Prince Edward Island
CANADA

Education, Early Learning and Culture
English Programs

Prince Edward Island Mathematics Curriculum

Mathematics

MAT521K

CURRICULUM

Acknowledgments

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- The following specialist from the Prince Edward Island Department of Education and Early Childhood Development:

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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for Grades 10-12 Mathematics* (2008) has been adopted as the basis for a revised high school mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential Graduation Learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

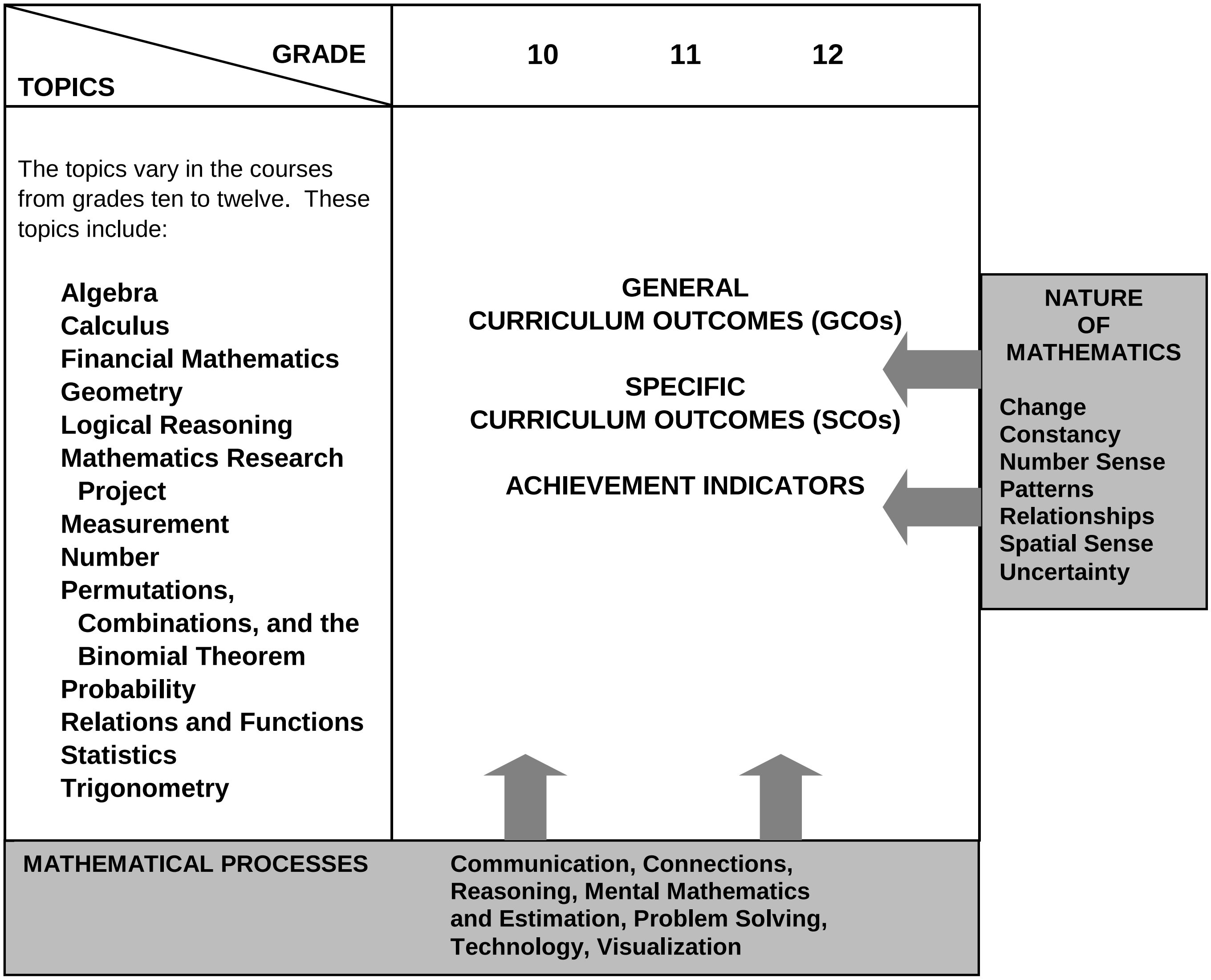
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practice mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for 10-12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



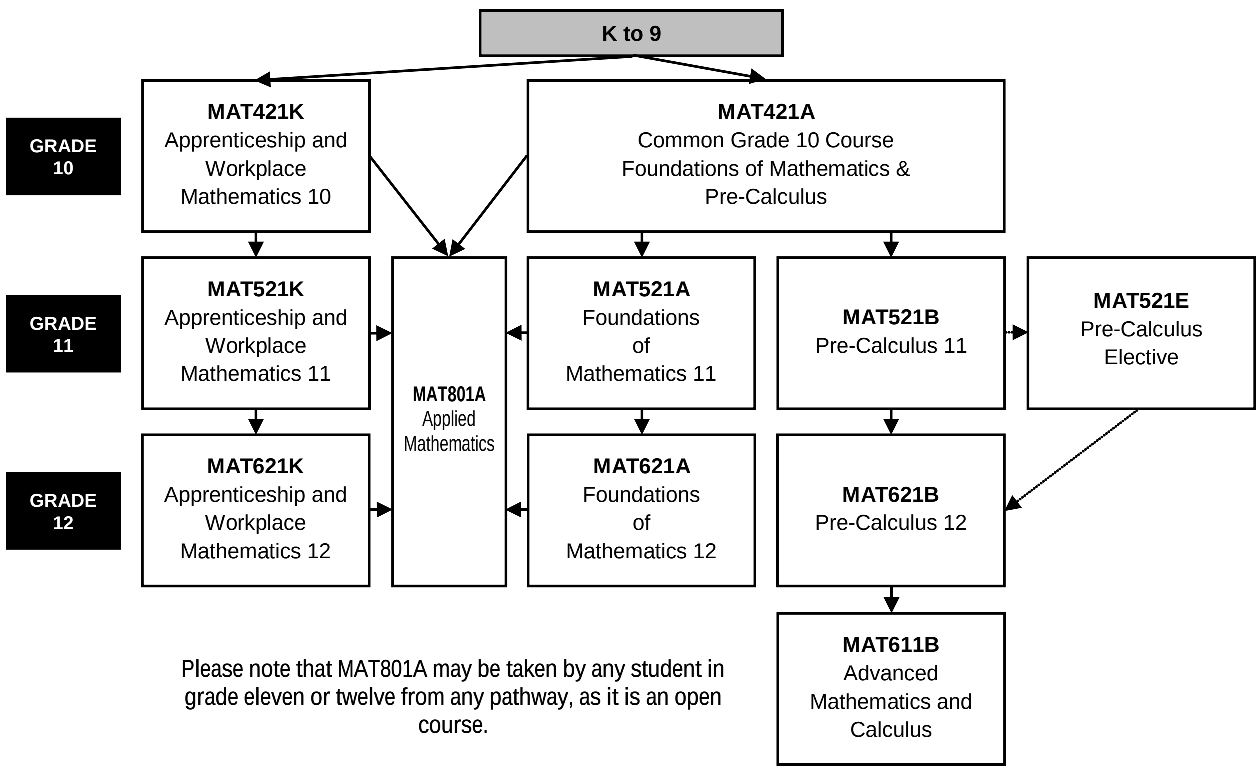
The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into a number of topics, as described above. These topics are not intended to be discrete units of instruction. The integration of outcomes across topics makes mathematical experiences meaningful. Students should make the connections among concepts both within and across topics. Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.

- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Pathways and Topics

The Prince Edward Island 10-12 mathematics curriculum includes pathways with corresponding topics rather than strands, which are found in the Prince Edward Island K-9 mathematics curriculum. Three pathways are available: **Apprenticeship and Workplace Mathematics**, **Foundations of Mathematics**, and **Pre-Calculus**. A common grade ten course (Foundations of Mathematics and Pre-Calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-Calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings. These pathways are illustrated in the diagram below:



The goals of all three pathways are to provide the prerequisite knowledge, skills, understandings, and attitudes for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study or for direct entry into the work force. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Apprenticeship and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics, and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, algebra and number, logical reasoning, relations and functions, statistics, probability, and a mathematics research project.

Pre-Calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, combinatorics, and introductory calculus.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

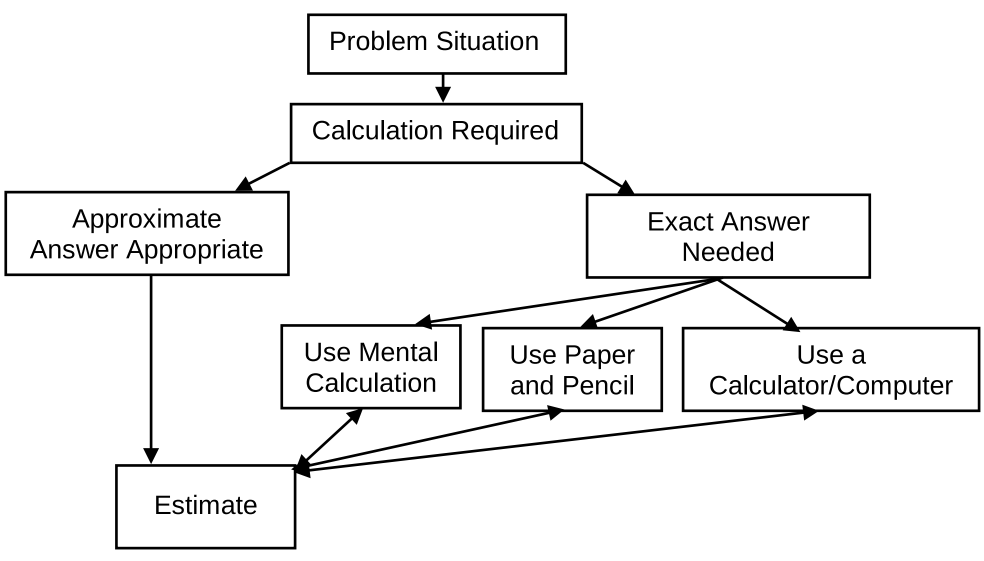
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?" or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.

- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all topics in mathematics and it is important that connections are made among topics. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical topics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate “communicating to learn mathematics and learning to communicate mathematically” (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

➢ Inquiry-Based Learning and Project Based Learning

Inquiry-based learning (IBL) allows students to explore, investigate, and construct new meaning from prior knowledge and from new information that is retrieved from other sources. It is not linear in nature, but promotes a continual looping back and forth throughout the process as students gather and process new information, redirect their inquiries, and continue through the process. Mathematical inquiry will require students to practise and refine their critical and creative-thinking skills. The terms *inquiry* and *research* are often used interchangeably within an educational context. While research often becomes the end-result of an inquiry process, it is the process itself that should be emphasized within an educational context. More information regarding the development of a mathematics research project is included in the appendix at the end of this document.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, teaching has been effective, or how best to address student learning needs. The quality of assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

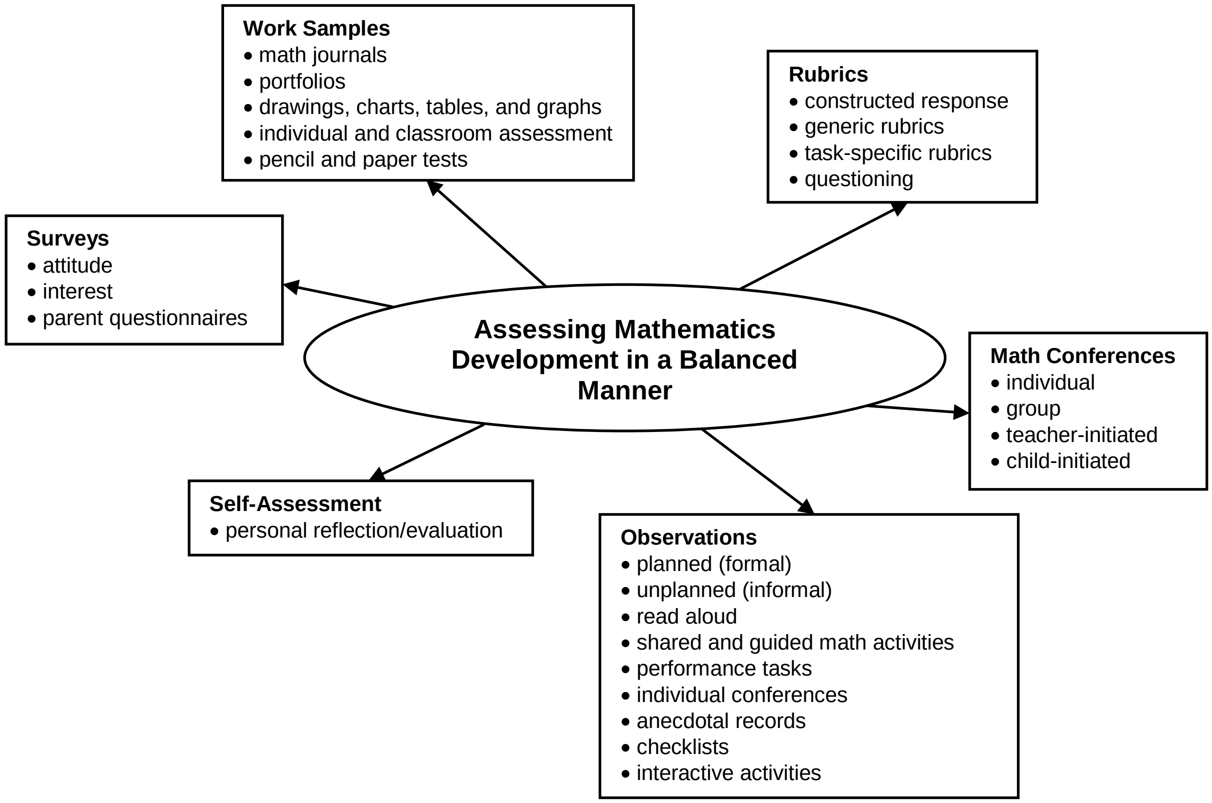
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children’s learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;

- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, phone calls, and electronic methods.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;

- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he or she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island high school mathematics curriculum are organized into a number of topics across the grades from ten to twelve. Each topic has associated with it a general curriculum outcome (GCO). They are overarching statements about what students are expected to learn in each topic from grades ten to twelve.

Topic	General Curriculum Outcome (GCO)
Algebra (A)	Develop algebraic reasoning.
Algebra and Number (AN)	Develop algebraic reasoning and number sense.
Calculus (C)	Develop introductory calculus reasoning.
Financial Mathematics (FM)	Develop number sense in financial applications.
Geometry (G)	Develop spatial sense.
Logical Reasoning (LR)	Develop logical reasoning.
Mathematics Research Project (MRP)	Develop an appreciation of the role of mathematics in society.
Measurement (M)	Develop spatial sense and proportional reasoning. **(*Foundations of Mathematics and Pre-Calculus*)**
	Develop spatial sense through direct and indirect measurement. **(*Apprenticeship and Workplace Mathematics*)**
Number (N)	Develop number sense and critical thinking skills.
Permutations, Combinations and Binomial Theorem (PC)	Develop algebraic and numeric reasoning that involves combinatorics.
Probability (P)	Develop critical thinking skills related to uncertainty.
Relations and Functions (RF)	Develop algebraic and graphical reasoning through the study of relations.
Statistics (S)	Develop statistical reasoning.
Trigonometry (T)	Develop trigonometric reasoning.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific curriculum outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding topic and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades ten to twelve which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;

- a list of the sections in *Math at Work 11* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

In the second half of this document, a curriculum guide supplement is presented which follows the primary resource, *Math at Work 11*. As well, an appendix is included which outlines the steps to follow in the development of an effective mathematics research project.

MEASUREMENT

SPECIFIC CURRICULUM OUTCOMES

M1 – Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.

M2 – Solve problems that involve SI and imperial units in volume and capacity measurements.

MAT521K – Topic: Measurement (M)

GCO: Develop spatial sense through direct and indirect measurement.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
M4 Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.	**M1** Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.	

SCO: **M1 – Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.** [C, CN, ME, PS, V]

Students who have achieved this outcome should be able to:

A. Explain, using examples, the difference between volume and surface area.

B. Explain, using examples, including nets, the relationship between area and surface area.

C. Explain how a referent can be used to estimate surface area.

D. Estimate the surface area of a 3-D object.

E. Illustrate, using examples, the effect of dimensional changes on surface area.

F. Solve a contextual problem that involves the surface area of 3-D objects, including spheres, and that requires the manipulation of formulas.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (B F)

1.2 (C D F)

1.3 (E F)

1.4 (F)

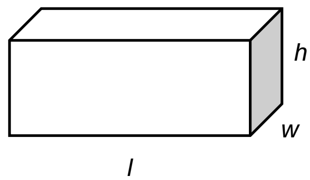
3.1 (A)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

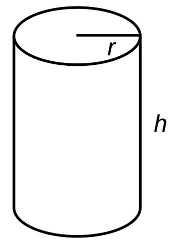
SCO: M1 – Solve problems that involve SI and imperial units in surface area measurements and verify the solutions. [C, CN, ME, PS, V]

Elaboration

The surface area of a right rectangular prism and of a right cylinder can be calculated by adding the area of the bases (top and bottom) plus the areas of the other faces.

