SCO: **5.SP1 Differentiate between first-hand and second-hand data.**
[C, R, T, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide questions and have students determine how best to collect the data in order for students to recognize the difference between first-hand and second-hand data.
- Have students generate which can best be answered using first-hand data, describe how that data could be collected.
- Have students generate questions that can best be answered using second-hand data, and describe how that data could be collected.

Suggested Activities

- Provide examples of data relevant to self, family, or community and categorize the data, with explanation, as first-hand or second-hand data.
- Have students formulate questions that can best be answered using first-hand data (e.g., “What game will we play at home tonight?” “I can survey everyone at home to find out what games everyone wants to play.”).
- Have students formulate a question related to self, family, or community, which can best be answered using second-hand data (e.g., “Which has the larger population – my community or my friend’s community?”). Then students should describe how this data could be collected (e.g., find the data on the StatsCan website: http://www.statcan.gc.ca), and finally answer the question.
- Have students find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the Internet, and compare different ways in which the data might be interpreted and used (e.g., statistics about health-related issues, sports data, or votes for favourite websites).

SCO: **5.SP1 Differentiate between first-hand and second-hand data.**
[C, R, T, V]

Assessment Strategies

- Ask students to write a question about preferred types of books that can be answered using first-hand data and explain why. How and from whom would the data be collected?
- Ask students to write a question about the populations of the cities in New Brunswick, and have them explain why the question is best answered using second-hand data. Where can the data be found?
- Have students work in groups to generate questions for which the data would be collected first- and second-hand.

SCO: **5.SP2 Construct and interpret double bar graphs to draw conclusions.**
[C, PS, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Four	Grade Five	Grade Six
4.SP2 Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.	**5.SP2** Construct and interpret double bar graphs to draw conclusions.	**6.SP1** Create, label and interpret line graphs to draw conclusions. **6.SP2** Graph collected data and analyze the graph to solve problems.

Achievement Indicators

Students who have achieved this outcome should be able to:

A. determine the attributes (title, axes, intervals and legend) of double bar graphs by comparing a given set of double bar graphs;

B. represent a given set of data by creating a double bar graph, label the title and axes, and create a legend without the use of technology;

C. draw conclusions from a given double bar graph to answer questions.;

D. provide examples of double bar graphs used in a variety of print and electronic media, such as newspapers, magazines and the Internet; and

E. solve a given problem by constructing and interpreting a double bar graph.

SCO: **5.SP2 Construct and interpret double bar graphs to draw conclusions.**
[C, PS, R, T, V]

Elaboration

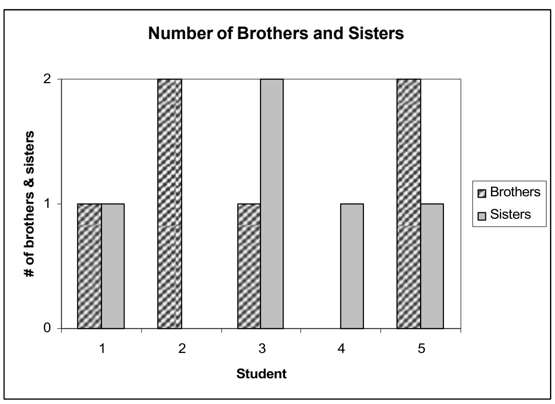
Students should be aware that sometimes when two pieces of data are collected about a certain population, it is desirable to display both of them side by side, using the same scale. For example, census data often shows male and female data separately for different years. This is usually done using a **double bar graph**. A **legend** is used to help the reader interpret a double bar graph. An example is presented below. Five students in the class have been asked how many brothers and sisters they have.

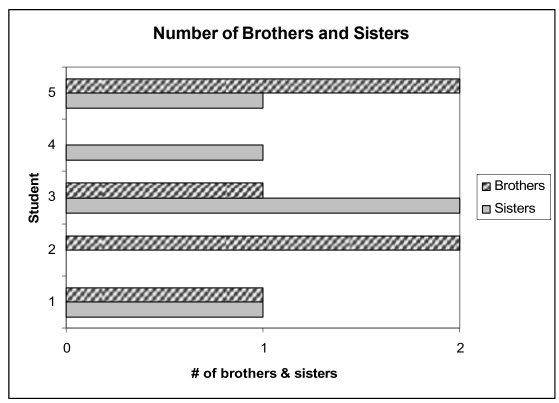
This type of graph allows students to be compared not only in terms of how many brothers they have, or how many sisters they have, but also to compare the number of brothers versus the number of sisters.

	Brothers	Sisters
Student 1	1	1
Student 2	2	0
Student 3	1	2
Student 4	0	1
Student 5	2	1

The data may be displayed horizontally or vertically as shown below.

Graphs should include a **title, labelled categories (including units), labelled horizontal** and **vertical axes, an appropriate scale, correctly plotted points** and **legend** (if needed). The pairs of bars should be separated. A common mistake made by students is to place the incremental numbers in the space rather than on the line.





This specific curriculum outcome is addressed in *Math Makes Sense 5* in the following units:

- Unit 7, Lesson 2, pp. 261-265
- Unit 7, Lesson 3, pp. 266-269
- Unit 7, Technology Lesson, pp. 270, 271

SCO: **5.SP2 Construct and interpret double bar graphs to draw conclusions.**
[C, PS, R, T, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students determine when it is appropriate to display data in a double bar graph.
- Provide students with sets of data and have them determine appropriate scales.
- Provide students with two double bar graphs displaying the same data using a different scale, and have students determine which they prefer and why.
- Have students collect first-hand and second-hand data and create double bar graphs making sure to include appropriate title, axis labels, intervals and legend.
- Have students use second-hand data collected from sites, such as Statistics Canada that use large numbers. (http://www.statcan.gc.ca and Census at School: www.censusatschool.ca)
- Have students generate sets of questions that can be answered by reading various double bar graphs.
- Have students compare data in the double bar graph within and among the pairs.

Suggested Activities

- Provide examples of double bar graphs from a variety of media sources, and ask students to bring in examples from similar sources.
- Have students examine double bar graph samples and determine the attributes (title, axes, legend, intervals). Ask them to compare and share the information displayed.
- Have students collect and graph first-hand data, such as girls' and boys' favourite activity in gym.
- Have students collect information on the length and mass of various animals and display the data in a double bar graph. Ask what conclusions they might draw.
- Have students create double bar graphs on subjects that are of personal interest, such as comparing hockey players' salaries from two different teams.

SCO: **5.SP2 Construct and interpret double bar graphs to draw conclusions.**
[C, PS, R, T, V]

Assessment Strategies

- Ask the student to describe some data that would be appropriate to display using a double bar graph.
- Have students generate a double bar graph from given sets of data without the use of technology. Rubrics might include appropriate scales and labeling as well as accuracy.
- Have students draw conclusions from a given double bar graph to answer questions.

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