Prince Edward Island
CANADA

Education and Early Years

Prince Edward Island Mathematics Curriculum

Mathematics

Grade 8

CURRICULUM

Acknowledgments

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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focussed to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

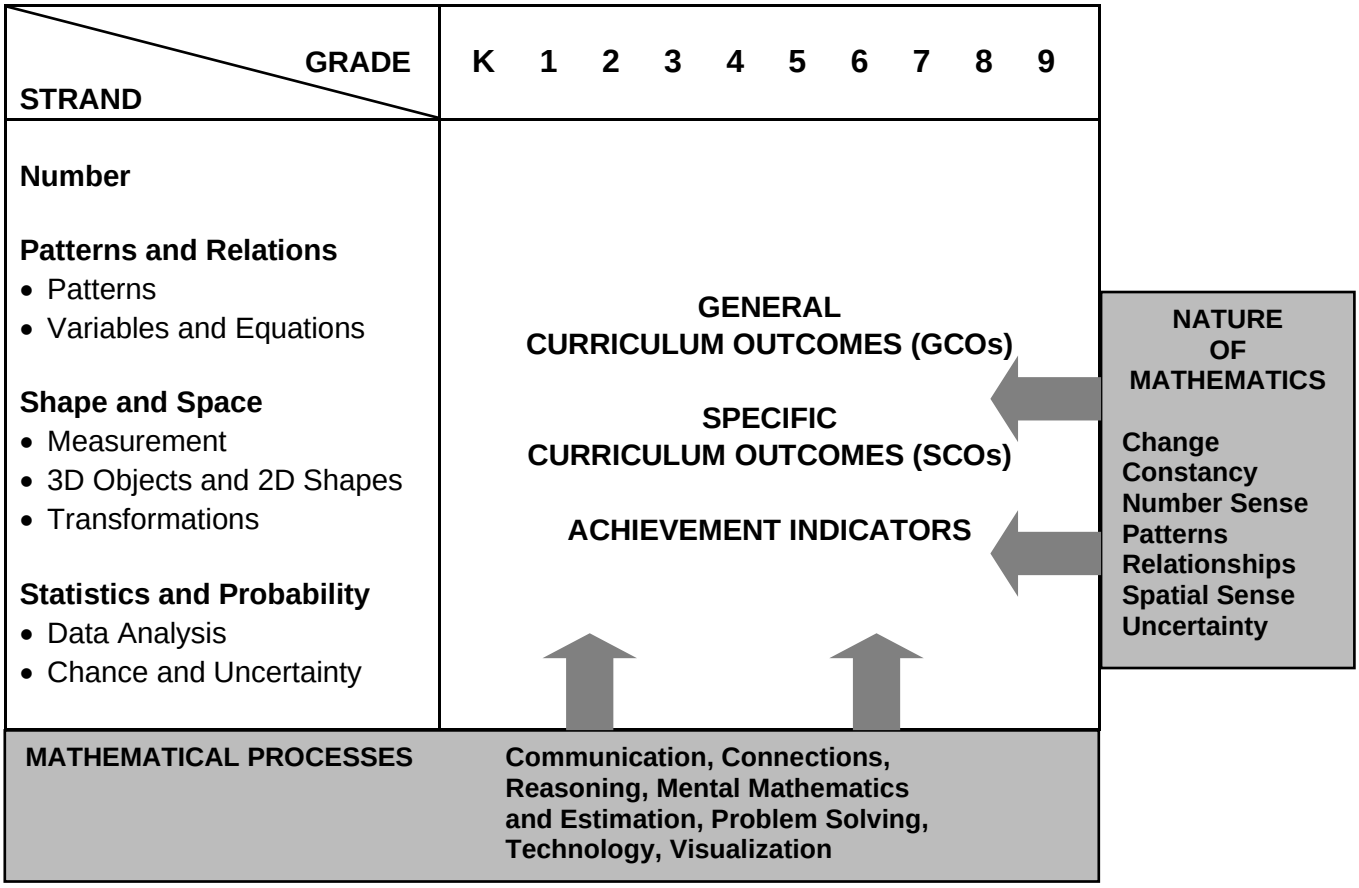
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for K-9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connections among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.
- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

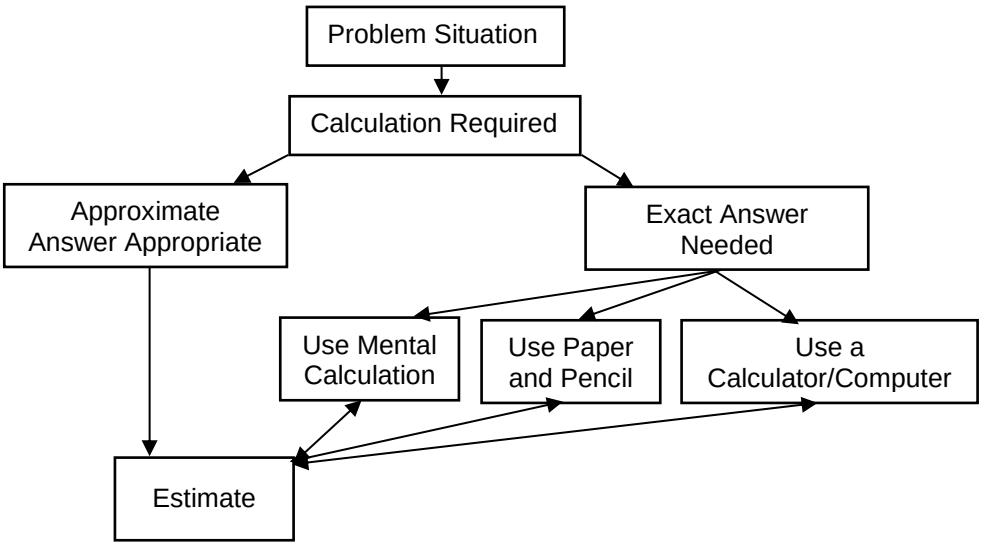
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001). Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you. . . ?" or "How could you. . . ?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not