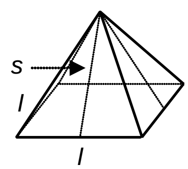


$SA = 2lw + 2lh + 2wh$

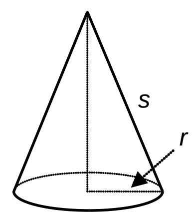


$SA = 2\pi r^2 + 2\pi rh$

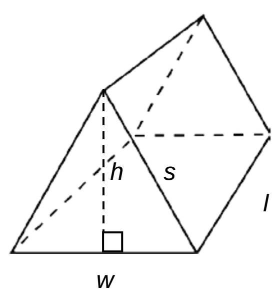
The surface area of a right rectangular pyramid, a right triangular pyramid, and a right cone can be calculated by adding the area of the base plus the areas of the other faces.



$SA = l^2 + 2ls$

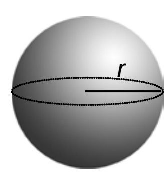


$SA = \pi r^2 + \pi rs$



$SA = lw + wh + 2ls$

The surface area of a sphere depends on the only the radius.



$SA = 4\pi r^2$

MAT521K – Topic: Measurement (M)

GCO: Develop spatial sense through direct and indirect measurement.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
M4 Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.	**M2** Solve problems that involve SI and imperial units in volume and capacity measurements.	

SCO: **M2 – Solve problems that involve SI and imperial units in volume and capacity measurements.** [C, CN, ME, PS, V]

Students who have achieved this outcome should be able to:

A. Explain, using examples, the difference between volume and capacity.

B. Identify and compare referents for volume and capacity measurements in SI and imperial units.

C. Estimate the volume or capacity of a 3-D object or container, using a referent.

D. Identify a situation where a given SI or imperial volume unit would be used.

E. Solve problems that involve the volume of 3-D objects and composite 3-D objects in a variety of contexts.

F. Solve a problem that involves the capacity of containers.

G. Write a given volume measurement expressed in one SI unit cubed in another SI unit cubed.

H. Write a given volume measurement expressed in one imperial unit cubed in another imperial unit cubed.

I. Determine the volume of prisms, cones, cylinders, pyramids, spheres and composite 3-D objects, using a variety of measuring tools such as rulers, tape measures, calipers and micrometers.

J. Determine the capacity of prisms, cones, pyramids, spheres and cylinders, using a variety of measuring tools and methods, such as graduated cylinders, measuring cups, measuring spoons and displacement.

K. Describe the relationship between the volumes of:

- cones and cylinders with the same base and height;
- pyramids and prisms with the same base and height.

L. Illustrate, using examples, the effect of dimensional changes on volume.

M. Solve a contextual problem that involves the volume of a 3-D object, including composite 3-D objects, or the capacity of a container.

N. Solve a contextual problem that involves the volume of a 3-D object and requires the manipulation of formulas.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (B C D I J M)

3.2 (A B C D F G H I J M)

3.3 (E F I J K L M N)

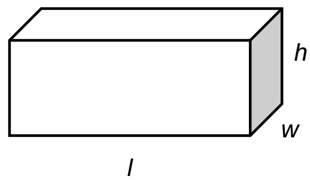
3.4 (F I M N)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

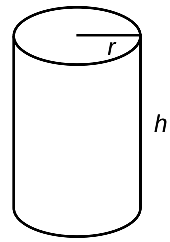
SCO: M2 – Solve problems that involve SI and imperial units in volume and capacity measurements.
[C, CN, ME, PS, V]

Elaboration

The volume of a right rectangular prism and a right cylinder can be calculated by multiplying the area of the base by the height.

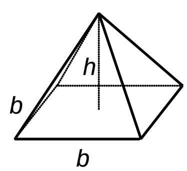


$V = lwh$

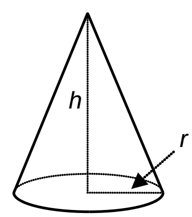


$V = \pi r^2 h$

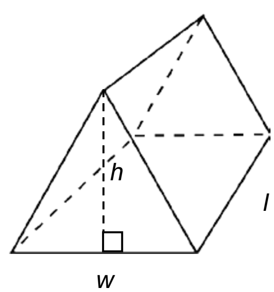
The volume of a square-based pyramid is found by calculating one-third of the volume of its related right prism. The volume of a right cone is found by calculating one-third of the volume of its related right cylinder. The volume of a triangular prism is found by calculating one-half of the volume of its related right rectangular prism.



$V = \frac{1}{3} b^2 h$

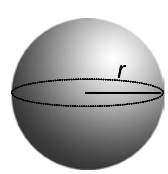


$V = \frac{1}{3} \pi r^2 h$



$V = \frac{1}{2} lwh$

The volume of a sphere depends on the only the radius.



$V = \frac{4}{3} \pi r^3$

GEOMETRY

SPECIFIC CURRICULUM OUTCOMES

G1 – Solve problems that involve two and three right triangles.

G2 – Solve problems that involve scale.

G3 – Model and draw 3-D objects and their views.

G4 – Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.

MAT521K – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
G2 Demonstrate an understanding of the Pythagorean theorem by: • identifying situations that involve right triangles; • verifying the formula; • applying the formula; • solving problems. **G4** Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: • applying similarity to right triangles; • generalizing patterns from similar right triangles; • applying the primary trigonometric ratios; • solving problems.	**G1** Solve problems that involve two and three right triangles.	**G1** Solve problems by using the sine law and cosine law, excluding the ambiguous case. **G2** Solve problems that involve: • triangles; • quadrilaterals; • regular polygons;

SCO: **G1 – Solve problems that involve two and three right triangles.** [CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Identify all of the right triangles in a given illustration for a context.

B. Determine if a solution to a problem that involves two or three right triangles is reasonable.

C. Sketch a representation of a given description of a problem in a 2-D or 3-D context.

D. Solve a contextual problem that involves angles of elevation or angles of depression.

E. Solve a contextual problem that involves two or three right triangles, using the primary trigonometric ratios.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (A C)

7.2 (C D)

7.3 (B C E)

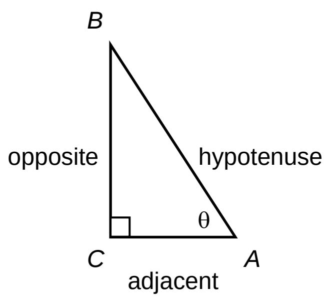
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

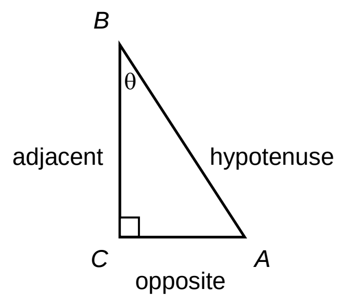
SCO: G1 – Solve problems that involve two and three right triangles. [CN, PS, T, V]

Elaboration

This outcome continues the work on trigonometry that was begun in MAT431A, except that the problems may involve more than one right triangle. An effective method of solving trigonometric problems involving more than one right triangle is to separate the triangles and label them. This will make the problem much easier to solve.

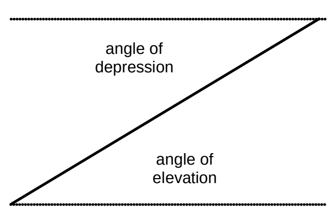
In similar triangles, corresponding angles are equal, and corresponding sides are in proportion. Therefore, the ratios of the lengths of corresponding sides are equal. The sides of a right triangle are labelled according to a reference angle, θ.





A trigonometric ratio is a ratio of the measures of two sides of a right triangle. The three primary trigonometric ratios are tangent, sine and cosine. The short form for the tangent ratio of angle *A* is tan *A*. It is defined as $\tan A = \dfrac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$. The short form for the sine ratio of angle *A* is sin *A*. It is defined as $\sin A = \dfrac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$. The short form for the cosine ratio of angle *A* is cos *A*. It is defined as $\cos A = \dfrac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$.

The line of sight is an invisible line from one person or object to another person or object. Some applications of trigonometry involve an angle of elevation or an angle of depression. An angle of elevation is the angle formed by the horizontal and a line of sight above the horizontal. An angle of depression is the angle formed by the horizontal and a line of sight below the horizontal. As the diagram below shows, the angle of elevation and the angle of depression along the same line of sight are equal.



MAT521K – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
G3 Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.	**G2** Solve problems that involve scale.	

SCO: **G2 – Solve problems that involve scale.** [PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Describe contexts in which a scale representation is used.

B. Determine, using proportional reasoning, the dimensions of an object from a given scale drawing or model.

C.

D. Construct a model of a 3-D object, given the scale.

E. Draw, with and without technology, a scale diagram of a given object.

F. Solve a contextual problem that involves scale.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B C D E)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: G2 – Solve problems that involve scale. [PS, R, T, V]

Elaboration

A scale is a comparison between the actual size of an object and the size of its image, therefore a scale diagram would be a drawing that is similar to the actual figure. These scale diagrams can be either an enlargement or reduction of the actual object depending on the context. If the scale factor is bigger than one, this will result in an enlargement whereas if the scale factor is less than one, it is a reduction.

Students have an intuitive sense of shapes that are enlargements or reductions of each other. Students have experienced maps and pictures that have been drawn to scale, and images produced by photocopiers and computer software. The use of computer software can allow for a great deal of flexibility in the investigation of enlargement and reduction. Other methods, such as using graph paper, or a protractor and ruler can also be used to draw scale diagrams. It should be noted that when a ratio is used to represent an enlargement or reduction, the format of the ratio is $\text{New} : \text{Original}$. A ratio of $2:1$ means the new figure is an enlargement to twice the size of the original. Likewise, a ratio of $1:3$ means that the new figure is a reduction to $\frac{1}{3}$ of the size of the original, or the original is three times the size of the new figure.

MAT521K – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
M4 Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.	**G3** Model and draw 3-D objects and their views.	**G3** Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including: • translations; • rotations; • reflections; • dilations.

SCO: **G3 – Model and draw 3-D objects and their views.** [CN, R, V]

Students who have achieved this outcome should be able to:

A. Draw a 2-D representation of a given 3-D object.

B. Draw, using isometric dot paper, a given 3-D object.

C. Draw to scale top, front and side views of a given 3-D object.

D. Construct a model of a 3-D object, given the top, front and side views.

E. Draw a 3-D object, given the top, front and size views.

F. Determine if the given views of a 3-D object represent a given object, and explain the reasoning.

G. Identify the point of perspective of a given one-point perspective drawing of a 3-D object.

H. Draw a one-point perspective view of a given 3-D object.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.2 (A B C D E F)

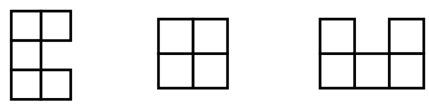
2.3 (G H)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: G3 – Model and draw 3-D objects and their views. [CN, R, V]

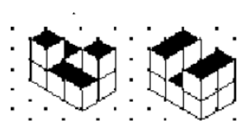
Elaboration

Observing and learning to represent 2-D and 3-D figures in various positions by drawing and construction helps students to develop spatial sense. Students' mathematical experience with 3-D is often derived from 2-D pictures. It is important that students be able to interpret information from 2-D pictures of the world, as well as to represent real-world information in 2-D. Students can be given a series of 2-D views of a 3-D object, such as in the example below, and be asked to construct, using cubes, a building that adheres to the plans. Such plans are often referred to as orthographic plans or drawings.

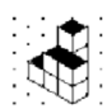


Top Front Right

This outcome can also be explored using isometric drawings. For example, the orthographic plans at the top are satisfied by each of the following isometric drawings.



Students can discover that, when they are given only one view of an isometric drawing, they often cannot see all of the cubes because some are hidden. Students can be given an isometric drawing, such as the one shown below, and be asked to create a building from it. Generally, not all students make the same structure, and they realize that one drawing can lead to more than one 3-D object. They can again explore the maximum, minimum, and variety of structures which can support a given drawing.



Any perspective representation of a scene that includes parallel lines has one or more vanishing points in a perspective drawing. A one-point perspective drawing means that the drawing has a single vanishing point, usually (though not necessarily) directly opposite the viewer's eye and usually (though not necessarily) on the horizon line. All lines parallel with the viewer's line of sight recede to the horizon towards this vanishing point. This is the standard "receding railroad tracks" phenomenon, as shown in the picture at the right. One vanishing point is typically used for roads, railway tracks, hallways, or buildings viewed so that the front is directly facing the viewer. Any objects that are made up of lines either directly parallel with the viewer's line of sight or directly perpendicular (the railroad slats) can be represented with one-point perspective.

When the point of perspective is above an object, the object appears to go upward into the distance; when the point of perspective is below an object, the object appears to go downward into the distance; and the same phenomenon happens when the point of perspective is to the left or right of an object.

MAT521K – Topic: Geometry (G)

GCO: Develop spatial sense.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
M4 Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.	**G4** Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.	**G3** Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including: • translations; • rotations; • reflections; • dilations.

SCO: **G4 – Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.** [CN, V]

Students who have achieved this outcome should be able to:

A. Draw the components of a given exploded diagram, and explain their relationship to the original 3-D object.

B. Sketch an exploded view of a 3-D object to represent the components.

C. Draw to scale the components of a 3-D object.

D. Sketch a 2-D representation of a 3-D object, given its exploded view.

Note: It is intended that the simple 3-D objects come from contexts such as flat-packed furniture or sewing patterns.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

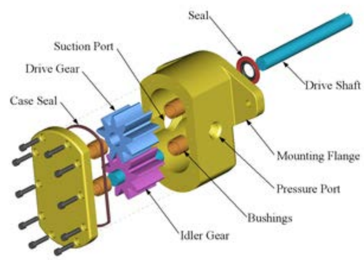
2.3 (A B C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: G4 – Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects.
[CN, V]

Elaboration

An exploded view drawing is a type of drawing that shows the intended assembly of mechanical or other parts. It shows all parts of the assembly and how they fit together. In mechanical systems usually the component closest to the center are assembled first, or is the main part in which the other parts get assembled. This drawing can also help to represent the disassembly of parts, where the parts on the outside normally get removed first.



Exploded diagrams are common in descriptive manuals showing parts placement, or parts contained in an assembly or sub-assembly. Usually such diagrams have the part identification number and a label indicating which part fills the particular position in the diagram. Many spreadsheet applications can automatically create exploded diagrams, such as exploded pie charts.

NUMBER

SPECIFIC CURRICULUM OUTCOMES

N1 – Analyse puzzles and games that involve numerical reasoning, using problem-solving strategies.

N2 – Solve problems that involve personal budgets.

N3 – Demonstrate an understanding of compound interest.

N4 – Demonstrate an understanding of financial institution services used to access and manage finances.

N5 – Demonstrate an understanding of credit options, including:
- **credit cards;**
- **loans.**

MAT521K – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
	N1 Analyse puzzles and games that involve numerical reasoning, using problem-solving strategies.	**N1** Analyse puzzles and games that involve logical reasoning, using problem-solving strategies.

SCO: **N1 – Analyse puzzles and games that involve numerical reasoning, using problem-solving strategies.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Determine, explain and verify a strategy to solve a puzzle or to win a game; *e.g.,*
- guess and check;
- look for a pattern;
- make a systematic list;
- draw or model;
- eliminate possibilities;
- simplify the original problem;
- work backward;
- develop alternate approaches.

B. Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

C. Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Note: It is intended that this outcome be integrated throughout the course by using puzzles and games such as cribbage, magic squares and Kakuro.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

Integrated throughout the text.

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: N1 – Analyse puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]

Elaboration

This particular outcome is integrated throughout the course. Each chapter has a section called *Games and Puzzles* that can be used to meet this particular SCO. They are found on pages 55, 101, 149, 201, 257, 303, and 359 of the textbook.

MAT521K – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
N2 Demonstrate an understanding of income, including: • wages; • salary; • contracts; • commission; • piecework to calculate gross and net pay.	**N2** Solve problems that involve personal budgets.	**N2** Solve problems that involve the acquisition of a vehicle by: • buying; • leasing; • planning to buy.

SCO: **N2 – Solve problems that involve personal budgets.** [CN, PS, R, T]

Students who have achieved this outcome should be able to:

A. Identify income and expenses that should be included in a personal budget.

B. Explain considerations that must be made when developing a budget, *e.g.*, prioritizing, recurring and unexpected expenses.

