

2023/24 PEI Math 621B DRAFT Curriculum Document

The following document streamlines the Math 621B curriculum, by narrowing the instructional focus to key content while maintaining its cognitive integrity. This document represents a research-supported, collaborative initiative between Department of Education and Early Years personnel and mathematics teachers across the province.

This document is intended to supplement (not replace) the current curriculum guide, to aid in an efficient delivery of the curriculum. Teachers should continue to consult the curriculum guide and other resources located on the Learn site.

There are three main parts to the following document:

I. Outcomes

Highlighting and bolding have been added to identify key content.

Any text without bolding indicates content that does not need to be directly instructed or summatively assessed. The items not bolded can be used to provide additional instruction or extension opportunities and will be added to the elaborations section of the curriculum guide.

Some wording has been struck through and replaced with terminology to provide clarity.

Note the example outcome from Math 521B provided below:

RF4 - Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, x and y intercepts to solve problems.
Sections: 3.2, 3.3, 4.1, 4.3, and 4.4

Achievement Indicators:

- A. Explain the reasoning for **Apply the process of completing the square.**
- B. **Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y = a(x - p)^2 + q$ by completing the square.**
- C. Identify, explain and ~~correct~~ errors in an example of completing the square.
- D. **Determine the characteristics of a quadratic function given in the form, $y = ax^2 + bx + c$ and explain the strategy used.**
- E. **Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$.**
- F. **Verify, with or without technology, that a quadratic function in the form $y = ax^2 + bx + c$ represents the same function as a given quadratic function, in the form $y = a(x - p)^2 + q$.**
- G. **Write a quadratic function that models a given situation and explain any assumptions.**
- H. **Solve a problem, with or without technology, by analyzing a quadratic function.**

Annotations:

- Terminology replaced to provide outcome clarity.** (Points to "Apply the process of completing the square" in indicator A)
- Items in regular font communicate they will be moved to the elaborations.** (Points to "Identify, explain and correct errors" in indicator C)
- Highlighted and bolded achievement indicators to communicate instructional focus.** (Points to indicators A, B, D, E, F, G, and H)

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II. Suggested Sequencing and Timeline

A table providing a recommended sequencing of topics, the outcomes related to that topic, and the suggested instructional time is included.

III. PowerSchool Reporting Structure

A table of the relative and absolute weights in PowerSchool is included.

Text	Indicators to be retained
Text	Replaced text to improve clarity
Text	Indicators to be relocated to instructional elaboration

I. Math 621B Outcomes

T1 - Demonstrate an understanding of angles in standard position, expressed in degrees and radians.

Section: 4.1

Achievement Indicators:

- A. Sketch, in standard position, an angle (positive or negative) when the measure is given in degrees.
- B. Describe the relationship among different systems of angle measurement, with emphasis on radians and degrees.
- C. Sketch, in standard position, an angle with a measure of 1 radian.
- D. Sketch, in standard position, an angle with a measure expressed in the form $k\pi$ radians where $k \in \mathbb{Q}$.
- E. Express the measure of an angle in radians (exact value or decimal approximation), given its measure in degrees.
- F. Express the measure of an angle in degrees, given its measure in radians (exact value or decimal approximation).
- G. Determine the measures, in degrees or radians, of all angles in a given domain that are coterminal with a given angle in standard position.
- H. Determine the general form of the measures, in degrees or radians, of all angles that are coterminal with a given angle in standard position.
- I. Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius r and solve problems based upon that relationship.

T2 - Develop and apply the equation of the unit circle.

Sections: 4.2 and 4.3

Achievement Indicators:

- A. Derive the equation of the unit circle from the Pythagorean theorem.
- B. Describe the six trigonometric ratios, using a point $P(x, y)$, that is the intersection of the terminal arm of an angle and the unit circle.
- C. Generalize the equation of a circle with centre $(0, 0)$ and radius r .

T3 - Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees.

Section: 4.3

Achievement Indicators:

- A. Determine, with technology, the approximate value of a trigonometric ratio for any angle with a measure expressed in either degrees or radians.**
- B. Determine, using a unit circle or reference triangle, the exact value of a trigonometric ratio for angles expressed in degrees that are multiples of 0° , 45° , 60° , 90° , or for angles expressed in radians that are multiples of 0 , $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, or $\frac{\pi}{2}$, and explain the strategy.**
- C. Determine, with or without technology, the measures, in degrees or radians, of the angles in a specified domain, given the value of a trigonometric ratio.**
- D. Explain how to determine the exact values of the six trigonometric ratios, given the coordinates of a point on the terminal arm of an angle in standard position.**
- E. Determine the measures of the angles in a specified domain in degrees or radians, given a point on the terminal arm of an angle in standard position.**
- F. Determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain.**
- G. Sketch a diagram to represent a problem that involves trigonometric ratios.**
- H. Solve a problem, using trigonometric ratios.**

T4 - Graph and analyse the trigonometric functions sine, cosine, and tangent to solve problems.