a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3D objects and 2D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.
- The theoretical probability of flipping a coin and getting heads is 0.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2D shapes and 3D objects, and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2D shapes and 3D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3D or 2D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of

probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education (p.60)." The *Standards* elaborate that all

students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate "communicating to learn mathematics and learning to communicate mathematically" (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, or teaching has been effective, or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

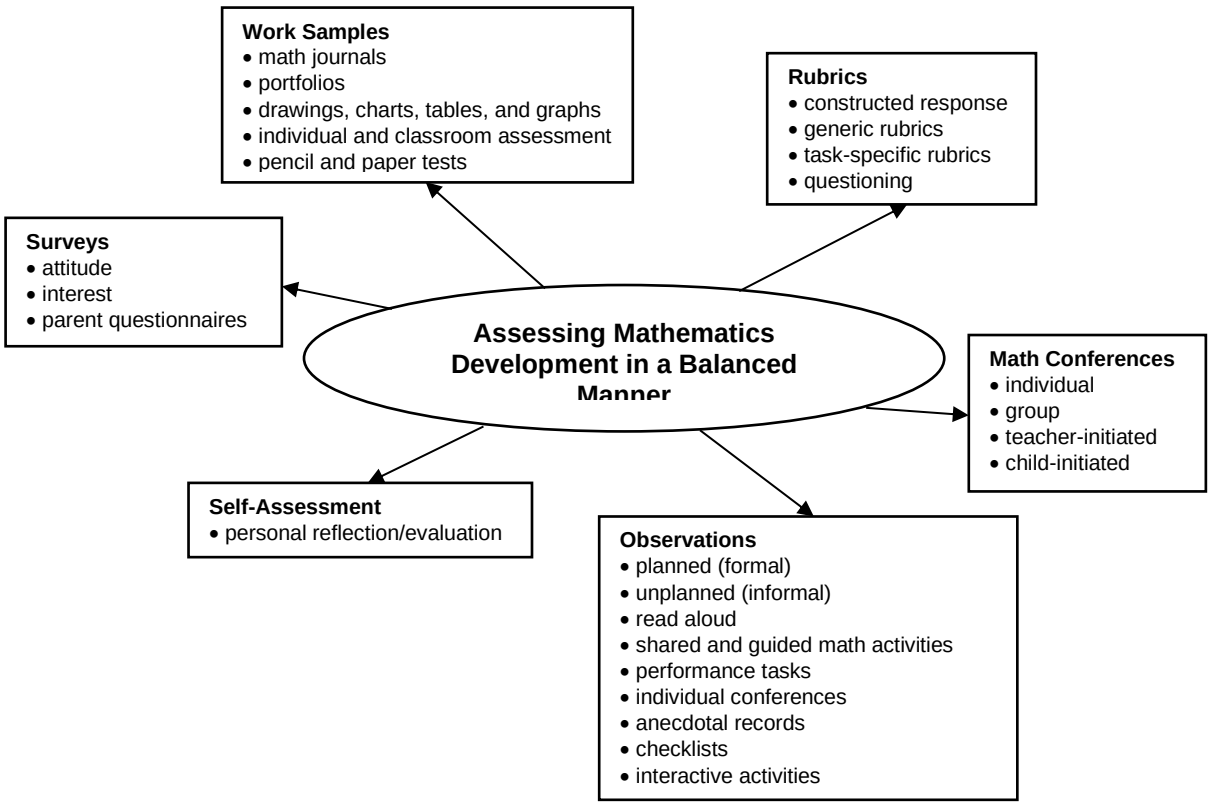
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;
- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, and phone calls.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense.
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems.
	3D Objects and 2D Shapes: Describe the characteristics of 3D objects and 2D shapes, and analyse the relationships among them.
	Transformations: Describe and analyse position and motion of objects and shapes.
Statistics and Probability (SP)	**Data Analysis**: Collect, display, and analyse data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding strand and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades seven to nine which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;
- a list of the sections in *MathLinks 8* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

NUMBER

SPECIFIC CURRICULUM OUTCOMES

8.N1 – Demonstrate an understanding of percents greater than or equal to 0%.

8.N2 – Demonstrate an understanding of ratio and rate.

8.N3 – Solve problems that involve rates, ratios and proportional reasoning.

8.N4 – Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).

8.N5 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
7.N3 Solve problems involving percents from 1% to 100%.	**8.N1** Demonstrate an understanding of percents greater than or equal to 0%.	