C. Create a personal budget based on given income and expense data.

D. Collect income and expense data, and create a budget.

E. Modify a budget to achieve a set of personal goals.

F. Investigate and analyse, with or without technology, "what if …" questions related to personal budgets.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.2 (A B C D E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: N2 – Solve problems that involve personal budgets. [CN, PS, R, T]

Elaboration

A budget is a list of all planned expenses and revenues. It is a plan for saving and spending. A budget is an important concept in microeconomics, which uses a budget line to illustrate the trade-offs between two or more goods. In other words, a budget is an organizational plan stated in monetary terms. It can be applied to a business, a household, or an individual.

MAT521K – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
	N3 Demonstrate an understanding of compound interest.	

SCO: **N3 – Demonstrate an understanding of compound interest.** [CN, ME, PS, T]

Students who have achieved this outcome should be able to:

A. Solve a problem that involves simple interest, given three of the four values in the formula $I = Prt$.

B. Compare simple and compound interest, and explain their relationship.

C. Solve, using a formula, a contextual problem that involves compound interest.

D. Explain, using examples, the effect of different compounding periods on calculations of compound interest.

E. Estimate, using the Rule of 72, the time required for a given investment to double in value.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.3 (A B C D E)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: N3 – Demonstrate an understanding of compound interest. [CN, ME, PS, T]

Elaboration

Interest is a fee paid on borrowed assets. It is the price paid for the use of borrowed money, or money earned by deposited funds. Assets that are sometimes lent with interest include money, shares, consumer goods through hire purchase, major assets such as aircraft, and even entire factories in finance lease arrangements. The interest is calculated upon the value of the assets in the same manner as upon money.

Interest can be thought of as the “rent of money”. It is defined as the compensation paid by the borrower of money to the lender of money. When money is deposited in a bank, interest is typically paid to the depositor as a percentage of the amount deposited; when money is borrowed, interest is typically paid to the lender as a percentage of the amount owed. The percentage of the principal that is paid as a fee over a certain period of time (typically one month or year), is called the interest rate.

Simple interest is calculated only on the principal amount, or on that portion of the principal amount that remains unpaid. It is calculated using the formula $I = Prt$, where I is the interest, P is the principal, r is the annual rate of interest, expressed as a decimal, and t is the time, in years.

Compound interest arises when interest is added to the principal, so that from that moment on, the interest that has been added also earns interest. This addition of interest to the principal is called compounding. A bank account, for example, may have its interest compounded every year. As an example, an account with $1000 initial principal and 5% interest per year would have a balance of $1050 at the end of the first year, $1102.50 at the end of the second year, and so on. It is calculated using the formula $A = P(1+i)^n$, where A is the amount, P is the principal, i is the rate of interest per compounding period, expressed as a decimal, and n is the number of compounding periods.

MAT521K – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
	N4 Demonstrate an understanding of financial institution services used to access and manage finances.	

SCO: **N4 – Demonstrate an understanding of financial institution services used to access and manage finances.** [C, CN, R, T]

Students who have achieved this outcome should be able to:

A. Describe the type of banking services available from various financial institutions, such as online services.

B. Describe the types of accounts available at various financial institutions.

C. Identify the type of account that best meets the needs for a given set of criteria.

D. Identify and explain various automated teller machines (ATM) service charges.

E. Describe the advantages and disadvantages of online banking.

F. Describe the advantages and disadvantages of debit card purchases.

G. Describe ways that ensure the security of personal and financial information, *e.g.*, passwords, encryption, protection of personal identification number (PIN) and other personal identity information.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.1 (A B C D E F G)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: N4 – Demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T]

Elaboration

A bank is a financial institution and a financial intermediary that accepts deposits and channels those deposits into lending activities, either directly or through capital markets. A bank connects customers that have capital deficits to customers with capital surpluses. Examples of services that banks offer are

- chequing and savings accounts;
- loans;
- online banking;
- paying bills;
- issuing debit and credit cards;
- issuing traveller's cheques;
- investment services;
- registered retirement savings plans;
- investment plans.

MAT521K – Topic: Number (N)

GCO: Develop number sense and critical thinking skills.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
	N5 Demonstrate an understanding of credit options, including: • credit cards; • loans.	**N2** Solve problems that involve the acquisition of a vehicle by: • buying; • leasing; • planning to buy.

SCO: N5 – Demonstrate an understanding of credit options, including:

- **credit cards;**
- **loans.**

[CN, ME, PS, R]

Students who have achieved this outcome should be able to:

A. Compare advantages and disadvantages of different types of credit options, including bank and store credit cards, personal loans, lines of credit, and overdraft.

B. Make informed decisions and plans related to the use of credit, such as service charges, interest, payday loans and sales promotions, and explain the reasoning.

C. Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt and timing purchases.

D. Compare credit card options from various companies and financial institutions.

E. Solve a contextual problem that involves credit cards or loans.

F. Solve a contextual problem that involves credit linked to sales promotions.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.4 (A B C D E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: N5 – Demonstrate an understanding of credit options, including:
- **credit cards;**
- **loans.**

[CN, ME, PS, R]

Elaboration

A credit card is a small plastic card issued to users as a system of payment. It allows its holder to buy goods and services based on the holder's promise to pay for these goods and services at a later time. The issuer of the card creates a revolving account and grants a line of credit to the user from which the user can borrow money for payment to a merchant or as a cash advance to the user.

Credit cards are issued by a credit card issuer, such as a bank or credit union, after an account has been approved by the credit provider, after which cardholders can use it to make purchases at merchants accepting that card. When a purchase is made, the credit card user agrees to pay the card issuer. The cardholder indicates consent to pay by signing a receipt with a record of the card details. Electronic verification systems allow merchants to verify in a few seconds that the card is valid and the credit card customer has sufficient credit to cover the purchase, allowing the verification to happen at time of purchase. The verification is performed using a credit card payment terminal or point-of-sale (POS) system with a communications link to the merchant's acquiring bank by using data from the card that is obtained from a magnetic stripe or chip on the card.

The main benefit to each customer is convenience. Compared to debit cards and cheques, a credit card allows small short-term loans to be quickly made to a customer who need not calculate a balance remaining before every transaction, provided the total charges do not exceed the maximum credit line for the card. Credit cards also provide more fraud protection than debit cards.

The main detriment to customers is the high interest rate, often between 15% and 20% per year. When credit card bills are not paid quickly, the cost can easily grow out of control to the point where the customer is not able to pay off the bills.

ALGEBRA

SPECIFIC CURRICULUM OUTCOMES

A1 – Solve problems that require the manipulation and application of formulas related to:

- **volume and capacity;**
- **surface area;**
- **slope and rate of change;**
- **simple interest;**
- **finance charges.**

A2 – Demonstrate an understanding of slope:

- **as rise over run;**
- **as a rate of change;**
- **by solving problems.**

A3 – Solve problems by applying proportional reasoning and unit analysis.

MAT521K – Topic: Algebra (A)

GCO: Develop algebraic reasoning.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
A1 Solve problems that require the manipulation and application of formulas related to: • perimeter; • area; • the Pythagorean theorem; • primary trigonometric ratios; • income.	**A1** Solve problems that require the manipulation and application of formulas related to: • volume and capacity; • surface area; • slope and rate of change; • simple interest; • finance charges.	

SCO: A1 – Solve problems that require the manipulation and application of formulas related to:

- **volume and capacity;**
- **surface area;**
- **slope and rate of change;**
- **simple interest;**
- **finance charges.**

[CN, PS, R]

Students who have achieved this outcome should be able to:

A. Solve a contextual problem that involves the application of a formula that does not require manipulation.

B. Solve a contextual problem that involves the application of a formula that requires manipulation.

C. Explain and verify why different forms of the same formula are equivalent.

D. Describe, using examples, how a given formula is used in a trade or an occupation.

E. Create and solve a contextual problem that involves a formula.

F. Identify and correct errors in a solution to a problem that involves a formula.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

Integrated throughout the text.

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: A1 – Solve problems that require the manipulation and application of formulas related to:

- **volume and capacity;**
- **surface area;**
- **slope and rate of change;**
- **simple interest;**
- **finance charges.**

[CN, PS, R]

Elaboration

This particular outcome is integrated throughout the course. Other important formulas from MAT431A have also been included below. Specifically, students will be expected to manipulate formulas, such as:

Surface Area of a Rectangular Prism	$SA = 2lw + 2lh + 2wh$
Surface Area of a Triangular Prism	$SA = lw + wh + 2ls$
Surface Area of a Square-Based Pyramid	$SA = l^2 + 2ls$
Surface Area of a Cylinder	$SA = 2\pi r^2 + 2\pi rh$
Surface Area of a Cone	$SA = \pi r^2 + \pi rs$
Surface Area of a Sphere	$SA = 4\pi r^2$
Volume of a Rectangular Prism	$V = lwh$
Volume of a Triangular Prism	$V = \frac{1}{2}lwh$
Volume of a Square-Based Pyramid	$V = \frac{1}{3}b^2h$
Volume of a Cylinder	$V = \pi r^2 h$
Volume of a Cone	$V = \frac{1}{3}\pi r^2 h$
Volume of a Sphere	$V = \frac{4}{3}\pi r^3$
Slope	$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
Simple Interest	$I = Prt$
Compound Interest	$A = P(1+i)^n$
Perimeter of a Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference of a Circle	$C = 2\pi r$ or $C = \pi d$
Area of a Rectangle	$A = lw$
Area of a Triangle	$A = \frac{bh}{2}$
Area of a Circle	$A = \pi r^2$
Pythagorean Theorem	$c^2 = a^2 + b^2$
Sine of Angle *A*	$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$
Cosine of Angle *A*	$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$
Tangent of Angle *A*	$\tan A = \frac{\text{opposite}}{\text{adjacent}}$

MAT521K – Topic: Algebra (A)

GCO: Develop algebraic reasoning.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
	A2 Demonstrate an understanding of slope: • as rise over run; • as a rate of change; • by solving problems.	**A1** Demonstrate an understanding of linear relations by: • recognizing patterns and trends; • graphing; • creating tables of values; • writing equations; • interpolating and extrapolating; • solving problems.

SCO: **A2 – Demonstrate an understanding of slope:**

- **as rise over run;**
- **as a rate of change;**
- **by solving problems.**

[C, CN, PS, V]

Students who have achieved this outcome should be able to:

A. Describe contexts that involve slope; *e.g.*, ramps, roofs, road grade, flow rates within a tube, skateboard parks, ski hills.

B. Explain, using diagrams, the difference between two given slopes (*e.g.*, a $3:1$ and a $1:3$ roof pitch), and describe the implications.

C. Describe the conditions under which a slope will be either 0 or undefined.

D. Explain, using examples and illustrations, slope as rise over run.

E. Verify that the slope of an object, such as a ramp or a roof, is constant.

F. Explain, using illustrations, the relationship between slope and angle of elevation, *e.g.*, for a ramp with a slope of $7:100$, the angle of elevation is approximately 4^0.

G. Explain the implications, such as safety and functionality, of different slopes in a given context.

H. Explain, using examples and illustrations, slope as a rate of change.

I. Solve a contextual problem that involves slope or rate of change.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A B)

6.2 (C D E F G)

6.3 (H I)

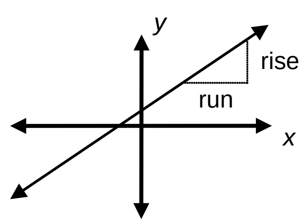
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: A2 – Demonstrate an understanding of slope:
- **as rise over run;**
- **as a rate of change;**
- **by solving problems.**

[C, CN, PS, V]

Elaboration

The slope of a line or line segment indicates how steep the line is. The slope of a line is the ratio of the rise to the run.



$$m = \frac{\text{rise}}{\text{run}}$$

The slope of a line can be determined using two points on the line, (x_1, y_1) and (x_2, y_2), by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$. The slope indicates the rate of change of a linear relation.

All horizontal lines, which have zero slope, are parallel to each other, and all vertical lines, which have undefined slope, are parallel to each other.

MAT521K – Topic: Algebra (A)

GCO: Develop algebraic reasoning.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT621K
N1 Solve problems that involve unit pricing and currency exchange, using proportional reasoning.	**A3** Solve problems by applying proportional reasoning and unit analysis.	

SCO: **A3 – Solve problems by applying proportional reasoning and unit analysis.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Explain the process of unit analysis used to solve a problem (*e.g.*, given km/h and time in hours, determine how many kilometres; given revolutions per minute, determine the number of seconds per revolution).

B. Solve a problem, using unit analysis.

C. Explain, using an example, how unit analysis and proportional reasoning are related; *e.g.*, to change km/h to km/min, multiply by $\frac{1\text{ h}}{60\text{ min}}$ because hours and minutes are proportional (constant relationship).

D. Solve a problem within and between systems, using proportions or tables; *e.g.*, km to m, or km/h to ft/sec.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: A3 – Solve problems by applying proportional reasoning and unit analysis. [C, CN, PS, R]

Elaboration

Proportional reasoning is the ability to think about and compare multiplicative relationships between quantities. These relationships are represented symbolically as ratios. A proportion is a statement of equality between two ratios. Different notations for proportions can be used:

$$2:5=4:10 \ \textit{or} \ 2 \text{ to } 5=4 \text{ to } 10 \ \textit{or} \ \frac{2}{5}=\frac{4}{10}$$

These can all be read as “two to five equals 4 to 10.”

Finding one number in a proportion when the other three numbers are known is called solving a proportion. For example, how many girls are in a class when the ratio of boys to girls in a class is $3:5$ and there are 12 boys. To solve, set up the proportion: $\frac{3}{5}=\frac{12}{?}$. The students must think multiplicatively to solve the proportion in the same way they would to determine equivalent fractions. Since 12 is equal to 3 times 4, we can find the missing term by multiplying 5 by 4, which gives an answer of 20. As well, a student who knows that a runner who runs at a rate of $\frac{1\text{ km}}{7\text{ min}}$ will win a 10 km race over a runner who runs at a rate of $\frac{1\text{ km}}{8\text{ min}}$ is thinking proportionally.

STATISTICS

SPECIFIC CURRICULUM OUTCOMES

S1 – Solve problems that involve creating and interpreting graphs, including:
- **bar graphs;**
- **histograms;**
- **line graphs;**
- **circle graphs.**

MAT521K – Topic: Statistics (S)

GCO: Develop statistical reasoning.

GRADE 10 – MAT421K	GRADE 11 – MAT521K	GRADE 12 – MAT631A
	S1 Solve problems that involve creating and interpreting graphs, including: • bar graphs; • histograms; • line graphs; • circle graphs.	**S1** Solve problems that involve measures of central tendency, including: • mean; • median; • mode; • weighted mean; • trimmed mean.

SCO: **S1 – Solve problems that involve creating and interpreting graphs, including:**

- **bar graphs;**
- **histograms;**
- **line graphs;**
- **circle graphs.**

[C, CN, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Determine the possible graphs that can be used to represent a given data set, and explain the advantages and disadvantages of each.

B. Create, with and without technology, a graph to represent a given data set.

C. Describe the trends in the graph of a given data set.

D. Interpolate and extrapolate values from a given graph.

E. Explain, using examples, how the same graph can be used to justify more than one conclusion.

F. Explain, using examples, how different graphic representations of the same data set can be used to emphasize a point of view.

G. Solve a contextual problem that involves the interpretation of a graph.

Section(s) in Math at Work 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (A B)

4.2 (C D)

4.3 (E F G)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: S1 – Solve problems that involve creating and interpreting graphs, including:

- **bar graphs;**
- **histograms;**
- **line graphs;**
- **circle graphs.**

[C, CN, PS, R, T, V]

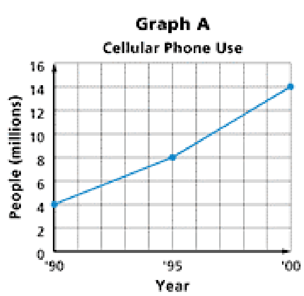
Elaboration

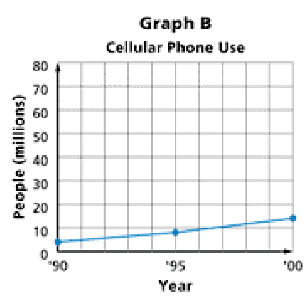
Students can compare various methods of displaying data and evaluating their effectiveness. Comparisons of scale adjustments to indicate such things as degree of growth or loss should be explored. Discussion should take place regarding how the choice of certain graphs can lead to inaccurate judgments. Students' understanding of statistics is enhanced by evaluating the arguments of others. This is particularly important since advertising, forecasting, and public policy are frequently based on data analysis. The media is full of representations of data to support statistical claims. These can be used to stimulate discussion.

It is important for students to be asked to evaluate various situations to determine and debate why a particular display is best suited to a specific type of data, or to a given context. Students should be able to discuss this in terms of continuous versus discrete data sets. For example, given a bar graph and a line graph, students should determine which is most appropriate to display the amount of water flowing into a container and justify their choice.