Sections: 5.1, 5.2, and 5.3

Achievement Indicators:

- A. Sketch, with or without technology, the graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$.
- B. Determine the characteristics (amplitude, asymptotes, domain, period, range and zeros) of the graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$.
- C. Determine how varying the value of a affects the graphs of $y = \sin x$ and $y = \cos x$.
- D. Determine how varying the value of d affects the graphs of $y = \sin x + d$ and $y = \cos x + d$.
- E. Determine how varying the value of c affects the graphs of $y = \sin(x - c)$ and $y = \cos(x - c)$.
- F. Determine how varying the value of b affects the graphs of $y = \sin bx$ and $y = \cos bx$.
- G. Sketch, without technology, graphs of the form of $y = a \sin(x - c) + d$ and $y = a \cos(x - c) + d$ using transformations and explain the strategies.
- H. Determine the characteristics (amplitude, asymptotes, domain, period, phase shift, range and zeros) of the graphs of trigonometric functions of the form $y = a \sin(x - c) + d$ and $y = a \cos(x - c) + d$.
- I. Determine the values of a , b , c , and d for the functions of the form $y = a \sin(x - c) + d$ and $y = a \cos(x - c) + d$ that correspond to a given graph and write the equation of the function.
- J. Determine a trigonometric function that models a situation to solve a problem.
- K. Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a problem situation.
- L. Solve a problem by analysing the graph of a trigonometric function.

T5 - Solve, algebraically and graphically, first and second-degree trigonometric equations with the domain expressed in degrees and radians.

Sections: 4.4, 5.4, and 6.4

Achievement Indicators:

- A. **Verify, with or without technology, that a given value is a solution to a trigonometric equation.**
- B. **Determine, algebraically, the solution of a trigonometric equation, stating the solution in exact form when possible.**
- C. **Determine, using technology, the approximate solution of a trigonometric equation in a restricted domain.**
- D. **Relate the general solution of a trigonometric equation to the zeros of the corresponding trigonometric function (restricted to sine and cosine functions).**
- E. Determine, using technology, the general solution of a given trigonometric equation.
- F. **Identify and correct errors in a solution for a trigonometric equation.**

T6 - Prove trigonometric identities, using reciprocal identities, quotient identities, Pythagorean identities, sum or difference identities (restricted to sine, cosine, and tangent), and double-angle identities (restricted to sine, cosine, and tangent).

Sections: 6.1, 6.2, and 6.3

Achievement Indicators:

- A. **Explain the difference between a trigonometric identity and trigonometric equation.**
- B. **Verify a trigonometric identity numerically for a given value in either degrees or radians.**
- C. **Explain why verifying the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid.**
- D. **Determine, graphically, the potential validity of a trigonometric identity, using technology.**
- E. Determine the non-permissible values of a trigonometric identity.
- F. **Prove, algebraically, that a trigonometric identity is valid.**
- G. **Determine, using the sum, difference and double-angle identities, the exact value of a trigonometric ratio.**

RF1 - Demonstrate an understanding of operations on, and composition of, functions.
Sections: 10.1, 10.2, and 10.3 (Composition of functions is the focus.)

Achievement Indicators:

A. Sketch the graph of a function that is the sum, difference, product or quotient of two functions, given their graphs.

B. Write the equations of a function that is the sum, difference, product or quotient of two or more functions, given their equations.

C. Determine the domain and range of a function that is the sum, difference, product or quotient of two functions.

D. Write a function $h(x)$ as the sum, difference, product, or quotient of two or more functions.

E. Determine the value of the composition of functions when evaluated at a point including:

$$\begin{aligned} f[f(a)] \\ f[g(a)] \\ g[f(a)] \end{aligned}$$

F. Determine, given the equations of two functions $f(x)$ and $g(x)$ the equations of the composite functions:

$$\begin{aligned} f[f(x)] \\ f[g(x)] \\ g[f(x)] \end{aligned}$$

G. Sketch, given the equations of two functions $f(x)$ and $g(x)$ the graph of the composite functions:

$$\begin{aligned} f[f(x)] \\ f[g(x)] \\ g[f(x)] \end{aligned}$$

H. Write the function $h(x)$ as the composition of two or more functions.

I. Write the function $h(x)$ by combining two or more functions through operations on, and composition of functions.

RF2 - Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations.