SCO: 8.N1 – Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V]

Students who have achieved this outcome should be able to:

A. provide a context where a percent may be more than 100% or between 0% and 1%;

B. represent a given fractional percent using grid paper;

C. represent a given percent greater than 100 using grid paper;

D. determine the percent represented by a given shaded region on a grid, and record it in decimal, fractional and percent form;

E. express a given percent in decimal or fractional form;

F. express a given decimal in percent or fractional form;

G. express a given fraction in decimal or percent form;

H. solve a given problem involving percents;

I. solve a given problem involving combined percents; and

J. solve a given problem that involves finding the percent of a percent, *e.g.,* A population increased by 10% one year and then increased by 15% the next year. Explain why there was not a 25% increase in population over the two years.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (A B C D)

4.2 (A D E F G)

4.3 (H)

4.4 (H I J)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: 8.N1 – Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V]

Elaboration

Percents are ratios or fractions where the second term or denominator is 100. The term percent is simply another name for *hundredths*. Percents can be written as low as 0, but can go higher than 100. In grade seven, students worked with percents from 1% to 100%. In grade eight, students examine contexts where percents can be greater than 100% or less than 1% (fractional percents).

Students should be able to move flexibly between percent, fraction and decimal equivalents in problem solving situations. For example, when finding 25% of a number, it is often much easier to use $\frac{1}{4}$ and then divide by 4 as a means of finding or estimating the percent. If students can express fractions and decimals as hundredths, the term percent can be substituted for the term hundredths. The fraction $\frac{3}{2}$ can be expressed in hundredths, $\frac{150}{100}$, which has a decimal equivalent of 1.5, and is equivalent to 150%.

Fractional and decimal percents can be related to benchmark percents. For example, 0.25% means a quarter of 1%. If you know that 1% of 400 is 4, then 0.25% of 400 would be a quarter of 4 or 1. It is also important to recognize that 1% can be a little or a lot depending on the size of the whole. For example, 1% of all of the population of a city is a lot of people compared to 1% of the students in a class.

Students will continue to create and solve problems that they explored in grade seven, which involve finding a, b or c in the relationship of $a\%$ of $b = c$ using estimation and calculation. They will also be required to apply percentage increase and decrease in problem situations in which percents greater than 100 or fractional percents are meaningful. In these situations, is important for students to recognize that 100% is still the whole.

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
7.N3 Solve problems involving percents from 1% to 100%. **7.SP3** Express probabilities as ratios, fractions and percents.	**8.N2** Demonstrate an understanding of ratio and rate.	**9.N3** Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers; • solving problems that involve arithmetic operations on rational numbers.

SCO: 8.N2 – Demonstrate an understanding of ratio and rate. [C, CN, V]

Students who have achieved this outcome should be able to:

A. express a two-term ratio from a given context in the forms $3:5$ or 3 to 5;

B. express a three-term ratio from a given context in the forms $4:7:3$ or 4 to 7 to 3;

C. express a part to part ratio as a part to whole fraction, *e.g.,* frozen juice to water; 1 can of concentrate to 4 cans of water can be represented as $\frac{1}{5}$, which is the ratio of concentrate to solution, or $\frac{4}{5}$, which is the ratio of water to solution;

D. identify and describe ratios and rates from real-life examples, and record them symbolically;

E. express a given rate using words or symbols, *e.g.,* 20 L per 100 km or 20 L/100 km; and

F. express a given ratio as a percent and explain why a rate cannot be represented as a percent.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B C D F)

2.2 (D E F)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 8.N2 – Demonstrate an understanding of ratio and rate. [C, CN, V]

Elaboration

A ratio is a comparison of at least two quantities. Ratios can express comparisons of a part to a whole (every fraction, percent, and probability is a ratio) or compare part of a whole to other parts of the same whole. Part-to-whole and part-to-part ratios compare two or more measures of the same type. A map scale is a common application of ratios.

A ratio that compares measures of two different types is called a rate (*e.g.*, a comparison of distance to time). A unit rate is an equivalent rate where the second term is one. This rate can be used to determine the better buy when comparing prices. Percent cannot be considered a rate because a percent is a ratio that compares quantities expressed in the same units.

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
7.N3 Solve problems involving percents from 1% to 100%. **7.SP3** Express probabilities as ratios, fractions and percents.	**8.N3** Solve problems that involve rates, ratios and proportional reasoning.	**9.N3** Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers; • solving problems that involve arithmetic operations on rational numbers.

SCO: 8.N3 – Solve problems that involve rates, ratios and proportional reasoning. **[C, CN, PS, R]**

Students who have achieved this outcome should be able to:

A. explain the meaning of $\frac{a}{b}$ within a given context;

B. provide a context in which $\frac{a}{b}$ represents a:

- fraction,
- rate,
- ratio,
- quotient,
- probability; and

C. solve a given problem involving rate, ratio or percent.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B C)

2.2 (A B C)

2.3 (A B C)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 8.N3 – Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Elaboration

Proportional reasoning is the ability to think about and compare multiplicative relationships between quantities. These relationships are represented symbolically as ratios. A proportion is a statement of equality between two ratios. Different notations for proportions can be used:

$$2:5=4:10 \text{ or } 2 \text{ to } 5=4 \text{ to } 10 \text{ or } \frac{2}{5}=\frac{4}{10}$$

These can be read as “two to five” and mean that for every 2 items there will be 5 items.

Finding one number in a proportion when the other three numbers are known is called solving a proportion. For example, how many girls are in a class when the ratio of boys to girls in a class is $3:5$ and there are 12 boys. To solve, set up the proportion: $\frac{3}{5}=\frac{12}{?}$. The students must think multiplicatively to solve the proportion in the same way they would to determine equivalent fractions. Also, a student who knows that a runner who runs at a rate of $\frac{1\text{ km}}{7\text{ min}}$ will win a 10 km race over a runner who runs at a rate of $\frac{1\text{ km}}{8\text{ min}}$ is thinking proportionally.