Students should also be aware of the characteristics of a good graph: it accurately shows the facts, complements or demonstrates arguments presented in the text, has a title and labels, shows data without altering the message of the data, and clearly shows any trends or differences in the data.

A common cause of misleading information on graphs stems from the choice of intervals on the vertical axis. Another cause is to begin the vertical axis numbering with something other than zero. Both situations may either over- or under-exaggerate increases or decreases. For example, the graphs below depict a situation where the choice of scale on the vertical axis impacts the effect of the graph.





Curriculum Guide Supplement

This supplement to the *Prince Edward Island MAT521K Mathematics Curriculum Guide* is designed to parallel the primary resource, *Math at Work 11*.

For each of the chapters in the text, an approximate timeframe is suggested to aid teachers with planning. The timeframe is based on a total of 80 classes, each with an average length of 75 minutes:

CHAPTER	SUGGESTED TIME
Chapter 1 – Surface Area	11 classes
Chapter 2 – Drawing and Design	10 classes
Chapter 3 – Volume and Capacity	11 classes
Chapter 4 – Interpreting Graphs	12 classes
Chapter 5 – Banking and Budgeting	13 classes
Chapter 6 – Slope	10 classes
Chapter 7 – Right Angles and Trigonometry	13 classes

Each chapter of the text is divided into a number of sections. In this document, each section is supported by a one-page presentation, which includes the following information:

- the name and pages of the section in *Math at Work 11*;
- the specific curriculum outcome(s) and achievement indicator(s) addressed in the section (see the first half of the curriculum guide for an explanation of symbols);
- the student expectations for the section, which are associated with the SCO(s);
- the new concepts introduced in the section;
- other key ideas developed in the section;
- suggested problems in *Math at Work 11*;
- possible instructional and assessment strategies for the section.

CHAPTER 1
SURFACE AREA

SUGGESTED TIME

11 classes

Section 1.1 – Nets and Surface Area of 3-D Objects (pp. 6-14)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **M1 (B F)**

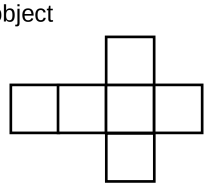
After this lesson, students will be expected to:

- draw nets of 3-D objects
- determine the area of the 2-D shapes that make up a 3-D object
- determine the surface area of 3-D objects

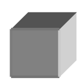
After this lesson, students should understand the following concepts:

- **rectangular prism –** a 3-D figure with two rectangular bases that are the same shape and size

- **net –** a 2-D diagram that can be folded to create a 3-D object

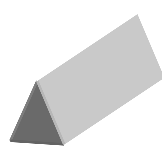


net



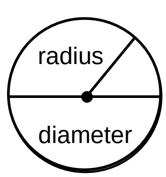
cube

- **surface area –** the sum of the areas of all the faces of a 3-D object; measured in square units or units^2
- **triangular prism** – a 3-D figure with only two triangular faces that are the same size and shape



- **cylinder –** a 3-D object with two circular faces that are the same size, and has a curved surface

- **diameter –** the distance across a circle through its centre
- **radius –** the distance from the centre of a circle to a point on the circumference



Suggested Problems in *Math at Work 11*:

- **pp. 9-10:** #1-8
- **p. 12:** #1-6
- **pp. 13-14:** #1-8

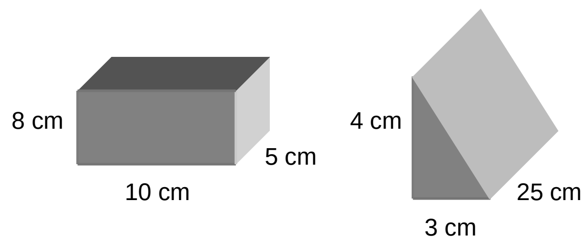
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

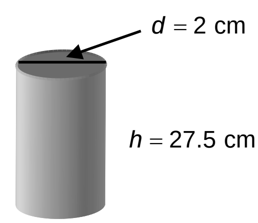
- Have students cut along the edge of various shaped boxes (cereal boxes, tennis ball canister, potato chip cans) and unfold them to form a net. Students should predict what the net will look like before they cut it and explore for it themselves.

Possible Assessment Strategies:

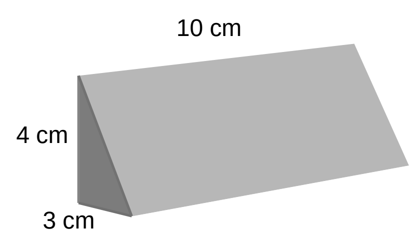
- The owners of a cracker factory are trying to choose a box to hold their new flavour of cracker. They want a box that uses the least amount of cardboard. Which box should they choose?



- Calculate the outside surface area of the paper towel tube with the following dimensions. Do not count the top or bottom circles. Round off the answer to one decimal place.



- What is the surface area of the wedge of cheese shown?



Section 1.2 – Estimating Surface Area (pp. 15-24)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **M1 (C D F)** **After this lesson, students will be expected to:** • use length references to estimate the dimensions and the surface area of an object • use area references to estimate the surface area of an object **Suggested Problems in *Math at Work 11*:** • **pp. 19-20**: #1-7 • **pp. 22-23:** #1-9 • **p. 24:** #1-7	**Possible Instructional Strategies:** • When solving problems, encourage students to estimate the surface area before calculating the exact answer in order to check the reasonableness of their calculations. **Possible Assessment Strategies:** • Complete each of the following. a. $10\text{ cm} \times 10\text{ cm} = \square\text{ cm}^2$ b. $5\text{ ft} \times 5\text{ ft} = \square\text{ ft}^2$ • What fraction of 1 m^2 is a Post-It note that is 10 cm by 10 cm? • Marilyn has 1 m^2 of paper to wrap a box 28 cm long, 24 cm wide and 12 cm high for a present. Does she have enough paper? • Brad is purchasing burlap to protect his three apple trees against the cold winter weather. He will wrap the burlap around the bottom 140 cm of each tree trunk. The trees are 22.1 cm, 24.7 cm and 33.2 cm in circumference. How much burlap will he need?

Section 1.3 – Using Formulas for Surface Area of 3-D Objects (pp. 25-39)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

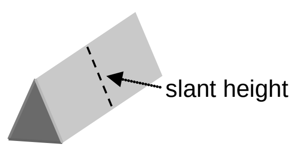
- **1.3 (E F)**

After this lesson, students will be expected to:

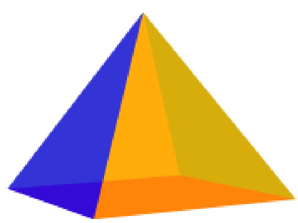
- use formulas to determine the surface area of rectangular and triangular prisms, pyramids, and cylinders

After this lesson, students should understand the following concepts:

- **slant height –** the shortest distance from the edge of the base of a 3-D figure to its highest point



- **square-based pyramid –** a 3-D figure with a square base and four triangular sides that connect at one point



SURFACE AREA FORMULAS	
Rectangular Prism	$SA = 2lw + 2lh + 2wh$
Triangular Prism	$SA = lw + wh + 2ls$
Square-Based Pyramid	$SA = l^2 + 2ls$
Cylinder	$SA = 2\pi r^2 + 2\pi rh$

Suggested Problems in *Math at Work 11*:

- **pp. 28-29:** #1-6
- **pp. 31-32:** #1-5
- **pp. 34-35:** #1-5
- **p. 37:** #1-6
- **pp. 38-39:** #1-11

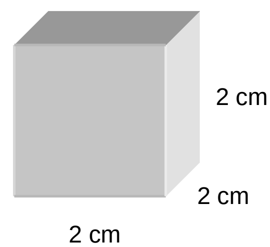
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Have the students practice working with the formulas for surface area.

Possible Assessment Strategies:

- a. Calculate the area of one face of this cube. Use it to calculate the surface area of the entire cube.



  b. What is the formula that relates the surface area of a cube (*SA*) to the area of one of its faces (*F*)?
- Suppose that all of the dimensions of a square-based prism are doubled. How does that affect its surface area?
- Calculate the surface area of a DVD case whose plastic covering measures 19 cm long, 12.5 cm wide and 1.7 cm thick.
- Calculate the surface area of the pencil sharpener on Kay's desk, if it is a cylinder with a diameter of 3.1 cm and a height of 5 cm. Round off the answer to one decimal place.
- The Great Pyramid of Gisa is a square-based pyramid with a base length 230 m and a slant height of 100 m. What is its surface area? Do not include the area of the base in the answer.

Section 1.4 – Surface Area of Cones and Spheres (pp. 40-49)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

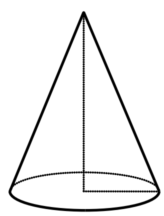
- **M1 (F)**

After this lesson, students will be expected to:

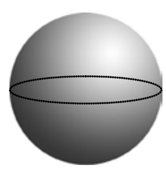
- use formulas to determine the surface area of cones and spheres

After this lesson, students should understand the following concepts:

- **cone –** a 3-D figure with a circular base and a curved surface that runs from the base to the highest point



- **sphere –** a round, ball-shaped figure



SURFACE AREA FORMULAS	
Cone	$SA = \pi r^2 + \pi rs$
Sphere	$SA = 4\pi r^2$

Suggested Problems in *Math at Work 11*:

- **p. 45:** #1-7
- **pp. 47-48:** #1-5
- **pp. 48-49:** #1-6

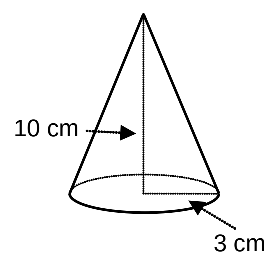
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Bring in models of cones to help explain surface area. Cut them out into their nets to help show the class how to derive their formulas.
- Remind students that to obtain the greatest possible accuracy, they should always use the π key on their calculators, and not an approximate value, such as 3.14 or $\frac{22}{7}$. Encourage students to estimate to check that their solutions are reasonable.

Possible Assessment Strategies:

- What is the slant height of the following cone? Round off the answer to one decimal place.



- If the diameter of a sphere triples, what happens to its surface area?
- Sketch a right cone with diameter 16 cm and slant height 12 cm. What is its surface area? Round off the answer to one decimal place.
- Calculate the surface area of a beach ball with radius 15 cm. Round off the answer to one decimal place.

CHAPTER 2
DRAWING AND DESIGN

SUGGESTED TIME

10 classes

Section 2.1 – Working with Scale (pp. 60-71)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G2 (A B C D E)**
- **A3 (A B C D)**

After this lesson, students will be expected to:

- use proportional reasoning to determine the dimensions of a scale drawing or model
- draw a scale diagram of a given object
- solve problems that involve scale
- describe when a scale representation might be used
- construct a model of a 3-D object

After this lesson, students should understand the following concepts:

- **scale –** the relationship between a distance on a drawing, model, or map and the actual distance; for example, a scale of 1 cm : 1 m means that 1 cm on the diagram, model, or map represents 1 m in actual size
- **scale drawing –** two-dimensional (2-D) drawing used to represent a place or object; uses scale to show the relationship between the distance on a drawing and the distance in real life

Suggested Problems in *Math at Work 11*:

- **pp. 64-65:** #1-7
- **pp. 68-69:** #1-7
- **pp. 70-71:** #1-7

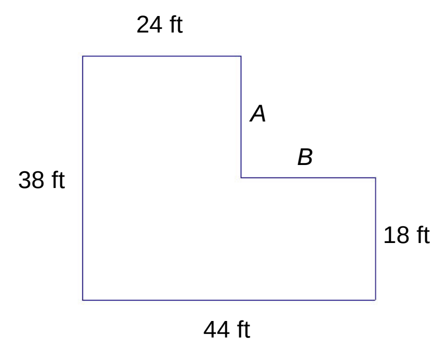
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

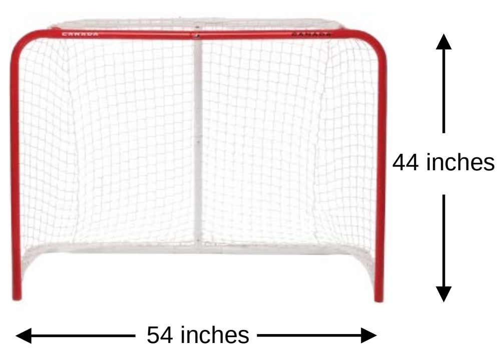
- Remind students to convert to the same units when determining the scale factor.
- Show students who struggle with the concept of scale an actual set of objects, in which one is double the size of the other. Have students verbalize what *double the size* means.
- Students can be given a 2-D shape on graph paper and then asked to come up with a procedure to either reduce or enlarge the shape.

Possible Assessment Strategies:

- Kelly is drawing a scale diagram of her house. She will let 1 inch represent 3 feet. If her house measures 30 feet by 42 feet, how big will her diagram be?
- Write each as a ratio whose first term is 1.
  a. 1 inch to 1 yard
  b. 5 cm to 1 m
- Determine the missing dimensions.



- This hockey net has dimensions as shown in the diagram.



Use a scale of 1 inch to 25 inches to create a scale drawing of the front of the net.

Section 2.2 – Representing Views of 3-D Objects (pp. 72-83)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G3 (A B C D E F)**

After this lesson, students will be expected to:

- draw top, front, and side views of a given 3-D object
- draw a 3-D object, given the top, front, and side views
- create an isometric drawing of a given 3-D object
- determine whether the given views of a 3-D object represent the object

After this lesson, students should understand the following concepts:

- **orthographic drawing –** a 2-D view of a 3-D object; often includes a front view, a top view, and a side view of the object
- **isometric drawing –** a view of a 3-D object in which
  - ➢ all horizontal edges of the object are drawn at a 30^0 angle;
  - ➢ all vertical edges of the object are drawn vertically
  - ➢ all lines are drawn to scale

Suggested Problems in *Math at Work 11*:

- **pp. 76-77:** #1-10
- **p. 81:** #1-6
- **pp. 82-83:** #1-8

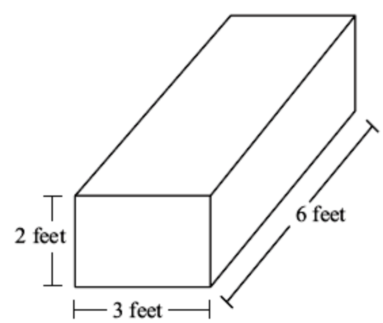
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

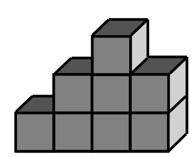
- Have students compare structures so they can come to realize that there can be more than one structure that fulfils the information in a set of plans. Have students explore such questions as the following: What is the minimum number of cubes that can be used to fulfill the plans provided? What is the maximum? How many different objects can be built to fulfill the plans?
- Use linking cubes as the basic building blocks for 3-D objects, as they are very versatile. With the front of the object facing the students, have them turn it 90^0 clockwise and sketch the object. Now have them turn it another 90^0 clockwise and sketch it again. Have them turn it one more time 90^0 clockwise and produce the third sketch. Have them continue rotating at 90^0 intervals until the sketch looks identical to the one already drawn.

Possible Assessment Strategies:

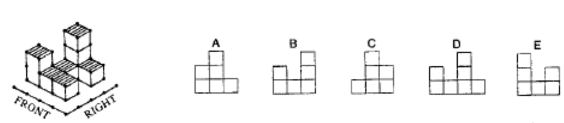
- State the dimensions for the front view, side view, and top view of the rectangular prism shown.



- Using blank paper, draw the top, front and side views of this object.



- Examine this picture of a building drawn from its right-front corner. Which one of A – E is the right orthographic view?



Section 2.3 – Representing Perspectives of 3-D Objects (pp. 84-95)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

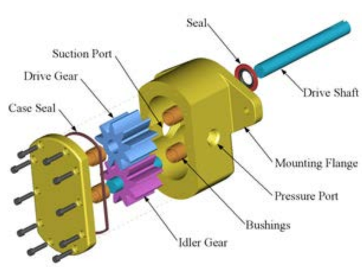
- **G3 (G H)**
- **G4 (A B C D)**

After this lesson, students will be expected to:

- draw a one-point perspective view of a 3-D object
- identify the point of perspective of a given one-point perspective drawing
- draw the components of an exploded view diagram
- sketch an exploded view diagram of a 3-D object
- sketch a 2-D representation of a 3-D object, given its exploded view diagram

After this lesson, students should understand the following concepts:

- **one-point perspective drawing –** a drawing in which it looks like objects in the background join at a vanishing point in the distance
- **vanishing point –** a point toward which parallel lines appear to join in the distance
- **point of perspective –** the position from which an object is being viewed
- **exploded view diagram –** a drawing that shows the components of an object with the parts slightly separated; often used to show the sequence of steps for assembling an object; usually shown as a set of isometric drawings



Suggested Problems in *Math at Work 11*:

- **pp. 88-90:** #1-11
- **pp. 93-94:** #1-6
- **p. 95:** #1-6

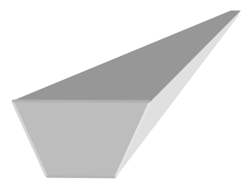
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- When drawing perspectives of 3-D objects, have students use a ruler to ensure that they create the most accurate drawing.
- Ensure that students understand how exploded view drawings are used to help put together the different components of an object to create a whole object.