Sections 1.1 and 2.1

Achievement Indicators:

- A. Compare the graphs of a set of functions of the form $y - k = f(x)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of k .
- B. Compare the graphs of a set of functions of the form $y = f(x - h)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of h .
- C. Compare the graphs of a set of functions of the form $y - k = f(x - h)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of h and k .
- D. Sketch the graph of $y - k = f(x)$, $y = f(x - h)$, or $y - k = f(x - h)$ for given values of h and k given the sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.
- E. Write the equation of a function whose graph is a vertical and/or horizontal translation of the graph of the function $y = f(x)$.

RF3 - Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations.

Sections: 1.2 and 2.1

Achievement Indicators:

- A. Compare the graphs of a set of functions of the form $y = af(x)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of a .
- B. Compare the graphs of a set of functions of the form $y = f(bx)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of b .
- C. Compare the graphs of a set of functions of the form $y = af(bx)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of a and b .
- D. Sketch the graph of $y = af(x)$, $y = f(bx)$, or $y = af(bx)$ for given values of a and b given a sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.
- E. Write the equation of a function, given its graph is a vertical and/or horizontal stretch of the graph of the function $y = f(x)$.

RF4 - Apply translations and stretches to the graphs and equations of functions.
Sections: 1.3 and 2.1

Achievement Indicators:

- A. Sketch the graph of the function $y - k = af[b(x - h)]$ for given values of a , b , h , and k given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.
- B. Write the equation of a function, given its graph which is a translation and/or stretch of the graph of the function $y = f(x)$.

RF5 - Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the x -axis, y -axis, and the line $y = x$.
Sections: 1.2, 1.4, and 2.1

Achievement Indicators:

- A. Generalize the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection through the x -axis, y -axis, or the line $y = x$.
- B. Sketch the reflection of the graph of a function $y = f(x)$ through the x -axis, y -axis, or the line $y = x$ given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.
- C. Generalize, using inductive reasoning, and explain rules for the reflection of the graph of a function $y = f(x)$, where the equation of $y = f(x)$ is not given.
- D. Sketch the graphs of $y = -f(x)$, $y = f(-x)$, and $x = f(y)$ given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.
- E. Write the equation of a function given its graph which is a reflection of the graph of the function $y = f(x)$ through the x -axis, y -axis, or the line $y = x$.

RF6 - Demonstrate an understanding of inverses of relations.

Section: 1.4 (This outcome can be addressed prior to logarithms.)

Achievement Indicators:

- A. Explain how the graph of the line $y = x$ can be used to sketch the inverse of a relation.
- B. Explain how the transformation $(x, y) \rightarrow (y, x)$ can be used to sketch the inverse of a relation.
- C. Sketch the graph of the inverse relation, given the graph of a relation.
- D. Determine if a relation and its inverse are functions.
- E. Determine restrictions on the domain of a function in order for its inverse to be a function.
- F. Determine the equation and sketch the graph of the inverse relation, given the equation a linear or quadratic relation.
- G. Explain the relationship between the domains and ranges of a relation and its inverse.
- H. Determine, algebraically or graphically, if two functions are inverses of each other.

RF7 - Demonstrate an understanding of logarithms.

Section: 8.1

Achievement Indicators:

- A. Explain the relationship between logarithms and exponents.
- B. Express a logarithmic expression as an exponential expression and vice versa.
- C. Determine, without technology, the exact value of a logarithm, such as $\log_2 8$.
- D. Estimate the value of a logarithm, using benchmarks, and explain the reasoning: e.g.

Since $\log_2 8 = 3$ and $\log_2 16 = 4$, $\log_2 9$ is approximately equal to 3.1

RF8 - Demonstrate an understanding of the product, quotient and power laws of logarithms.

Sections: 8.3 and 8.4

Achievement Indicators:

- A. **Develop and generalize the laws for logarithms, using numeric examples and exponent laws.**
- B. **Derive *Understand the derivation* of the laws of logarithms.**
- C. **Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression.**
- D. **Determine, with technology, the approximate value of a logarithmic expression, such as $\log_2 9$.**

Since $\log_2 8 = 3$ and $\log_2 16 = 4$, $\log_2 9$ is approximately equal to 3.1.