Students may need as much as three years worth of opportunities to reason in multiplicative situations to order to adequately develop proportional reasoning skills. Premature use of rules encourages students to apply rules without thinking and, thus, the ability to reason proportionally does not develop (Van de Walle & Lovin, vol. 3, 2006; p. 157).

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
7.N1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0. **7.N6** Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using: benchmarks; place value; equivalent fractions and/or decimals.	**8.N4** Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).	**9.N3** Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers; • solving problems that involve arithmetic operations on rational numbers.

SCO: **8.N4 – Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V]**

Students who have achieved this outcome should be able to:

A. model addition and subtraction of a given positive fraction or a given mixed number using concrete representations, and record symbolically;

B. determine the sum of two given positive fractions or mixed numbers with like denominators;

C. determine the difference of two given positive fractions or mixed numbers with like denominators;

D. determine a common denominator for a given set of positive fractions or mixed numbers;

E. determine the sum of two given positive fractions or mixed numbers with unlike denominators;

F. determine the difference of two given positive fractions or mixed numbers with unlike fractions;

G. simplify a given positive fraction or mixed number by identifying the common factor between the numerator and denominator;

H. simplify the solution to a given problem involving the sum or difference of two positive fractions or mixed numbers; and

I. solve a given problem involving the addition or subtraction of positive fractions or mixed numbers and determine if the solution is reasonable.

Section(s) in **MathLinks 7** *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.2 (A B G H I)

6.3 (A C G H I)

7.1 (D)

7.2 (A E F G H I)

7.3 (A B D G H I)

7.4 (C D F G H I)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 8.N4 – Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V]

Elaboration

Fractions are an extension of the whole number system used to represent values between the whole numbers. Since fractions include whole numbers, the same properties apply. Students need a solid conceptual foundation in fractions as a prerequisite for fraction computation. They must first understand the meaning of fractions, using different models – region, set and length or measurement. To help students add and subtract fractions correctly and with understanding, teachers must help them develop an understanding of the numerator and denominator, equivalence and the relation between mixed numbers and improper fractions (NCTM, 2000, p. 218).

- The meanings of each operation with fractions are the same as the meanings for the operations on whole numbers. Operations with fractions should begin by applying these same meanings to fractional parts. For addition and subtraction, it is critical to understand that the numerator tells the number of parts and the denominator the type of part.
- The estimation of fraction computations is tied almost entirely to both of the concepts of the operations used and of the fractions. A computation algorithm is not required for making estimates. Estimation should be an integral part of computation development to keep the attention of students on the meanings of the operations and the expected size of the results (Van de Walle and Lovin, 2006, p. 66).

An estimate is sometimes all that is required to satisfy a given situation. At other times, an estimate can be used to determine if an answer, found using an algorithm or a calculator, is reasonable. This skill permits students to make quick and efficient judgments about the reasonableness of answers acquired through algorithms. Developing benchmarks such as $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and 1 should be a focus of the estimation of sums and differences of fractions. It is necessary to encourage flexibility in thinking and provide learning opportunities in connecting:

- operations with whole numbers to operations with fractions;
- subtraction of fractions to addition of fractions;
- operations with fractions to real world problems (Alberta Education, 2004).

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
7.N6 Compare and order positive fractions, positive decimals and whole numbers by using: benchmarks; place value; equivalent fractions and/or decimals.	**8.N5** Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.	**9.N3** Demonstrate an understanding of rational numbers by: comparing and ordering rational numbers; solving problems that involve arithmetic operations on rational numbers.

SCO: 8.N5 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Students who have achieved this outcome should be able to:

A. identify the operation required to solve a given problem involving positive fractions;

B. provide a context that requires the multiplying of two given positive fractions;

C. provide a context that requires the dividing of two given positive fractions;

D. estimate the product of two given positive proper fractions to determine if the product will be closer to 0, $\frac{1}{2}$ or 1;

E. estimate the quotient of two given positive fractions and compare the estimate to whole number benchmark;

F. express a given positive mixed number as an improper fraction and a given positive improper fraction as a mixed number;

G. model multiplication of a positive fraction by a whole number concretely or pictorially and record the process;

H. model multiplication of a positive fraction by a positive fraction concretely or pictorially using an area model and record the process;

I. model division of a positive proper fraction by a whole number concretely or pictorially and record the process;

J. model division of a positive proper fraction by a positive proper fraction pictorially and record the process;

K. generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers; and

L. solve a given problem involving positive fractions, taking into consideration order of operations (limited to problems with positive solutions).

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A G L)

6.2 (A I L)

6.3 (B D H K L)

6.4 (B F H K)

6.5 (C E J K)

6.6 (K L)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 8.N5 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Elaboration

Multiplying and dividing fractions should be learned in relation to the addition and subtraction of fractions.

The following guidelines should be kept in mind when developing computational strategies for fractions. It is important to not rush to computational rules.

- Begin with simple contextual tasks (include sets, area models, distance).
- Connect the meaning of fraction computation with whole number computation.
- Let estimation and informal methods play a big role in the development of strategies.
- Explore each of the operations using models.

(Van de Walle & Lovin, vol. 3, 2006; p. 88)

It is important for students to understand what multiplication means: 3×5 means 3 groups of 5. The same is true for fractions. When multiplying a fraction by a whole number, it can be thought of as groups of the fraction or fractions of a group. For example, $3 \times \frac{1}{3}$ equals three groups of $\frac{1}{3}$ or $\frac{1}{3}$ of a group of three.