Possible Assessment Strategies:

- Sketch a triangle on a piece of paper. Locate a vanishing point that is below left of the triangle. Create a one-point perspective drawing of the triangle.
- Identify the point of perspective for this drawing.



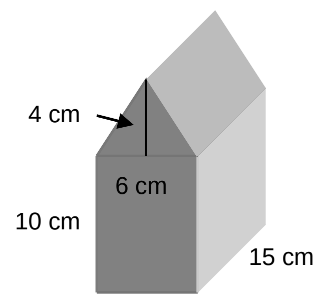
- Identify the item shown in the exploded view diagram. How many parts are shown in the diagram?

- Create and label an exploded view diagram of the following triple-scoop ice cream cone.

CHAPTER 3
VOLUME AND CAPACITY

SUGGESTED TIME
11 classes

Section 3.1 – Volume (pp. 106-117)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **M1 (A)** • **M2 (B C D I J M)** **After this lesson, students will be expected to:** • estimate the volume of cylinders, and of rectangular and triangular prisms • calculate the volume of cylinders, and of rectangular and triangular prisms **After this lesson, students should understand the following concept:** • **volume –** the amount of space an object occupies; measured in cubic units or units^3 **Suggested Problems in *Math at Work 11*:** • **pp. 110-111:** #1-14 • **pp. 114-115:** #1-7 • **pp. 116-117:** #1-10	**Possible Instructional Strategies:** • Some students may benefit from using diagrams labelled with the area of the base of a prism. Have students verbalize how the given base area is determined. Have them identify and label the dimensions of the prism, and then recalculate the volume. • Remind students that the units for volume are always in cubic units or units^3 **Possible Assessment Strategies:** • A certain cube has a surface area of 96 cm^2. What is the volume of the cube? • Find the volume of the following object.  • The Canola Oil Company is designing cans for its oil. Their cans hold 1 L, which is equivalent to a volume of 1000 cm^3. The area of the base of their can is 80 cm^2. How tall is the can?

Section 3.2 – Volume and Capacity (pp. 118-127)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **M2 (A B C D F G H I J M)** **After this lesson, students will be expected to:** • compare capacity and volume • convert between units of capacity • estimate capacity or volume using a reference **After this lesson, students should understand the following concept:** • **capacity –** the greatest amount that a container can hold; measured in cubic units or units^3 **CAPACITY CONVERSION FACTORS** 1 gallon = 4 quarts 1 quart = 2 pints 1 pint = 2 cups 1 cup = 8 fluid ounces **Suggested Problems in *Math at Work 11*:** • **pp. 121-122:** #1-9 • **pp. 124-125:** #1-8 • **pp. 126-127:** #1-9	**Possible Instructional Strategies:** • Ensure that students understand the difference between volume and capacity. **Possible Assessment Strategies:** • What is the relationship between the area of the base of a right cylinder and its volume? • Complete each of the following. a. 4 cups = $\square$ fl oz b. $\frac{1}{4}$ cup = $\square$ fl oz c. 24 fl oz = $\square$ cups d. 3 qt = $\square$ fl oz • Josh's car will hold 45 L of gas. If gas costs $1.27/L, how much does it cost to fill his gas tank?

Section 3.3 – Using Formulas for Volume and Capacity (pp. 128-136)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **M2 (E F I J K L M N)**

After this lesson, students will be expected to:

- use formulas to determine the volume or capacity of 3-D figures
- use formulas to determine the volume or capacity of composite figures

After this lesson, students should understand the following concept:

- **composite figure –** a 3-D object made up of two or more regular figures

VOLUME FORMULAS	
Rectangular Prism	$V = lwh$
Triangular Prism	$V = \frac{1}{2}lwh$
Square-Based Pyramid	$V = \frac{1}{3}b^2h$
Cylinder	$V = \pi r^2h$
Cone	$V = \frac{1}{3}\pi r^2h$

Suggested Problems in *Math at Work 11*:

- **pp. 131-132:** #1-11
- **pp. 134-135:** #1-7
- **p. 136:** #1-6

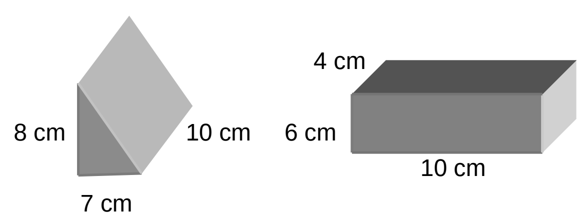
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Have the students practice working with the formulas for volume.
- Some students may not realize that the volume of a cube is simply one of its sides cubed. Encourage these students to continue using the formula $V = l \cdot w \cdot h$ to find the volume of a cube.
- Remind students that to obtain the greatest possible accuracy, they should always use the π key on their calculators, and not an approximate value, such as 3.14 or $\frac{22}{7}$.

Possible Assessment Strategies:

- Determine the height of each rectangular prism.
  a. volume = 108 cm^3, area of base = 12 cm^2
  b. volume = 80 cm^3, area of base = 16 cm^2
- Each of the following pieces of cheese costs $5.00. Which is the better deal? Why?



- a. A cylinder has a height of 22 cm and a radius of 10 cm. What is it volume? Round off the answer to one decimal place.
  b. What is the volume of its corresponding right cone with a height of 22 cm and radius 10 cm? Round off the answer to one decimal place.
- A school is having a fundraiser by selling popcorn and the students are making their own containers to save on expenses. If they have sheets of cardboard for the sides with dimensions 27 cm by 43 cm, would the volume be greater if the sheets were folded to make cylindrical containers with a height of 27 cm or with a height of 43 cm? Justify your answer mathematically.

Section 3.4 – Volume and Capacity of Spheres (pp. 137-143)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **M2 (F I M N)** **After this lesson, students will be expected to:** • determine the volume and capacity of spheres **VOLUME FORMULA** Sphere: $V=\frac{4}{3}\pi r^3$ **Suggested Problems in *Math at Work 11*:** • **pp. 140-141:** #1-7 • **pp. 142-143:** #1-7	**Possible Instructional Strategies:** • The volume of a sphere is two-thirds of the volume of a cylinder with the same radius and a height equal to the diameter of the sphere. Using the formula for the volume of a cylinder, ask students to derive the formula for the volume of a sphere. • Remind students that to obtain the greatest possible accuracy, they should always use the π key on their calculators, and not an approximate value, such as 3.14 or $\frac{22}{7}$. Encourage students to estimate to check that their solutions are reasonable. **Possible Assessment Strategies:** • Determine the volume of a sphere whose radius is 2.4 cm. Round off the answer to one decimal place. • Determine the volume of a soccer ball whose diameter is 24 cm. Round off the answer to one decimal place. • A sphere which is 12 cm in diameter fits exactly into a cube. Find the volume of the cube. • Find the diameter, correct to one decimal place, of a sphere with volume 500 cm^3. • A hot air balloon has a spherical shape with a diameter of 4 m. If 30 additional cubic metres of air are pumped into the balloon, what will be the new diameter? Round off the answer to one decimal place.

CHAPTER 4
INTERPRETING GRAPHS

SUGGESTED TIME

12 classes

Section 4.1 – Choosing a Graph (pp. 154-167)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

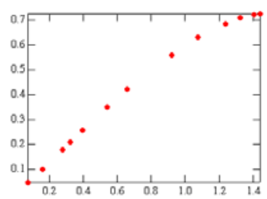
- **S1 (A B)**

After this lesson, students will be expected to:

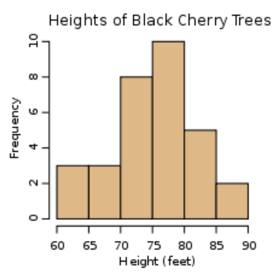
- create graphs
- choose possible graphs to represent a given data set
- explain the advantages and disadvantages of each type of graph

After this lesson, students should understand the following concepts:

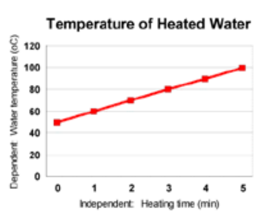
- **discrete –** data values that are distinct and can be counted; data values that fall into categories



- **histogram –** type of bar graph; shows the number of times data appear within a certain interval, such as the number of players between 170.5 and 173.5 cm in height; uses vertical bars without any gaps between them



- **continuous –** data values on a graph that are connected



Suggested Problems in *Math at Work 11*:

- **pp. 158-159:** #1-5
- **pp. 163-165:** #1-7
- **pp. 166-167:** #1-5

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- As a class, have students list advantages and disadvantages of the different types of graphs.
- Have students summarize what kinds of data each type of graph best displays. They could generate a context for using each type of graph.

Possible Assessment Strategies:

- The teacher of a math class recorded the number of students who had a final mark in each of the following ranges.

MARK	NUMBER OF STUDENTS
51 – 60	3
61 – 70	5
71 – 80	8
81 – 90	6
91 – 100	4

  a. Represent the data using a bar graph.
  b. Represent the data using a circle graph.

- Is the type of graph used to display the data in each scenario appropriate? Why or why not?
  a. Kate tracks her weight each week for a year. She represents the data using a line graph.
  b. Patrick records the time it takes 20 people to run one kilometre. He represents the data using a bar graph.
- Choose the type of graph that you would recommend to represent each situation. Justify your choice.
  a. Ray wants to compare the percent of students who go home for lunch to the percent of students who have lunch at school.
  b. Ben's mother wants to track his height since Ben was one year old.

Section 4.2 – Interpolating and Extrapolating Values (pp. 168-181)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S1 (C D)**

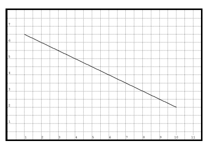
After this lesson, students will be expected to:

- describe the trends in a graph
- interpolate and extrapolate values from a graph
- determine if predictions and estimates are reasonable

After this lesson, students should understand the following concepts:

- **trend –** the general direction in which values in a data set tend to move; trends are either upward (for positive trends) or downward (for negative trends)

upward trend



downward trend

- **interpolate –** estimate a value that falls within a known range or graph of values
- **extrapolate –** estimate a value that falls outside a known range or graph of values

Suggested Problems in *Math at Work 11*:

- **pp. 174-175:** #1-4
- **pp. 178-179:** #1-5
- **pp. 180-181:** #1-4

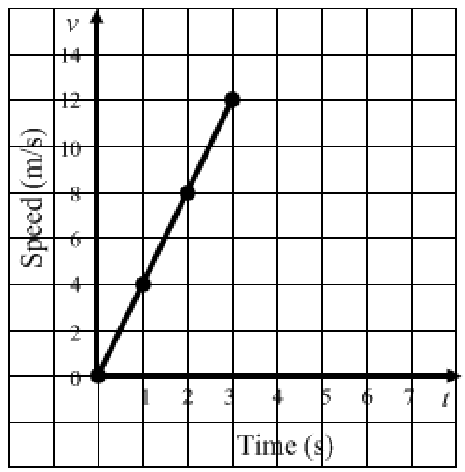
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

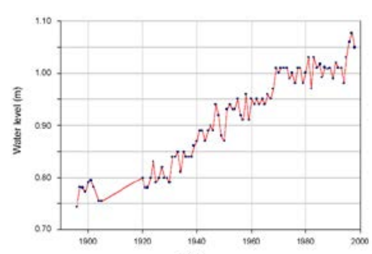
- Encourage students who need help to interpolate and extrapolate values to use a ruler and draw a vertical line from the *x*-axis to the graphed line, and then draw a horizontal line to the *y*-axis.
- Check that students understand when it is reasonable to use interpolation or extrapolation and when it is not reasonable. Remind students that both are used to estimate values.

Possible Assessment Strategies:

- Use the following graph to predict the speed after 6 seconds.



- Mary is getting in shape. The first day she does 9 sit-ups, the second day she does 13, the third day 17, and so on. Sketch a graph to represent this situation. If she continues in this way, how many sit-ups will she do on the 15th day? the 25th day? It is reasonable to continue this pattern forever?
- The following graph shows the mean water level in Charlottetown over the past 110 years. Describe the trend in the mean water level over that time.



Section 4.3 – Graphic Representations (pp. 182-195)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S1 (E F G)**

After this lesson, students will be expected to:

- determine if a graph accurately represents data
- explain how the same graph can show more than one conclusion
- explain how a graph can be misrepresented to emphasize a point of view

Suggested Problems in *Math at Work 11*:

- **pp. 186-187:** #1-5
- **pp. 190-193:** #1-7
- **pp. 194-195:** #1-4

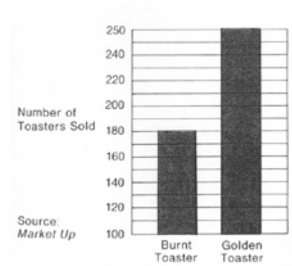
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Encourage students to think of ways and times that the media have published misleading graphs.

Possible Assessment Strategies:

- Ask students why the following statement is incorrect: “Sales of Golden Toaster were about double the sales of Burnt Toaster.” Discuss what could be changed on the graph, or added to make it less misleading.



- At an annual meeting for a certain company, the following information on profits was presented:

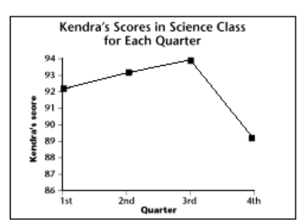
YEAR	PROFIT
1993	$340,000
1994	$350,000
1995	$370,000
1996	$410,000
1997	$450,000
1998	$465,000

  Make a graph to help support each of the following:

  a. The company for the past six years has experienced a very small profit increase.

  b. The company for the past six years shows a large profit increase.

- This graph shows that Kendra received a lower grade in science class during the fourth quarter of the year. Do you think Kendra should be worried by what appears to be such a large drop in her grades? Explain your reasoning.



CHAPTER 5
BANKING AND BUDGETING

SUGGESTED TIME

13 classes

Section 5.1 – Accounts (pp. 206-213)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **N4 (A B C D E F G)** **After this lesson, students will be expected to:** • describe a variety of banking services available • identify various banking service charges • describe ways that ensure the security of personal and financial information **After this lesson, students should understand the following concepts:** • **account –** a place at a financial institution to hold your money • **interest –** a fee paid for borrowing someone else's money • **service charges –** costs sometimes charged by financial institutions for providing services, such as banking by telephone, banking over the Internet, and using banking machines • **incentives –** in relation to banking, anything that financial institutions offer to customers, or potential customers, that makes doing business with that institution seem more attractive • **bank statement –** a record of all the transactions in an account over a period of time, usually one month • **credit –** in relation to banking, an amount of money added to a bank account • **debit –** in relation to banking, an amount of money subtracted from a bank account • **balance –** in relation to banking, the total amount of money in a bank account at a given point in time • **ATM –** stands for automated teller machine; machines people use to perform routine banking transactions **Suggested Problems in *Math at Work 11*:** • **pp. 210-211:** #1-9 • **pp. 212-213:** #1-8	**Possible Instructional Strategies:** • Discuss with the class the different types of services that are available at financial institutions. **Possible Assessment Strategies:** • A bank charges \$2 per month for managing an account. The account fee allows for 10 transactions each month. Additional transactions are \$0.50 each Determine the monthly account fee for the following number of transactions. a. 11 b. 20 c. 8 d. 0 • On April 30, John has \$2532.12 in his bank account. During the month, the following transactions were made: ➢ May 1 – Paid Rent \$680.00 ➢ May 5 – Withdrew \$50.00 ➢ May 12 – Direct Deposit Pay \$854.34 ➢ May 18 – Car Payment \$256.59 ➢ May 22 – Withdrew \$300.00 ➢ May 26 – Direct Deposit Pay \$854.34 How much did he have in his account on May 31? • Sam has \$300 in his bank account. He withdraws 15% of his money to buy a new jacket. How much money is left in the account after his withdrawal?

Section 5.2 – Budgets (pp. 214-227)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **N2 (A B C D E F)** **After this lesson, students will be expected to:** • identify income and expenses that should be included in a personal budget • create a budget • modify a budget to achieve a set of personal goals **After this lesson, students should understand the following concepts:** • **budget –** an organized plan for income and spending • **balanced budget –** a budget in which the total income equals the total expenses **Suggested Problems in *Math at Work 11*:** • **pp. 218-221:** #1-8 • **pp. 224-225:** #1-5 • **pp. 226-227:** #1-5	**Possible Instructional Strategies:** • Discuss with the class the process required in order to balance a budget. **Possible Assessment Strategies:** • One strategy for financial planning is to put away 10% of your net pay for investing in the future. Determine 10% of each of the following amounts. Round off the answers to the nearest cent. a. $785.40 b. $348.59 c. $1287.06 • Joanna has a net monthly income of $3000. Her expenses are: rent $840, electricity $60, food $400, transportation $300, phone and Internet $120, cable $50, clothing $200, and entertainment $200. How much is she able to save each month?