RF9 - Graph and analyse exponential and logarithmic functions.

Sections: 7.1, 7.2, 8.1, and 8.2

Achievement Indicators:

- A. **Sketch, with or without technology, the graph of an exponential function of the form:
 $y = a^x$, $a > 0$**
- B. **Identify the characteristics of the graph of an exponential function of the form
 $y = a^x$, $a > 0$ including the domain, range, horizontal asymptote and intercepts, and explain the significance of the horizontal asymptote.**
- C. **Sketch the graph of an exponential function by applying a set of transformations to the graph of $y = a^x$, $a > 0$ and state the characteristics of the graph.**
- D. **Sketch, with or without technology, the graph of a logarithmic function of the form:
 $y = \log_b x$, $b > 1$**
- E. **Identify the characteristics of the graph of a logarithmic function of the form
 $y = \log_b x$, $b > 1$ including the domain, range, vertical asymptote and intercepts, and explain the significance of the vertical asymptote.**
- F. **Sketch the graph of a logarithmic function by applying a set of transformations to the graph of $y = \log_b x$, $b > 1$, and state the characteristics of the graph.**
- G. **Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.**

RF10 - Solve problems that involve exponential and logarithmic equations.

Sections: 7.1, 7.2, 7.3, 8.3, and 8.4

Achievement Indicators:

- A. Determine the solution of an exponential equation in which the bases are powers of one another.
- B. Determine the solution of an exponential equation in which the bases are not power of one another, using a variety of strategies.
- C. Determine the solution of a logarithmic equation and verify the solution.
- D. Explain why a value obtained in solving a logarithmic equation may be extraneous.
- E. Solve a problem that involves exponential growth or decay.
- F. Solve a problem that involves the application of exponential equations to loans, mortgages and investments.
- G. Solve a problem that involves logarithmic scales, such as the Richter scale and the pH scale.
- H. Solve a problem by modelling a situation with an exponential or logarithmic equation.

RF11 - Demonstrate an understanding of factoring polynomials of degree > 2 (limited to polynomials of degree ≤ 5 with integral coefficients.)

Sections: 3.2 and 3.3

Achievement Indicators:

- A. Explain how long division of a polynomial expression by a binomial expression of the form $x - a$, $a \in I$ is related to synthetic division.
- B. Divide a polynomial expression by a binomial expression of the form $x - a$, $a \in I$ using long division or synthetic division.
- C. Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function.
- D. Explain the relationship between the remainder when a polynomial expression is divided by $x - a$, $a \in I$ and the value of the polynomial expression $x = a$ (remainder theorem).
- E. Explain and apply the factor theorem to express a polynomial expression as a product of factors.

RF12 - Graph and analyse polynomial functions (degree ≤ 5).
Sections: 3.1 and 3.4

Achievement Indicators:

- A. Identify the polynomial functions in a set of functions and explain the reasoning.
- B. Explain the role of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function.
- C. Generalize rules for graphing polynomial functions of odd or even degree.
- D. Explain the relationship between the zeros of a polynomial function, the roots of the corresponding polynomial equation, and the x-intercepts of the graph of the polynomial function.
- E. Explain how the multiplicity of a zero of a polynomial function affects the graph.
- F. Sketch, with or without technology, the graph of a polynomial function.
- G. Solve a problem by modeling a given situation with a polynomial function and analysing the graph of the function.

RF13 - Graph and analyse radical functions (limited to functions involving one radical).
Section: 2.1, 2.2 and 2.3

Achievement Indicators:

- A. Sketch the graph of the function $y = \sqrt{x}$ using a table of values and state the domain and the range.
- B. Sketch the graph of the function $y - k = a\sqrt{b(x - h)}$ by applying transformations to the graph of the function $y = \sqrt{x}$ and state the domain and range.
- C. Sketch the graph of the function $y = \sqrt{f(x)}$ given the graph of the function $y = f(x)$ and explain the strategies used.
- D. Compare the domain and range of the function $y = \sqrt{f(x)}$ to the domain and range of the function $y = f(x)$ and explain why the domains and ranges may differ.
- E. Describe the relationship between the roots of a radical equation and the x-intercepts of the graph of the corresponding radical function.
- F. Determine, graphically, an approximate solution of a radical equation.

RF14 - Graph and analyse rational functions (limited to numerators and denominators that are monomials, binomials, or trinomials).