When multiplying a proper fraction by another number, some students struggle with the product being less than one or both of the factors. Students need to keep in mind they are multiplying by a number that is less than one. Language is very important. Models should be explored consistently throughout the teaching of this outcome. The area model should be used as a key method for exploring fraction multiplication.

For division of a fraction by a whole number, ask students what each part represents (*e.g.*, $\frac{1}{2} \div 3$ is telling us to break the half into 3 equal parts, so the answer is $\frac{1}{6}$). For division of a whole number by a fraction, ask students how many parts there are in the whole number (*e.g.*, to find $4 \div \frac{1}{2}$, ask how many halves there are in 4, giving an answer of 8.)

When the denominators are the same, the numerators can be divided to find the answer. If the simple fractions do not have common denominators, one strategy is to make them common and then divide the numerators: $\frac{4}{3} \div \frac{1}{2} = \frac{8}{6} \div \frac{3}{6} = \frac{8}{3}$. This approach is easier for students to conceptualize rather than following the traditional method of inverting the second fraction and multiplying. The number line can provide a useful model for division to help students visualize division.

Estimation is important for students to determine whether their products and quotients are reasonable. There are many real world examples where students can apply these skills and they should be encouraged to estimate either before or after any computation.

PATTERNS AND RELATIONS

SPECIFIC CURRICULUM OUTCOMES

8.PR1 – Graph and analyse two-variable linear relations.

8.PR2 – Model and solve problems using linear equations of the form:

- $ax = b$;
- $\frac{x}{a} = b$, $a \neq 0$;
- $ax + b = c$;
- $\frac{x}{a} + b = c$, $a \neq 0$;
- $a(x + b) = c$

concretely, pictorially and symbolically, where *a*, *b* and *c* are integers.

Grade 8 – Strand: Patterns and Relations (PR)

GCO: Use patterns to describe the world and solve problems.

GRADE 7	GRADE 8	GRADE 9
7.PR1 Create a table of values from a linear relation, graph the table of values, and analyse the graph to draw conclusions and solve problems. **7.SS3** Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.	**8.PR1** Graph and analyse two-variable linear relations.	**9.PR1** Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.

SCO: 8.PR1 – Graph and analyse two-variable linear relations. [C, ME, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. determine the missing value in an ordered pair for a given equation;

B. create a table of values by substituting values for a variable in the equation of a given linear relation;

C. construct a graph from the equation of a linear relation (limited to discrete data); and

D. describe the relationship between the variables of a given graph.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

9.1 (A B D)

9.2 (B C D)

9.3 (B C D)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 8.PR1 – Graph and analyse two-variable linear relations. [C, ME, PS, R, T, V]

Elaboration

The graphs constructed from the equation of a given linear equation will be limited to discrete data. Discrete data can only have a finite or limited number of possible values. Generally discrete data are countable: for example, the number of students in class, the number of tickets sold or the number of Christmas trees that were purchased. Continuous data can have an infinite number of possible values within a selected range, such as the quantities of temperature or time. A graph of discrete data has plotted points, but they are not joined together.

Students should observe when looking at tabular data that, when an equal spacing between the values of one variable produces an equal spacing between values of the other variable, the relationship will be linear. Students should recognize that for linear relationships, the ratio of vertical change to horizontal change is consistent anywhere along the line. However, it is not necessary to discuss slope of a line at this point.

Many resources will show continuous data graphs (all points connected) displayed as though they are discrete (no points connected). For example, any graph with time on the horizontal axis is actually displaying continuous data. The analysis of graphs should include creating stories that describe the relationship depicted and constructing graphs based on a story which involves changes in related quantities. For example, as the temperature rises, the number of people at the beach increases.

When students are describing a relationship in a graph, they should use language like “as this increases that decreases” or “as one quantity drops, the other also drops.” When students are attempting to find a missing value in an ordered pair, they should use either patterning or substitution into the equation if the equation has been provided.

Students need to be able to transition between given information whether it is presented as a table of values, a graph, a linear relation or a set of ordered pairs.

Grade 8 – Strand: Patterns and Relations (PR)

GCO: Represent algebraic expressions in multiple ways.

GRADE 7	GRADE 8	GRADE 9
7.PR2 Demonstrate an understanding of preservation of equality by: • modeling preservation of equality, concretely, pictorially and symbolically; • applying preservation of equality to solve equations. **7.PR5** Model and solve problems that can be represented by one-step linear equations of the form $x+a=b$, concretely, pictorially and symbolically, where *a* and *b* are integers. **7.PR6** Model and solve problems that can be represented by linear equations of the form: • $ax+b=c$; • $ax=b$; • $\frac{x}{a}=b,\ a\neq 0$ concretely, pictorially and symbolically, where *a*, *b* and *c* are whole numbers.	**8.PR2** Model and solve problems using linear equations of the form: • $ax=b$; • $\frac{x}{a}=b,\ a\neq 0$; • $ax+b=c$; • $\frac{x}{a}+b=c,\ a\neq 0$; • $a(x+b)=c$ concretely, pictorially and symbolically, where *a*, *b* and *c* are integers.	**9.PR2** Model and solve problems using linear equations of the form: • $ax=b$; • $\frac{x}{a}=b,\ a\neq 0$; • $ax+b=c$; • $\frac{x}{a}+b=c,\ a\neq 0$; • $ax=b+cx$; • $a(x+b)=c$; • $ax+b=cx+d$; • $a(bx+c)=d(ex+f)$; • $\frac{a}{x}=b,\ x\neq 0$ where *a*, *b*, *c*, *d*, *e* and *f* are rational numbers.