Section 5.3 – Simple and Compound Interest (pp. 228-238)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **N3 (A B C D E)** **After this lesson, students will be expected to:** • solve a problem that involves simple interest • compare simple and compound interest • solve a problem that involves compound interest • estimate the time required for a given investment to double in value **After this lesson, students should understand the following concepts:** • **Guaranteed Investment Certificate (GIC) –** an investment that is very low risk because the investment and any interest earned are guaranteed by the bank; tend to pay higher rates of interest than bank accounts but lower rates than some other investments • **term deposit –** an amount of money deposited for a fixed length of time; there may be penalties for withdrawing the money before the end of the term • **simple interest –** interest that is paid once, generally at the end of the time period of the investment; sometimes called *regular interest*; the formula for simple interest is $I = Prt$ • **future value –** the value of an investment at the end of a certain time period; also called the *final amount* (*A*) • **present value –** the amount of money that is invested; also called the *principal* (*P*) • **compound interest –** interest that is earned on the original investment plus interest earned during the previous calculation period(s); the formula for compound interest is $A = P(1+i)^n$ **Suggested Problems in *Math at Work 11*:** • **pp. 232-233:** #1-9 • **pp. 236-237:** #1-6 • **pp. 237-238:** #1-7	**Possible Instructional Strategies:** • Ensure that students understand the difference between simple interest and compound interest. **Possible Assessment Strategies:** • Determine how much simple interest is earned in each case. a. $2500 at 3%/year for 5 years b. $800 at 2.5%/year for 2 years c. $1000 at 6%/year for 6 months • Determine how much compound interest is earned in each case. a. $5000 at 7%/year for 10 years b. $350 at 1.5%/year for 2 years c. $1250 at 3.25%/year for 3 years • A 25-year old woman plans to retire at age 50. She decided to invest an inheritance of $60,000 at 7% compounded annually. How much will her investment be worth at age 50? • Jane, who is 18 years old, won $1,000,000 in a lottery. She invested $500,000 of her winnings into a bank account for her retirement that pays 5% per year, when she turns 65, in 47 years. How much money will there be in her account when she turns 65?

Section 5.4 – Investing and Borrowing (pp. 239-251)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **N5 (A B C D E F)**

After this lesson, students will be expected to:

- use a formula to solve problems that involve compound interest
- compare different types of credit options
- make decisions and plans related to the use of credit
- describe strategies to use credit effectively
- solve a problem that involves credit cards or loans
- solve a problem that involves finance charges

After this lesson, students should understand the following concept:

- **mutual fund –** a collection of stocks and/or bonds; allows investors to pool their money

Suggested Problems in *Math at Work 11*:

- **pp. 243-245:** #1-12
- **pp. 247-249:** #1-11
- **pp. 250-251:** #1-11

Possible Instructional Strategies:

- It can be shown to students that the compound interest formula is an example of a geometric sequence, when *n* takes on whole number values.

Possible Assessment Strategies:

- Complete the following table:

	Annual Interest Rate	Frequency	*t*	*i*	*n*
a.	4%	annually	10 years		
b.	2%	quarterly	9 months		
c.	3%	monthly	18 months		

- For each of the following, determine the future value and the total interest earned.
  a. $200 invested for 9 years at 9% compounded monthly
  b. $750 invested for 12 years at 4% compounded quarterly
- Which is the better investment? Explain.
  - 4.5% compounded annually
  - 4.5% compounded quarterly
- A couple decides to set aside $5000 in a savings account for a second honeymoon trip. It is compounded quarterly at 9%. Find the amount of money they will have in 4 years.
- In order to pay for college, the parents of a child invest $20,000 in a bond that pays 8% interest compounded semi-annually. How much money will the bond be worth in 19 years?
- To pay for new machinery in 5 years, a company owner invests $10,000 at $7\frac{1}{2}\%$ compounded monthly. How much money will be available in 5 years?

CHAPTER 6

SLOPE

SUGGESTED TIME

10 classes

Section 6.1 – What Is Slope? (pp. 262-273)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

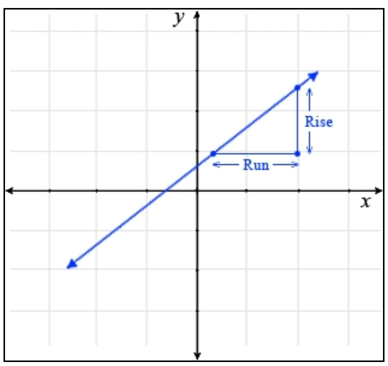
- **A2 (A B)**

After this lesson, students will be expected to:

- explain slope as rise divided by run
- verify that the slope is constant for a straight line
- solve problems that involve slope
- describe contexts that involve slope

After this lesson, students should understand the following concepts:

- **slope –** a measure of the steepness of a line; the ratio of the rise to the run of a line or line segment



- **rise –** the vertical distance between two points on a line
- **run –** the horizontal distance between two points on a line
- **constant –** a value that is always the same

Suggested Problems in *Math at Work 11*:

- **pp. 266-267:** #1-9
- **pp. 270-271:** #1-9
- **pp. 272-273:** #1-9

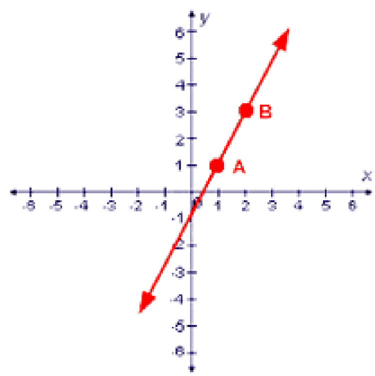
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Remind students that $\text{slope} = \frac{\text{rise}}{\text{run}}$. To help students remember this, remind them that the word *rise* occurs before the word *run* in the dictionary, so we place *rise* first in the numerator and *run* second in the denominator. Since rise represents a vertical distance, it is the change in *y*. Since run represents a horizontal distance, it is the change in *x*.

Possible Assessment Strategies:

- What is the slope of the line shown in the given graph?



- A garage is 24 ft wide. The vertical distance from the base of the roof to the peak is 5 ft.
  a. State the rise and run of the roof.
  b. What is the pitch of the roof?
- For safety, the slope of a ladder should be no greater than $4:1$. Using this slope, if the bottom of a ladder is $1\frac{1}{2}$ ft from the bottom of a wall, how high up the wall will it reach?
- Solve each proportion.

  a. $\frac{3}{7} = \frac{a}{28}$

  b. $\frac{b}{9} = \frac{3}{4}$

  c. $\frac{5}{c} = \frac{15}{33}$

Section 6.2 – Relationship Between Slope and Angle of Elevation (pp. 274-285)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

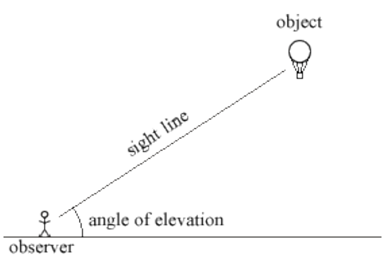
- **A2 (C D E F G)**

After this lesson, students will be expected to:

- describe the conditions for a slope that is zero or undefined
- explain the relationship between slope and angle of elevation
- explain the safety implications of different slopes
- solve problems by applying the slope formula

After this lesson, students should understand the following concepts:

- **angle of elevation –** an angle formed by the horizontal and a line of sight above the horizontal line



- **grade –** the slope of a road or railway track; usually expressed as a percent

SLOPE FORMULA

The formula for the slope of a line passing through the points (x_1, y_1) and (x_2, y_2) is

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

or

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Suggested Problems in *Math at Work 11*:

- **pp. 278-279:** #1-7
- **pp. 282-283:** #1-9
- **pp. 284-285:** #1-11

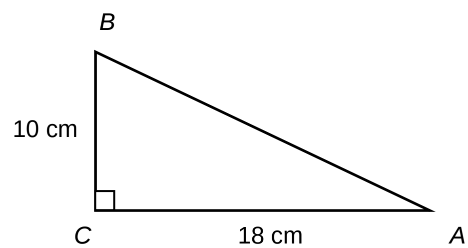
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students understand the relationships among slope, steepness, and the angle of elevation.
- Review with the class how to use trigonometry to solve problems involving the angle of elevation.

Possible Assessment Strategies:

- Determine the measure of angle *A* in the following triangle. Round off the answer to one decimal place.



- Determine the slope, *m*, of each line segment with the given end points.
  - a. $S(3,6)$ and $T(5,8)$
  - b. $H(4,3)$ and $K(4,8)$
  - c. $M(9,7)$ and $N(11,7)$
  - d. $W(2,2)$ and $S(4,5)$
- The bottom of a ladder is 3 m from a house. The angle between the ladder and the ground is 60^0.
  - a. How far up the house does the ladder reach? Round off the answer to one decimal place.
  - b. What is the length of the ladder? Round off the answer to one decimal place.
- The recommended slope of a particular ramp is $1:3$.
  - a. If a ramp is to have a vertical height of 15 inches, what must be its horizontal length in order to meet this $1:3$ ratio?
  - b. What angle does the ramp make with the ground? Round off the answer to one decimal place.

Section 6.3 – Slope as Rate of Change (pp. 286-297)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **6.3 (H I)**

After this lesson, students will be expected to:

- explain slope as a rate of change
- describe the difference between two slopes
- solve problems that involve slope and rate of change

After this lesson, students should understand the following concept:

- **rate of change –** a change in one quantity relative to the change in another quantity

Suggested Problems in *Math at Work 11*:

- **pp. 290-291:** #1-6
- **pp. 294-295:** #1-7
- **pp. 296-297:** #1-5

Possible Instructional Strategies:

- Ensure that students understand that a slope is an example of a rate of change.

Possible Assessment Strategies:

- Examine the following table of values.

x	*y*
1	2
4	8
7	14
10	20
13	26

a. What is the change in the *x*-values from one row to the next?

b. What is the change in the *y*-values from one row to the next?

c. What is the slope of the line that would connect these points on a graph?

- A jet ski rental operation charges a fixed insurance premium, plus an hourly rate. The total cost for 2 hours is $50 and for 5 hours is $110. Determine the hourly rate to rent the jet ski.
- Would the graph of the data in this table show a constant slope?

x	*y*
1	2
3	3
5	5
7	8
9	12

CHAPTER 7
RIGHT TRIANGLES AND TRIGONOMETRY

SUGGESTED TIME

13 classes

Section 7.1 – Right Triangles (pp. 308-321)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G1 (A C)**

After this lesson, students will be expected to:

- sketch a given scenario
- use trigonometric ratios to determine distances and lengths
- use trigonometric ratios to determine unknown angles
- solve problems using the three trigonometric ratios

After this lesson, students should understand the following concept:

- **primary trigonometric ratios –** the three ratios defined in a right triangle: sine, cosine, and tangent

PRIMARY TRIGONOMETRIC RATIOS	
Sine	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
Cosine	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
Tangent	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

Suggested Problems in *Math at Work 11*:

- **pp. 312-313:** #1-6
- **pp. 315-316:** #1-8
- **pp. 318-319:** #1-6
- **pp. 320-321:** #1-5

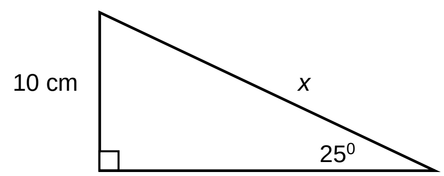
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Remind students that a diagram showing all of the given information should be the first step in the solution of any trigonometric problem. Their sketches need not be accurate, but reasonable representations of the given situation can help them decide on a strategy.
- When identifying the sides of a right triangle, it is a good idea to put **hyp** next to the hypotenuse, **opp** next to the opposite side and **adj** next to the adjacent side.
- Ensure that the calculators of all students are in degree mode.

Possible Assessment Strategies:

- Find the length of *x*. Round off the answer to one decimal place.



- A surveyor wants to determine the width of a river for a proposed bridge. The distance from the surveyor to the proposed bridge site is 400 m. The surveyor measures a 31^0 angle to the bridge site across the river. What is the width of the river, to the nearest metre?
- A small boat is 95 m from the base of a lighthouse that has a height of 36 m above sea level. Calculate the angle from the boat to the top of the lighthouse. Round off the answer to one decimal place.
- In the World Cup downhill held at Panorama Mountain in British Columbia, the skiers raced 3514 m down the mountain. If the vertical height of the course is 984 m, determine the average angle that the ski course makes with the ground. Round off the answer to one decimal place.
- A pilot starts his takeoff and climbs steadily at an angle of 12.2^0. Determine the horizontal distance the plane has travelled when it is 5.4 km above the ground. Round off the answer to one decimal place.

Section 7.2 – Angles of Elevation and Depression (pp. 322-337)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

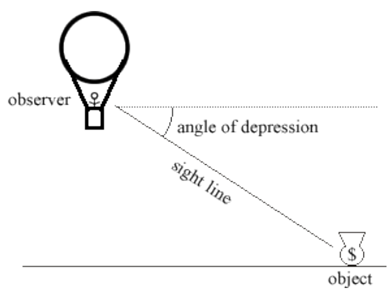
- **G1 (C D)**

After this lesson, students will be expected to:

- identify and sketch a given scenario
- use angles of elevation and depression to determine distances and lengths
- use trigonometric ratios to determine unknown angles of elevation and depression
- solve problems using angles of elevation and depression

After this lesson, students should understand the following concept:

- **angle of depression –** angle formed by the horizontal and a line of sight below the horizontal line



Suggested Problems in *Math at Work 11*:

- **pp. 326-327:** #1-7
- **pp. 330-331:** #1-6
- **pp. 334-335:** #1-6
- **pp. 336-337:** #1-7

Possible Instructional Strategies:

- Ensure that students understand that the angle of elevation and the angle of depression along the same line of sight are always equal.

Possible Assessment Strategies:

- Determine each angle measure. Round off the answers to one decimal place.
  a. $\cos A = 0.3890$
  b. $\sin A = 0.9820$
  c. $\tan A = 1.4529$
- A 30-m long line is used to hold a helium weather balloon. Due to a breeze, the line makes a 75^0 angle with the ground. Determine the height of the balloon. Round off the answer to one decimal place.
- Natalie is rock climbing and Aaron is belaying. When Aaron pulls the rope taut to the ground, the angle of depression is 73^0. If Aaron is standing 8 ft from the wall, what length of the rope is off the ground?

Section 7.3 – Multiple Right Triangles (pp. 338-353)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G1 (B C E)**

After this lesson, students will be expected to:

- identify right triangles in a given scenario
- use trigonometric ratios to determine unknown angles and lengths
- determine whether a solution to a problem is reasonable

Suggested Problems in *Math at Work 11*:

- **pp. 342-343:** #1-7
- **pp. 346-347:** #1-5
- **pp. 350-351:** #1-5
- **pp. 352-353:** #1-5

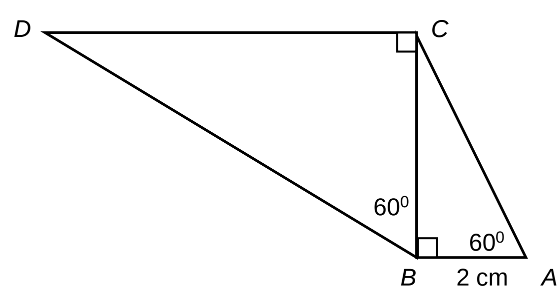
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Remind students that a diagram showing all of the given information should be the first step in the solution of any trigonometric problem. Their sketches need not be accurate, but reasonable representations of the given situation can help them decide on a strategy.
- When presented with a problem that involves two right triangles, encourage students to draw the two triangles separately with the sides and angles labelled in both triangles.

Possible Assessment Strategies:

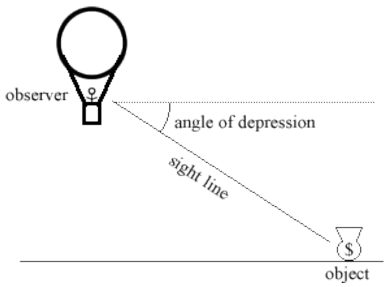
- From a height of 50 m in his fire tower near the lake, a ranger observes the beginnings of two fires. One fire is due west at an angle of depression of 9^0. The other fire is due east at an angle of depression of 7^0. What is the distance between the two fires, to the nearest tenth of a metre?
- From his hotel window overlooking the street, Ken observes a bus moving away from the hotel. The angle of depression to the bus changes from 46^0 to 22^0. Determine the distance the bus travels in that time if Ken's window is 100 m above street level. Round off the answer to one decimal place.
- The triangles *ABC* and *BCD* have right angles at *B* and *C*, respectively. Calculate the length of side *CD*. Round off the answer to one decimal place.