Sections: 9.1, 9.2, and 9.3

Achievement Indicators:

- A. Graph, with or without technology, a rational function.
- B. Analyse the graphs of a set of rational functions to identify common characteristics.
- C. Explain the behaviour of the graph of a rational function for values of the variable near a non-permissible value.
- D. Determine if the graph of a rational function will have an asymptote or a hole for a non-permissible value.
- E. Match a set of rational functions to their graphs and explain the reasoning.
- F. Describe the relationship between the roots of a rational equation and the x-intercepts of the graph of the corresponding rational function.
- G. Determine, graphically, an approximate solution of a rational equation.

PC1 - Apply the fundamental counting principle to solve problems.

Section: 11.1

Achievement Indicators:

- A. Count the total number of possible choices that can be made using graphic organizers such as lists or tree diagrams.
- B. Explain, using examples, why the total number of possible choices is found by multiplying rather than adding the number of ways the individual choices can be made.
- C. Solve a simple counting problem by applying the fundamental counting principle.

PC2 - Determine the number of permutations of n elements taken r at a time to solve problems.

Section: 11.1

Achievement Indicators:

- A. Count, using graphic organizers such as lists and tree diagrams, the number of ways of arranging the elements of a set in a row.
- B. **Determine, in factorial notation, the number of permutations of n different elements taken n at a time to solve a problem.**
- C. **Determine, using a variety of strategies, the number of permutations of n different elements taken r at a time to solve a problem.**
- D. Explain why n must be greater than or equal to r in the notation ${}_nP_r$.
- E. Solve an equation that involves ${}_nP_r$ notation, such as ${}_nP_r = 30$.
- F. Explain, using examples, the effect on total number of permutations when two or more elements are identical.

PC3 - Determine the number of combinations of n different elements taken r at a time to solve problems.

Section: 11.2

Achievement Indicators:

- A. **Explain, using examples, the difference between a permutation and a combination.**
- B. Determine the number of ways that a subset of k elements can be selected from a set of n different elements.
- C. **Determine, the number of combinations of n different elements taken r at a time to solve a problem.**
- D. Explain why n must be greater than or equal to r in the notation ${}_nC_r$ or $\binom{n}{r}$.
- E. Explain, using examples why ${}_nC_r = {}_nC_{n-r}$ or $\binom{n}{r} = \binom{n}{n-r}$.
- F. Solve an equation that involves ${}_nC_r$ or $\binom{n}{r}$, such as ${}_nC_2 = 15$ or $\binom{n}{2} = 15$.

PC4 - Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers).

Section: 11.3

Achievement Indicators:

- A. **Explain the patterns found in the expanded form of $(x + y)^n$, $n \leq 4$ and $n \in N$ by multiplying n factors of $(x + y)$.**
- B. **Explain how to determine the subsequent row in Pascal's triangle, given any row.**
- C. **Relate the coefficients of the terms in the expansion of $(x + y)^n$ to the $(n + 1)$ st row in Pascal's triangle.**
- D. Explain, using examples, how the coefficients of the terms in the expansion of $(x + y)^n$ are determined by combinations.
- E. **Expand $(x + y)^n$ using the binomial theorem.**
- F. Determine a specific term in the expansion of $(x + y)^n$.

II. Suggested Sequencing and Timeline

The following table contains a suggested topic sequencing as well as an approximate time frame to cover each of the topics. The time frame is based on a total of 80 classes, each with an average length of 75 minutes:

Topic	Outcomes	Suggested Time
Function Transformations (Including Section 2.1 - Radical Functions)	RF2, RF3, RF4, RF5, RF6, RF13	11 classes
Polynomial Functions	RF11, RF12	8 classes
Exponential Functions	RF9, RF10	6 classes
Logarithmic Functions	RF7, RF8, RF9, RF10	10 classes
Trigonometry and the Unit Circle	T1, T2, T3	9 classes
Trigonometric Functions and Their Graphs	T4, T5	10 classes
Trigonometric Identities	T6	8 classes
Rational Functions	RF14	7 classes
Function Operations (Composition of Functions requires the most focus)	RF1	6 classes
Permutations, Combinations, and Binomial Theorem	PC1, PC2, PC3, PC4	5 classes
	Total	80 classes

III. PowerSchool Reporting Structure

Topic	Relative (%)	Absolute (%)
Function Transformations and Polynomial Functions	28.6	20
Exponential and Logarithmic Functions	18.6	13
Trigonometry	32.9	23
Rational Functions and Function Operations	15.6	11
Permutations, Combinations, and Binomial Theorem	4.3	3
Major Assessment		30