SCO: 8.PR2 – Model and solve problems using linear equations of the form:

- $\boldsymbol{ax=b}$;
- $\boldsymbol{\frac{x}{a}=b,\ a\neq 0}$;
- $\boldsymbol{ax+b=c}$;
- $\boldsymbol{\frac{x}{a}+b=c,\ a\neq 0}$;
- $\boldsymbol{a(x+b)=c}$

concretely, pictorially and symbolically, where *a*, *b* and *c* are integers. [C, CN, PS, V]

Students who have achieved this outcome should be able to:

A. model a given problem with a linear equation and solve the equation using concrete models, *e.g.,* counters, integer tiles;

B. verify the solution to a given linear equation using a variety of methods, including concrete materials, diagrams and substitution;

C. draw a visual representation of the steps used to solve a given linear equation and record each step symbolically;

D. solve a given linear equation symbolically;

E. identify and correct an error in a given incorrect solution of a linear equation;

F. apply the distributive property to solve a given linear equation, *e.g.,* $2(x+3)=8$ becomes $2x+6=8$; and

G. solve a given problem using a linear equation and record the process.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

10.1 (A B C D E G) **10.2 (A B C D E G)**
10.3 (A B C D E G) **10.4 (A B C D E F G)**

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 8.PR2 – Model and solve problems using linear equations of the form:

- $ax = b$;
- $\frac{x}{a} = b$, $a \neq 0$;
- $ax + b = c$;
- $\frac{x}{a} + b = c$, $a \neq 0$;
- $a(x + b) = c$

concretely, pictorially and symbolically, where *a*, *b* and *c* are integers. [C, CN, PS, V]

Elaboration

Students have experience solving one-step equations in the form of $x + a = b$ and two-step equations in the forms of $ax + b = c$, $ax = b$, and $\frac{x}{a} = b\ (a \neq 0)$, where *a*, *b* and *c* are whole numbers. In grade eight, students will continue to solve equations that will now include integers as well as fractions for the values of *a*, *b* and *c*.

In problem solving situations, students should consider in advance what might be a reasonable solution, and be aware that once they acquire a solution, it can be checked for accuracy by substitution into the original equation.

In order for students to solve linear equations of the forms $ax = b$, $\frac{x}{a} = b\ (a \neq 0)$, $ax + b = c$, $\frac{x}{a} + b = c\ (a \neq 0)$, and $a(x + b) = c$, they must recognize that the idea of "balancing" or "moving from one side to another" by using opposite operation(s) is required. The zero principle is an important aspect of finding equality between the two sides. This, in fact, allows for the preservation of balance and equality in the equation (where left side equals right side). In the form $ax + b = c$, for example, students need to perform a two-step elimination process to solve for the variable whereas in some other equations, only a single-step process is needed.

There are many methods for solving linear equation such as inspection, systematic trial (guess and test), rewriting the equation, creating models using algebra tiles and using illustrations of balances to show equality. Students should be encouraged to choose the most appropriate method for solving a given problem. Emphasis at this level should be on solving problems concretely, pictorially, and symbolically.

SHAPE AND SPACE

SPECIFIC CURRICULUM OUTCOMES

8.SS1 – Draw and construct nets for 3D objects.

8.SS2 – Determine the surface area of:
- **right rectangular prisms;**
- **right triangular prisms;**
- **right cylinders**

to solve problems.

8.SS3 – Develop and apply formulas for determining the volume of right prisms and right cylinders.

Grade 8 – Strand: Shape and Space (SS)

GCO: Use direct or indirect measurement to solve problems.

GRADE 7	GRADE 8	GRADE 9
	8.SS1 Draw and construct nets for 3D objects.	

SCO: 8.SS1 – Draw and construct nets for 3D objects. [C, CN, PS, V]

Students who have achieved this outcome should be able to:

A. match a given net to the 3D object it represents;

B. construct a 3D object from a given net;

C. draw nets for a given right circular cylinder, right rectangular prism and right triangular prism, and verify by constructing the 3D objects from the nets; and

D. predict 3D objects that can be created from a given net and verify the prediction.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.2 (A B C D)

5.3 (C)

5.4 (C)

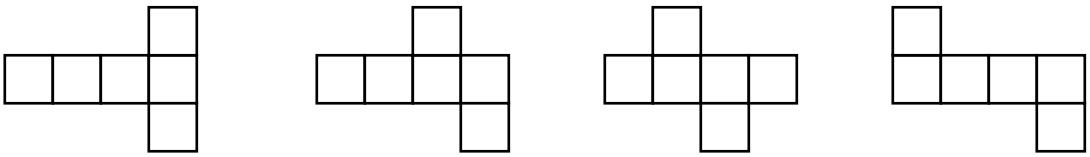
[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 8.SS1 – Draw and construct nets for 3D objects. [C, CN, PS, V]

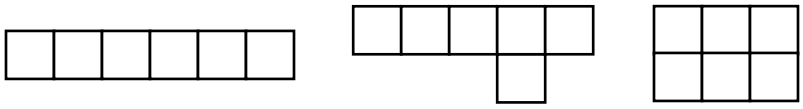
Elaboration

A net is a 2D representation of a 3D object that can be folded to recreate the shape. A net shows all of the faces of an object. A net can be used to make an object called a polyhedron. Two faces meet at an edge. Three or more faces meet at a vertex. When students are making nets, they should focus on the faces, and how the faces fit together to form the shape.

It is important for students to realize that there can be many different nets for a single shape. Even though the faces do not change, they can be connected in different ways. Please note that it is not a different net if it is a reflection or rotation of one you already have. For example, these four nets will all produce a cube.