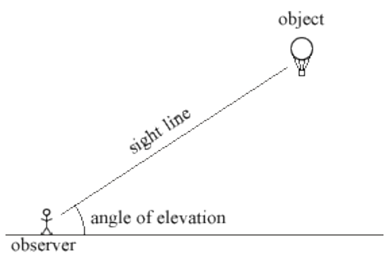
GLOSSARY OF MATHEMATICAL TERMS

A

- **account –** a place at a financial institution to hold your money
- **angle of depression –** angle formed by the horizontal and a line of sight below the horizontal line



- **angle of elevation –** an angle formed by the horizontal and a line of sight above the horizontal line



- **ATM –** stands for automated teller machine; machines people use to perform routine banking transactions
- **average –** the sum of all values in a data set divided by the number of values in the data set

B

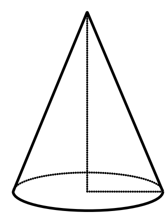
- **balance –** in relation to banking, the total amount of money in a bank account at a given point in time
- **balanced budget –** a budget in which the total income equals the total expenses
- **bank statement –** a record of all the transactions in an account over a period of time, usually one month
- **budget –** an organized plan for income and spending

C

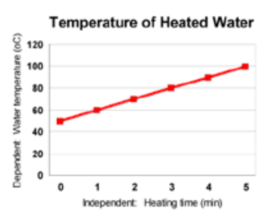
- **capacity –** the greatest amount that a container can hold; measured in cubic units or units^3
- **complementary angles –** two angles that add to 90^0 and form a right angle
- **composite figure –** a 3-D object made up of two or more regular figures
- **compound interest –** interest that is earned on the original investment plus interest earned during the previous calculation period(s); the formula for compound interest is

$$A = P\left(1+i\right)^n$$

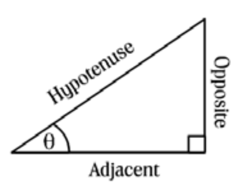
- **cone –** a 3-D figure with a circular base and a curved surface that runs from the base to the highest point



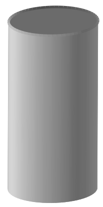
- **constant –** a value that is always the same
- **continuous –** data values on a graph that are connected; for continuous data, the top end of one interval needs to be at the bottom end of the next interval



- **cosine ratio –** the ratio of the length of the adjacent side to the length of the hypotenuse for an angle in a right triangle; $\cos\ \theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

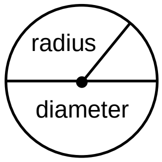


- **credit –** in relation to banking, an amount of money added to a bank account
- **cylinder –** a 3-D object with two circular faces that are the same size, and has a curved surface

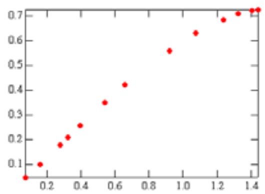


D

- **debit –** in relation to banking, an amount of money subtracted from a bank account
- **diameter –** the distance across a circle through its centre

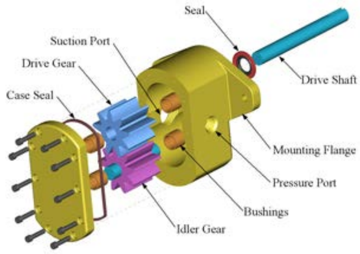


- **discrete –** data values that are distinct and can be counted; data values that fall into categories



E

- **exploded view diagram –** a drawing that shows the components of an object with the parts slightly separated; often used to show the sequence of steps for assembling an object; usually shown as a set of isometric drawings



- **extrapolate –** estimate a value that falls outside a known range or graph of values

F

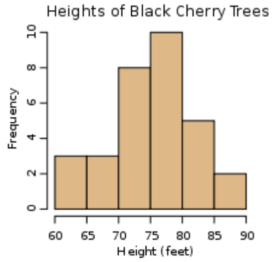
- **fixed expenses –** expenses that are unlikely to change from month to month
- **fixed term investments –** require investors to keep money invested for a specific period of time
- **frequency –** number of values in each category in a table of values; record each response using a tally mark; count the number of tally marks to find the frequency
- **future value –** the value of an investment at the end of a certain time period; also called the *final amount* (*A*)

G

- **grade –** the slope of a road or railway track; usually expressed as a percent
- **gross pay –** the total amount that an employee earns, before taxes are deducted
- **Guaranteed Investment Certificate (GIC) –** an investment that is very low risk because the investment and any interest earned are guaranteed by the bank; tend to pay higher rates of interest than bank accounts but lower rates than some other investments

H

- **histogram –** type of bar graph; shows the number of times data appear within a certain interval, such as the number of players between 170.5 and 173.5 cm in height; uses vertical bars without any gaps between them



I

- **incentives –** in relation to banking, anything that financial institutions offer to customers, or potential customers, that makes doing business with that institution seem more attractive
- **interest –** a fee paid for borrowing someone else's money
- **interpolate –** estimate a value that falls within a known range or graph of values
- **isometric drawing –** a view of a 3-D object in which
  - all horizontal edges of the object are drawn at a 30^0 angle;
  - all vertical edges of the object are drawn vertically
  - all lines are drawn to scale

L

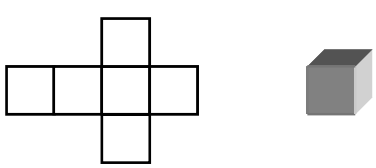
- **line of credit –** an arrangement between a financial institution and a customer for a maximum amount that can be borrowed at any time

M

- **median –** the middle number in a data set after the data has been arranged in order
- **mutual fund –** a collection of stocks and/or bonds; allows investors to pool their money

N

- **net –** a 2-D diagram that can be folded to create a 3-D object



net cube

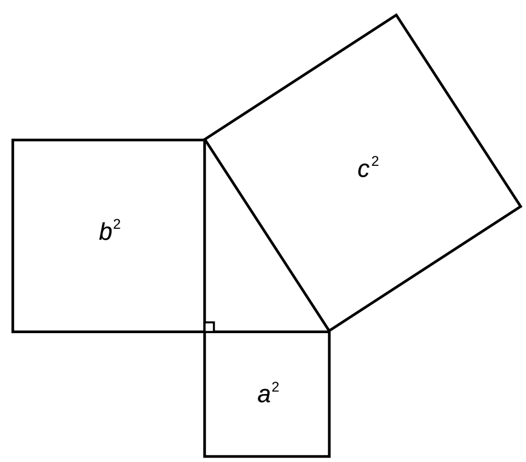
- **net pay –** the total amount that an employee gets to take home, after taxes are deducted

O

- **one-point perspective drawing –** a drawing in which it looks like objects in the background join at a vanishing point in the distance
- **orthographic drawing –** a 2-D view of a 3-D object; often includes a front view, a top view, and a side view of the object
- **overdraft protection –** a service that financial institutions may offer to temporarily cover a customer's next few purchases if the amount of money in the customer's account falls to $0.00

P

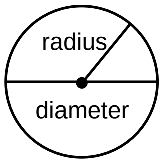
- **personal loan –** a loan that is given for personal use
- **PIN –** a pass code selected by the user of a bank card to gain access to the user's bank accounts; stands for personal identification number
- **point of perspective –** the position from which an object is being viewed
- **present value –** the amount of money that is invested; also called the *principal* (*P*)
- **primary trigonometric ratios –** the three ratios defined in a right triangle: sine, cosine, and tangent
- **Pythagorean relationship –** the relationship among the lengths of the sides of a right triangle; the sum of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse



$$a^2 + b^2 = c^2$$

R

- **radius –** the distance from the centre of a circle to a point on the circumference; the radius is equal to half of the diameter



- **rate of change –** a change in one quantity relative to the change in another quantity
- **rectangular prism –** a 3-D figure with two rectangular bases that are the same shape and size

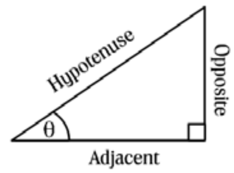
- **return –** the profit on an investment
- **rise –** the vertical distance between two points on a line
- **run –** the horizontal distance between two points on a line

S

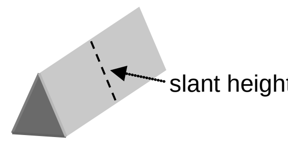
- **scale –** the relationship between a distance on a drawing, model, or map and the actual distance; for example, a scale of 1 cm : 1 m means that 1 cm on the diagram, model, or map represents 1 m in actual size
- **scale drawing –** two-dimensional (2-D) drawing used to represent a place or object; uses scale to show the relationship between the distance on a drawing and the distance in real life
- **service charges –** costs sometimes charged by financial institutions for providing services, such as banking by telephone, banking over the Internet, and using banking machines
- **simple interest –** interest that is paid once, generally at the end of the time period of the investment; sometimes called *regular interest*; the formula for simple interest is

$$I = Prt$$

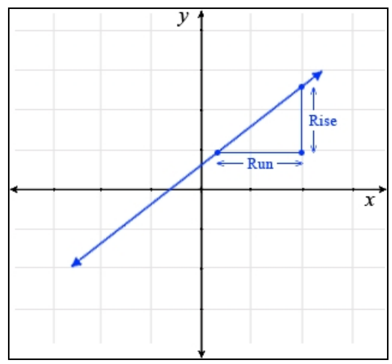
- **sine ratio –** the ratio of the length of the opposite side to the length of the hypotenuse for an angle in a right triangle; $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$



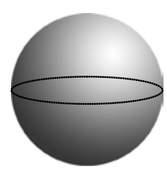
- **slant height –** the shortest distance from the edge of the base of a 3-D figure to its highest point



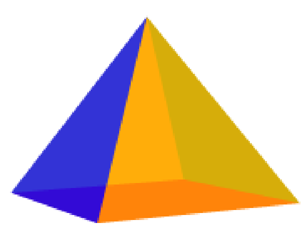
- **slope –** a measure of the steepness of a line; the ratio of the rise to the run of a line or line segment



- **sphere –** a round, ball-shaped figure



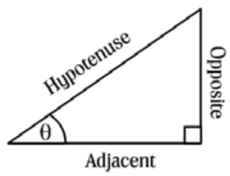
- **square-based pyramid –** a 3-D figure with a square base and four triangular sides that connect at one point



- **surface area –** the sum of the areas of all the faces of a 3-D object; measured in square units or units^2

T

- **tangent ratio –** the ratio of the length of the opposite side to the length of the adjacent side for an angle in a right triangle; $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

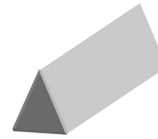


- **term deposit –** an amount of money deposited for a fixed length of time; there may be penalties for withdrawing the money before the end of the term
- **transaction –** a transfer of money, such as a deposit or a withdrawal from a bank account
- **trend –** the general direction in which values in a data set tend to move; trends are either upward (for positive trends) or downward (for negative trends)

upward trend

downward trend

- **triangular prism** – a 3-D figure with only two triangular faces that are the same size and shape

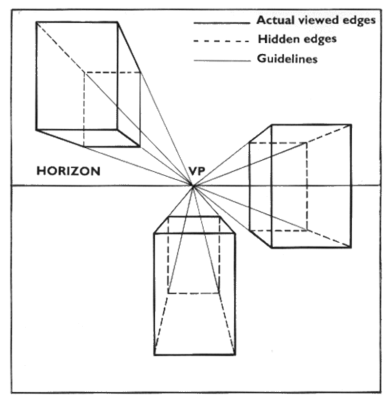


U

- **utilities –** basic household services, such as heat, electricity, and water

V

- **vanishing point –** a point toward which parallel lines appear to join in the distance



- **variable expenses –** expenses that are likely to change from month to month
- **volume –** the amount of space an object occupies; measured in cubic units or units^3

SOLUTIONS TO POSSIBLE ASSESSMENT STRATEGIES

SECTION 1.1

- The triangular prism is the better box, with a surface area of 312 cm^2, as compared with 340 cm^2 for the rectangular prism.
- 172.8 cm^2
- 132 cm^2

SECTION 1.2

- a. $10 \text{ cm} \times 10 \text{ cm} = 100 \text{ cm}^2$

  b. $5 \text{ ft} \times 5 \text{ ft} = 25 \text{ ft}^2$
- $\frac{1}{100}$
- The surface area of the box is 2592 cm^2. Since 1 m^2 is equivalent to 10,000 cm^2, she has enough paper.
- 11,200 cm^2

SECTION 1.3

- a. $F = 4 \text{ cm}^2$, $SA = 24 \text{ cm}^2$

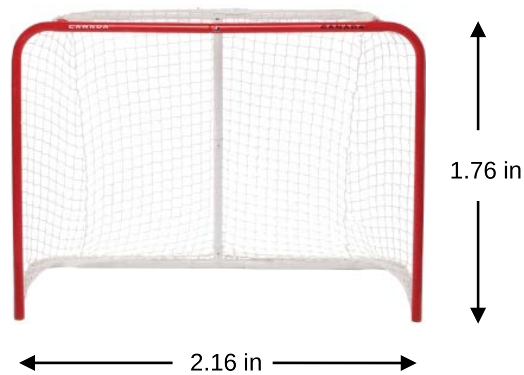
  b. $SA = 6 \times F$
- The surface area will be 4 times larger.
- 582.1 cm^2
- 63.8 cm^2
- 46,000 m^2

SECTION 1.4

- 10.4 cm
- The surface area will be 9 times larger.
- 502.7 cm^2
- 2827.4 cm^2

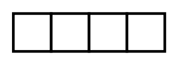
SECTION 2.1

- 10 in $\times$ 14 in
- a. 1 : 36

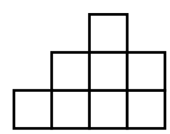
  b. 1 : 20
- $A = 20$ ft, $B = 20$ ft
- 

SECTION 2.2

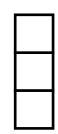
- Front view: $2 \text{ ft} \times 3 \text{ ft}$; Side view: $2 \text{ ft} \times 6 \text{ ft}$; Top view: $3 \text{ ft} \times 6 \text{ ft}$
- Top View:

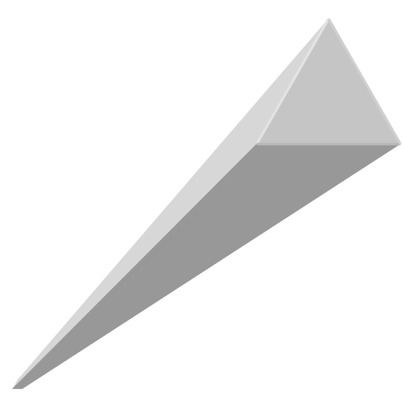
  Front View:

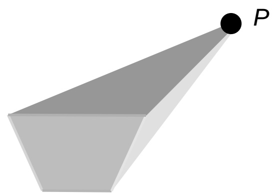
  

  Side View:

  
- B

SECTION 2.3

- 

- 

- sneaker; 6 parts
- 

SECTION 3.1

- 64 cm^3
- 1080 cm^3
- 12.5 cm

SECTION 3.2

- The volume of a right cylinder equals the area of its base times its height.
- a. 4 cups = 32 fl oz

  b. $\frac{1}{4}$ cup = 2 fl oz

  c. 24 fl oz = 3 cups

  d. 3 qt = 96 fl oz
- $57.15

SECTION 3.3

- a. 9 cm

  b. 5 cm
- The triangular block of cheese has a volume of 280 cm^3 and the rectangular block of cheese has a volume of 240 cm^3. The triangular block is the better buy, because you get more cheese for the same price.
- a. 6911.5 cm^3

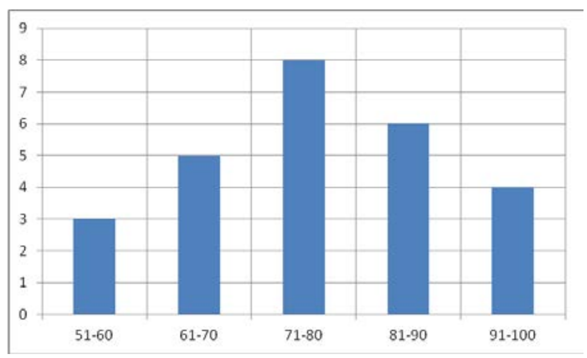
  b. 2303.8 cm^3
- The cylinder with a height of 27 cm will have an approximate volume of 3973 cm^3. The cylinder with a height of 43 cm will have an approximate volume of 2495 cm^3. Therefore, the height should be 27 cm to get the greater volume.