Students cannot assume that because a cube has six square faces, any grouping of six squares will create a net. For example, the following are not nets for a cube:



A regular pyramid has a regular polygon as its base. The other faces are triangles. Many students are surprised to find that pyramids with different heights can be created on the same base.

Please note that this outcome is closely related to 8.SS2.

Grade 8 – Strand: Shape and Space (SS)

GCO: Use direct or indirect measurement to solve problems.

GRADE 7	GRADE 8	GRADE 9
7.SS1 Demonstrate an understanding of circles by: • describing the relationships among radius, diameter and circumference of circles; • relating circumference to π; • determining the sum of the central angles; • constructing circles with a given radius or diameter; • solving problems involving the radii, diameters and circumferences of circles. **7.SS2** Develop and apply a formula for determining the area of: • triangles; • parallelograms; • circles.	**8.SS2** Determine the surface area of: • right rectangular prisms; • right triangular prisms; • right cylinders to solve problems.	**9.SS1** Develop and apply the Pythagorean theorem to solve problems. **9.SS2** Determine the surface area of composite 3D objects to solve problems.

SCO: 8.SS2 – Determine the surface area of:

- **right rectangular prisms;**
- **right triangular prisms;**
- **right cylinders**

to solve problems. [C, CN, PS, R, V]

Students who have achieved this outcome should be able to:

A. explain, using examples, the relationship between the area of 2D shapes and the surface area of a given 3D object;

B. identify all the faces of a given prism, including right rectangular and right triangular prisms;

C. describe and apply strategies for determining the surface area of a given right rectangular or right triangular prism;

D. describe and apply strategies for determining the surface area of a given right cylinder; and

E. solve a given problem involving surface area.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.3 (A B C E)

5.4 (D E)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 8.SS2 – Determine the surface area of:

- **right rectangular prisms;**
- **right triangular prisms;**
- **right cylinders**

to solve problems. [C, CN, PS, R, V]

Elaboration

It is important for students to realize that there can be many different nets for a single shape. Even though the faces do not change, they can be connected in different ways. Please note that it is not a different net if it is a reflection or rotation of one you already have. For example, these four nets will all produce a cube.

In calculating surface area, students should start with objects such as cereal or cracker boxes for rectangular prisms, boxes from some types of chocolate bars for triangular prisms, and cylinders such as those found in bathroom tissue, paper towels or gift paper. These objects can be cut open, and the shape of each net determined. Students can then estimate the area of each face and total the areas to find the surface area. Students can compare and discuss similarities and differences in their approaches. Teachers should validate the different methods but encourage students to realize that some methods can do the job more efficiently.

One possible net for a cylinder is shown below. One dimension of the rectangle is the circumference of the circle, and the other is the height of the cylinder.

Please note that this outcome is closely related to 8.SS1 and 8.SS3.

Grade 8 – Strand: Shape and Space (SS)

GCO: Use direct or indirect measurement to solve problems.

GRADE 7	GRADE 8	GRADE 9
7.SS1 Demonstrate an understanding of circles by: • describing the relationships among radius, diameter and circumference of circles; • relating circumference to π; • determining the sum of the central angles; • constructing circles with a given radius or diameter; • solving problems involving the radii, diameters and circumferences of circles. **7.SS2** Develop and apply a formula for determining the area of: • triangles; • parallelograms; • circles.	**8.SS3** Develop and apply formulas for determining the volume of right prisms and right cylinders.	

SCO: 8.SS3 – Develop and apply formulas for determining the volume of right prisms and right cylinders. [C, CN, PS, R, V]

Students who have achieved this outcome should be able to:

A. determine the volume of a given right prism, given the area of the base;

B. generalize and apply a rule for determining the volume of right cylinders;

C. explain the connection between the area of the base of a given right 3D object and the formula for the volume of that object;

D. demonstrate that the orientation of a given 3D object does not affect its volume; and

E. apply a formula to solve a given problem involving the volume or a right cylinder or a right prism.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (A B C D)

7.2 (A C E)

7.3 (A B C E)

7.4 (E)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

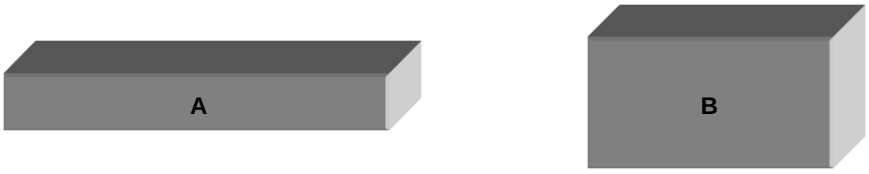
SCO: 8.SS3 – Develop and apply formulas for determining the volume of right prisms and right cylinders.[C, CN, PS, R, V]

Elaboration

The volume of a shape is a measure that describes the amount of space that an object occupies. Connections should be made between calculating the area of an object's base and calculating its volume. The volume of a shape should be thought of as the area of the base multiplied by its height rather than having students memorize different formulas for different shapes. Objects should be placed in various orientations so students can see that the volume is not affected by orientation of the shape.

Estimation of volume can be done in a variety of situations. For example, it may be desirable to find out how many cans or smaller packages will fit into a larger box or to estimate the volume of a package when the dimensions are not accurately known. Often, for rough estimations, cylinders can be treated as if they were rectangular prisms so that a rough estimation of volume would be $\text{length} \times \text{width} \times \text{height}$. In this case, the diameter of the circular base is treated as both length and width, and all dimensions are rounded for convenience of calculating mentally.

Some students will only use one dimension to calculate volume. For example, many students will say that prism A has more volume than prism B because prism A is longer.



Please note that this outcome is closely related to 8.SS2.

STATISTICS AND PROBABILITY

SPECIFIC CURRICULUM OUTCOMES

8.SP1 – Critique ways in which data is presented.

8.SP2 – Solve problems involving the probability of independent events.

Grade 8 – Strand: Statistics and Probability (SP)

GCO: Collect, display and analyse data to solve problems.

GRADE 7	GRADE 8	GRADE 9
7.SP1 Demonstrate an understanding of central tendency and range by: • determining the measures of central tendency (mean, median, mode) and range; • determining the most appropriate measures of central tendency to report findings. **7.SP2** Determine the effect on the mean, median and mode when an outlier is included in a data set.	**8.SP1** Critique ways in which data is presented.	

SCO: 8.SP1 – Critique ways in which data is presented. [C, R, T, V]

Students who have achieved this outcome should be able to:

A. compare the information that is provided for the same data set by a given set of graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, to determine the strengths and limitations of each graph;

B. identify the advantages and disadvantages of different graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, in representing a specific given set of data;

C. justify the choice of a graphical representation for a given situation and its corresponding data set;

D. explain how the format of a given graph, such as the size of the intervals, the width of bars and the visual representation, may lead to misinterpretation of the data;

E. explain how a given formatting choice could misinterpret the data; and

F. identify conclusions that are inconsistent with a given data set or graph and explain the misinterpretation.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A B C)

1.2 (D E F)

1.3 (A B C D E F)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 8.SP1 – Critique ways in which data is presented. [C, R, T, V]

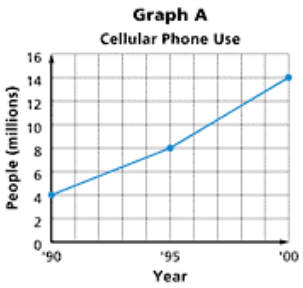
Elaboration

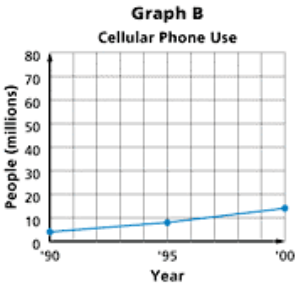
Students can compare various methods of displaying data and evaluating their effectiveness. Comparisons of scale adjustments to indicate such things as degree of growth or loss should be explored. Discussion should take place regarding how the choice of certain graphs can lead to inaccurate judgments. Students' understanding of statistics is enhanced by evaluating the arguments of others. This is particularly important since advertising, forecasting, and public policy are frequently based on data analysis. The media is full of representations of data to support statistical claims. These can be used to stimulate discussion.

It is important for students to be asked to evaluate various situations to determine and debate why a particular display is best suited to a specific type of data, or to a given context. Students should be able to discuss this in terms of continuous versus discrete data sets. For example, given a bar graph and a line graph, students should determine which is most appropriate to display the amount of water flowing into a container and justify their choice.

Students should also be aware of the characteristics of a good graph: it accurately shows the facts, complements or demonstrates arguments presented in the text, has a title and labels, shows data without altering the message of the data and clearly shows any trends or differences in the data.

A common cause of misleading information on graphs stems from the choice of intervals on the vertical axis. Another cause is to begin the vertical axis numbering with something other than zero. Both situations may either over- or under-exaggerate increases or decreases. For example, the graphs below depict a situation where the choice of scale on the vertical axis impacts the effect of the graph.





Grade 8 – Strand: Statistics and Probability (SP)

GCO: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

GRADE 7	GRADE 8	GRADE 9
7.SP3 Express probabilities as ratios, fractions and percents. **7.SP4** Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.	**8.SP2** Solve problems involving the probability of independent events.	

SCO: 8.SP2 – Solve problems involving the probability of independent events. [C, CN, PS, T]

Students who have achieved this outcome should be able to:

A. determine the probability of two given independent events and verify the probability using a different strategy;

B. generalize and apply a rule for determining the probability of independent events; and

C. solve a given problem that involves determining the probability of independent events.

Section(s) in MathLinks 8 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

11.1 (A)

11.2 (A)

11.3 (A B C)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

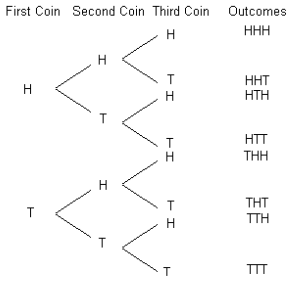
SCO: 8.SP2 – Solve problems involving the probability of independent events. [C, CN, PS, T]

Elaboration

Probability questions presented will be limited to those involving independent events. Tossing heads on a coin and rolling a 5 on a number cube are independent events, where the outcome of one event has no effect on the outcome of another. Although the focus is on independent events, it is still important for students to understand the difference between independent and dependent events. Selecting a heart from a deck of cards, not replacing the card, and then selecting another heart would be an example of dependent events, where the outcome of the second event is affected by the first.

Students should already be familiar with constructing tree diagrams (for two or more events) and tables (limited to two events) for determining the sample space of all possible outcomes for an event, such as the examples given below.

All possible outcomes when 3 coins are tossed



All possible sums generated when two number cubes are rolled

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The probability that an event will occur is denoted as $P(E)$, and is found by:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Students should be adept at expressing probability outcomes as fractions, decimals or percents.

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