SECTION 3.4

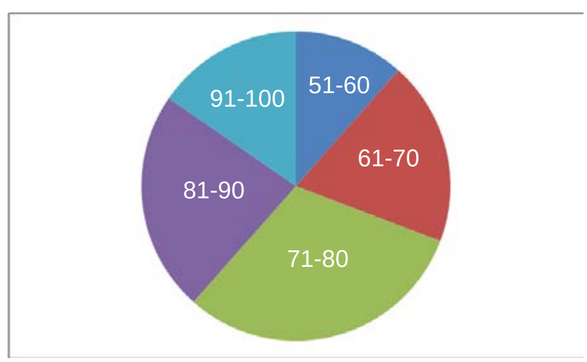
- 57.9 cm^3
- 7238.2 cm^3
- 1728 cm^3
- 9.8 cm
- 5.0 m

SECTION 4.1

- a.



  b.



- a. A line graph is appropriate because it shows her change in weight over time.

   b. A bar graph is appropriate because it shows how many people there are in each category, and makes a comparison among the categories.
- a. A circle graph or a bar graph should be used to make a comparison between the two groups of students.
  b. A line graph should be used to show the change in Ben's height over time.

SECTION 4.2

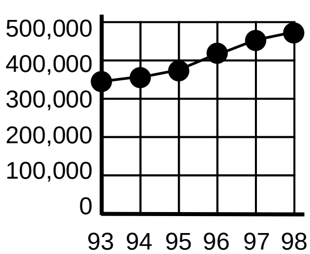
- 24 m/s
-

65 sit-ups, 105 sit-ups; It is not reasonable to continue this pattern forever because there are limits to the amount of exercise that the human body can endure.
- upward trend

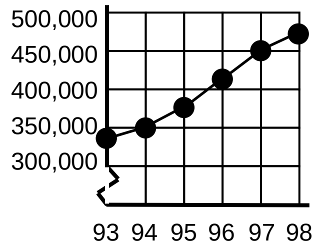
SECTION 4.3

- The statement is incorrect because the vertical scale does not begin at 0. In order to make the graph less misleading, the vertical scale should start at 0, which means that the right bar would not be double the length of the left bar.
- Answers may vary. Possible solutions are:

  a.



  b.



- Kendra should not be worried about her fourth quarter mark because the vertical axis begins at the relatively high mark of 86, making the differences in her grades seem larger than they are.

SECTION 5.1

- a. $2.50
  b. $7.00
  c. $2.00
  d. $2.00
- $2954.21
- $255

SECTION 5.2

- a. $78.54
  b. $34.86
  c. $128.71
- $830

SECTION 5.3

- a. $375
  b. $40
  c. $30
- a. $4835.76
  b. $10.58
  c. $125.88
- $325,645.96
- $4,952,985.55

SECTION 5.4

-

	Annual Interest Rate	Frequency	*t*	*i*	*n*
a.	4%	annually	10 years	0.04	10
b.	2%	quarterly	9 months	0.005	3
c.	3%	monthly	18 months	0.0025	18

- a. $A = \$448.22$, $I = \$248.22$
  b. $A = \$1209.17$, $I = \$459.17$
- The first option pays 1.045 times the principal after one year, and the second option pays approximately 1.0458 times the principal after one year, so the second option is better.
- $7138.11
- $88,776.27
- $14,532.94

SECTION 6.1

- 2
- a. Rise: 5; Run: 12
  b. $\frac{5}{12}$
- 6 ft
- a. $a = 12$
  b. $b = \frac{27}{4} = 6.75$
  c. $c = 11$

SECTION 6.2

- 29.1^0
- a. 1
  b. undefined
  c. 0
  d. $\frac{3}{2}$
- a. 5.2 m
  b. 6.0 m
- a. 45 inches
  b. 18.4^0

SECTION 6.3

- a. 3
  b. 6
  c. 2
- $20/h
- no

SECTION 7.1

- 23.7 cm
- 240 m
- 20.8^0
- 16.3^0
- 25.0 km

SECTION 7.2

- a. 67.1^0
  b. 79.1^0
  c. 55.5^0
- 29.0 m
- 27.4 ft

SECTION 7.3

- 722.9 m
- 150.9 m
- 6 cm

APPENDIX

MATHEMATICS RESEARCH PROJECT

➢ Introduction

A research project can be a very important part of a mathematics education. Besides the greatly increased learning intensity that comes from personal involvement with a project, and the chance to show universities, colleges, and potential employers the ability to initiate and carry out a complex task, it gives the student an introduction to mathematics as it is – a living and developing intellectual discipline where progress is achieved by the interplay of individual creativity and collective knowledge. A major research project must successfully pass through several stages. Over the next few pages, these stages are highlighted, together with some ideas that students may find useful when working through such a project.

➢ Creating an Action Plan

As previously mentioned, a major research project must successfully pass though several stages. The following is an outline for an action plan, with a list of these stages, a suggested time, and space for students to include a probable time to complete each stage.

STAGE	SUGGESTED TIME	PROBABLE TIME
Select the topic to explore.	1 – 3 days	
Create the research question to be answered.	1 – 3 days	
Collect the data.	5 – 10 days	
Analyse the data.	5 – 10 days	
Create an outline for the presentation.	2 – 4 days	
Prepare a first draft.	3 – 10 days	
Revise, edit and proofread.	3 – 5 days	
Prepare and practise the presentation.	3 – 5 days	

Completing this action plan will help students organize their time and give them goals and deadlines that they can manage. The times that are suggested for each stage are only a guide. Students can adjust the time that they will spend on each stage to match the scope of their projects. For example, a project based on primary data (data that they collect) will usually require more time than a project based on secondary data (data that other people have collected and published). A student will also need to consider his or her personal situation – the issues that each student deals with that may interfere with the completion of his or her project. Examples of these issues may include:

- a part-time job;
- after-school sports and activities;
- regular homework;
- assignments for other courses;
- tests in other courses;
- time they spend with friends;
- family commitments;
- access to research sources and technology.

➢ Selecting the Research Topic

To decide what to research, a student can start by thinking about a subject and then consider specific topics. Some examples of subjects and topics may be:

SUBJECT	TOPIC
Entertainment	• effects of new electronic devices • file sharing
Health care	• doctor and/or nurse shortages • funding
Post-secondary education	• entry requirements • graduate success
History of Western and Northern Canada	• relations among First Nations • immigration

It is important to take the time to consider several topics carefully before selecting a topic for a research project. The following questions will help a student determine if a topic that is being considered is suitable.

- **Does the topic interest the student?**

  Students will be more successful if they choose a topic that interests them. They will be more motivated to do research, and they will be more attentive while doing the research. As well, they will care more about the conclusions they make.

- **Is the topic practical to research?**

  If a student decides to use first-hand data, can the data be generated in the time available, with the resources available? If a student decides to use second-hand data, are there multiple sources of data? Are the sources reliable, and can they be accessed in a timely manner?

- **Is there an important issue related to the topic?**

  A student should think about the issues related to the topic that he or she has chosen. If there are many viewpoints on an issue, they may be able to gather data that support some viewpoints but not others. The data they collect from all viewpoints should enable them to come to a reasoned conclusion.

- **Will the audience appreciate the presentation?**

  The topic should be interesting to the intended audience. Students should avoid topics that may offend some members of their audience.

➢ Creating the Research Question or Statement

A well-written research question or statement clarifies exactly what the project is designed to do. It should have the following characteristics:

- The research topic is easily identifiable.

- The purpose of the research is clear.
- The question or statement is focused. The people who are listening to or reading the question or statement will know what the student is going to be researching.

A good question or statement requires thought and planning. Below are three examples of initial questions or statements and how they were improved.

UNACCEPTABLE QUESTION OR STATEMENT	WHY?	ACCEPTABLE QUESTION OR STATEMENT
Is mathematics used in computer technology?	Too general	What role has mathematics played in the development of computer animation?
Water is a shared resource.	Too general	Homes, farms, ranches, and businesses east of the Rockies all use runoff water. When there is a shortage, that water must be shared.
Do driver's education programs help teenagers parallel park?	Too specific, unless the student is generating his or her own data	Do driver's education programs reduce the incidence of parking accidents?

The following checklist can be used to determine if the research question or statement is effective.

- Does the question or statement clearly identify the main objective of the research? After the question or statement is read to someone, can they tell what the student will be researching?
- Is the student confident that the question or statement will lead him or her to sufficient data to reach a conclusion?
- Is the question or statement interesting? Does it make the student want to learn more?
- Is the topic that the student chose purely factual, or is that student likely to encounter an issue, with different points of view?

➢ Carrying Out the Research

As students continue with their projects, they will need to conduct research and collect data. The strategies that follow will help them in their data collection.

There are two types of data that students will need to consider – primary and secondary. Primary data is data that the student collects himself or herself using surveys, interviews and direct observations. Secondary data is data that the student obtains through other sources, such as online publications, journals, magazines, and newspapers.

Both primary and secondary data have their advantages and disadvantages. Primary data provide specific information about the research question or statement, but may take time to collect and process. Secondary data is usually easier to obtain and can be analysed in less time. However, because the data was originally gathered for other purposes, a student may need to sift through it to find exactly what he or she is looking for.

The type of data chosen can depend on many factors, including the research question, the skills of the student, and available time and resources. Based on these and other factors, the student may chose to use primary data, secondary data, or both.

When collecting primary data, the student must ensure the following:

- For surveys, the sample size must be reasonably large and the random sampling technique must be well designed.
- For surveys and interviews, the questionnaires must be designed to avoid bias.
- For experiments and studies, the data must be free from measurement bias.
- The data must be compiled accurately.

When collecting secondary data, the student should explore a variety of resources, such as:

- textbooks, and other reference books;
- scientific and historical journals, and other expert publications;
- the Internet;
- library databases.

After collecting the secondary data, the student must ensure that the source of the data is reliable:

- If the data is from a report, determine what the author's credentials are, how recent the data is, and whether other researchers have cited the same data.
- Be aware that data collection is often funded by an organization with an interest in the outcome or with an agenda that it is trying to further. Knowing which organization has funded the data collection may help the student decide how reliable the data is, or what type of bias may have influenced the collection or presentation of the data.
- If the data is from the Internet, check it against the following criteria:
    - authority – the credentials of the author should be provided;
    - accuracy – the domain of the web address may help the student determine the accuracy;
    - currency – the information is probably being accurately managed if pages on a site are updated regularly and links are valid.

➢ Analysing the Data

Statistical tools can help a student analyse and interpret the data that is collected. Students need to think carefully about which statistical tools to use and when to use them, because other people will be scrutinizing the data. A summary of relevant tools follows.

Measures of central tendency will give information about which values are representative of the entire set of data. Selecting which measure of central tendency (mean, median, or mode) to use depends on the distribution of the data. As the researcher, the student must decide which measure most accurately describes the tendencies of the population. The following criteria should be considered when deciding upon which measure of central tendency best describes a set of data.

- Outliers affect the mean the most. If the data includes outliers, the student should use the median to avoid misrepresenting the data. If the student chooses to use the mean, the outliers should be removed before calculating the mean.

- If the distribution of the data is not symmetrical, but instead strongly skewed, the median may best represent the set of data.
- If the distribution of the data is roughly symmetrical, the mean and median will be close, so either may be appropriate to use.
- If the data is not numeric (for example, colour), or if the frequency of the data is more important than the values, use the mode.

Both the range and the standard deviation will give the student information about the distribution of data in a set. The range of a set of data changes considerably because of outliers. The disadvantage of using the range is that it does not show where most of the data in a set lies – it only shows the spread between the highest and the lowest values. The range is an informative tool that can be used to supplement other measures, such as standard deviation, but it is rarely used as the only measure of dispersion.

Standard deviation is the measure of dispersion that is most commonly used in statistical analysis when the mean is used to calculate central tendency. It measures the spread relative to the mean for most of the data in the set. Outliers can affect standard deviation significantly. Standard deviation is a very useful measure of dispersion for symmetrical distributions with no outliers. Standard deviation helps with comparing the spread of two sets of data that have approximately the same mean. For example, the set of data with the smaller standard deviation has a narrower spread of measurement around the mean, and therefore has comparatively fewer high or low scores, than a set of data with a higher standard deviation.

When working with several sets of data that approximate normal distributions, you can use *z*-scores to compare the data values. A *z*-score table enables a student to find the area under a normal distribution curve with a mean of zero and a standard deviation of one. To determine the *z*-score for any data value in a set that is normally distributed, the formula $z = \frac{x - \bar{x}}{s}$ can be used where x is any observed data value in the set, $\bar{x}$ is the mean of the set, and is s is the standard deviation of the set.

When analysing the results of a survey, a student may need to interpret and explain the significance of some additional statistics. Most surveys and polls draw their conclusions from a sample of a larger group. The margin of error and the confidence level indicate how well a sample represents a larger group. For example, a survey may have a margin of error of plus or minus 3% at a 95% level of confidence. This means that if the survey were conducted 100 times, the data would be within 3 percent points above or below the reported results in 95 of the 100 surveys.

The size of the sample that is used for a poll affects the margin of error. If a student is collecting data, he or she must consider the size of the sample that is needed for a desired margin of error.

➢ Identifying Controversial Issues

While working on a research project, a student may uncover some issues on which people disagree. To decide on how to present an issue fairly, he or she should consider some questions to ask as the research proceeds.

- **What is the issue about?**

  The student should identify which type of controversy has been uncovered. Almost all controversy revolves around one or more of the following:
  - Values – What should be? What is best?
  - Information – What is the truth? What is a reasonable interpretation?

- Concepts – What does this mean? What are the implications?

- **What positions are being taken on the issue?**

  The student should determine what is being said and whether there is reasonable support for the claims being made. Some questions to ask are:
  - Would you like that done to you?
  - Is the claim based on a value that is generally shared?
  - Is there adequate information?
  - Are the claims in the information accurate?
  - Are those taking various positions on the issue all using the same term definitions?

- **What is being assumed?**

  Faulty assumptions reduce legitimacy. The student can ask:
  - What are the assumptions behind an argument?
  - Is the position based on prejudice or an attitude contrary to universally held human values, such as those set out in the United Nations Declaration of Human Rights?
  - Is the person who is presenting a position or an opinion an insider or an outsider?

- **What are the interests of those taking positions?**

  The student should try to determine the motivations of those taking positions on the issue. What are their reasons for taking their positions? The degree to which the parties involved are acting in self-interest could affect the legitimacy of their opinions.

➢ The Final Product and Presentation

The final presentation should be more than just a factual written report of the information gathered from the research. To make the most of the student's hard work, he or she should select a format for the final presentation that suits his or her strengths, as well as the topic.

To make the presentation interesting, a format should be chosen that suits the student's style. Some examples are:

- a report on an experiment or an investigation;
- a short story, musical performance, or play;
- a web page;
- a slide show, multimedia presentation, or video;
- a debate;
- an advertising campaign or pamphlet;
- a demonstration or the teaching of a lesson.

Sometimes, it is also effective to give the audience an executive summary of the presentation. This is a one-page summary of the presentation that includes the research question and the conclusions that were made.

Before giving the presentation, the student can use these questions to decide if the presentation will be effective.

- Did I define my topic well? What is the best way to define my topic?
- Is my presentation focused? Will my classmates find it focused?
- Did I organize my information effectively? Is it obvious that I am following a plan in my presentation?
- Am I satisfied with my presentation? What might make it more effective?
- What unanswered questions might my audience have?

➢ Peer Critiquing of Research Projects

After the student has completed his or her research for the question or statement being studied, and the report and presentation have been delivered, it is time to see and hear the research projects developed by other students. However, rather than being a passive observer, the student should have an important role – to provide feedback to his or her peers about their projects and presentations.

Critiquing a project does not involve commenting on what might have been or should have been. It involves reacting to what is seen and heard. In particular, the student should pay attention to:

- strengths and weaknesses of the presentation;
- problems or concerns with the presentation.

While observing each presentation, students should consider the content, the organization, and the delivery. They should take notes during the presentation, using the following rating scales as a guide. These rating scales can also be used to assess the presentation.

Content

Shows a clear sense of audience and purpose.	1	2	3	4	5
Demonstrates a thorough understanding of the topic.	1	2	3	4	5
Clearly and concisely explains ideas.	1	2	3	4	5
Applies knowledge and skills developed in this course.	1	2	3	4	5
Justifies conclusions with sound reasoning.	1	2	3	4	5
Uses vocabulary, symbols and diagrams correctly.	1	2	3	4	5

Organization

Presentation is clearly focused.	1	2	3	4	5
Engaging introduction includes the research question, clearly stated.	1	2	3	4	5
Key ideas and information are logically presented.	1	2	3	4	5
There are effective transitions between ideas and information.	1	2	3	4	5
Conclusion follows logically from the analysis and relates to the question.	1	2	3	4	5

Delivery

Speaking voice is clear, relaxed, and audible.	1	2	3	4	5
Pacing is appropriate and effective for the allotted time.	1	2	3	4	5
Technology is used effectively.	1	2	3	4	5
Visuals and handouts are easily understood.	1	2	3	4	5
Responses to audience's questions show a thorough understanding of the topic.	1	2	3	4	5

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